Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

Tips from TIPS: the Informational Content of Treasury Inflation-Protected Security Prices

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2014-024

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First draft: April 23, 2006
This version: February 19, 2016

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Abstract

Treasury Inflation-Protected Securities (TIPS) are frequently thought of as risk-free real bonds. Using no-arbitrage term structure models, we show that TIPS yields exceeded risk-free real yields by as much as 100 basis points when TIPS were first issued and up to 300 basis points during the recent financial crisis. This spread reflects predominantly the poorer liquidity of TIPS relative to nominal Treasury securities. Other factors, including the indexation lag and the embedded deflation protection in TIPS, play a much smaller role. Ignoring this spread also significantly distorts the informational content of TIPS breakeven inflation, a widely-used proxy for expected inflation.

Keywords: TIPS; Breakeven; No-arbitrage term structure model; Liquidity; Expected inflation; Inflation risk premium; Survey forecasts; Indexation lag; Deflation floor

JEL classification: G12, G01, E43, E44

I. Introduction

Treasury Inflation-Protected Securities (TIPS) are fixed-income securities whose coupons and principal payments are indexed to the non-seasonally-adjusted consumption price index (CPI) for all urban consumers.[1] Since its inception in 1997, the market for TIPS has grown substantially and now comprises about 8.6% of the entire Treasury debt market. More than fifteen years of TIPS data provides a rich source of information to investors, policy makers, and researchers alike. TIPS yields can be viewed as rough measures of risk-free real interest rates, an important determinant of the costs for financing private investment and public debt and arguably a better gauge of the stance of monetary policy than nominal interest rates. Importantly, the spread between yields on nominal Treasury securities and on TIPS of comparable maturities—the "breakeven inflation rate" (BEI) or "inflation compensation"—is often used as a real-time proxy for market participants' inflation expectations.

Despite the potential usefulness of these assets, this paper presents evidence that it is essential to account for the lower liquidity of TIPS relative to their nominal counterparts when using them to measure real interest rates and inflation expectations. TIPS investors appear to demand extra compensations for holding these less liquid securities, thereby pushing up TIPS yields above the real yields that are consistent with nominal Treasury yields, and pushing down TIPS BEI below its fundamental levels. Treating the TIPS BEI as a clean proxy for inflation expectation can be especially problematic, since a combination of economically significant TIPS liquidity premiums and inflation risk premiums could potentially drive a notable wedge between the TIPS BEI and true inflation expectations. Indeed, we show that the early years of the TIPS market and the recent financial crisis provide two prominent examples when poor liquidity significantly distorted the information content of TIPS prices.

Here we refer to a broad concept of illiquidity that may originate from a variety of market imperfections, such as those associated with the introduction of a new financial instrument (as in

[1] Sack and Elsasser (2004) provides a detailed description of the TIPS market.

the early years of TIPS), various forms of trading costs and funding constraints (as experienced acutely during the recent financial crisis), and the demand imbalances between nominal Treasuries and TIPS (as during flight-to-safety episodes). Regardless of the source of illiquidity, we emphasize it is the expected future liquidity of TIPS relative to a comparable nominal Treasury security, rather than merely the current or the absolute level of liquidity, that should determine the liquidity premium embedded in the current TIPS yield. Recognizing the difficulty of capturing such a wide variety of current and expected future TIPS market conditions with a few observable measures, many of which had not been available until recently, we choose instead to model TIPS liquidity as driven by an unobserved TIPS-specific risk factor.

As a brief outline of the paper, we first present model-free evidence that a significant portion of the movement in TIPS yields is not spanned by nominal Treasury yields. We then introduce a no-arbitrage asset pricing framework that jointly models nominal Treasury yields, TIPS yields, and realized inflation, allowing for a wedge between TIPS yields and risk-free real yields adjusted for the indexation lag. This wedge is modeled as a function of the TIPS-specific risk factor mentioned above, and is referred to as the "TIPS-indexed bond spread" or the "TIPS spread" for short. Having obtained estimates of the TIPS spread, we examine how they are linked to observable measures of the relative illiquidity of TIPS versus nominal Treasury securities, while controlling for other technical factors such as CPI seasonality, the embedded deflation floor in TIPS, flight-to-safety demand for nominal Treasury securities, and Federal Reserve purchases of TIPS.

We obtain three main findings: First, a standard 3-factor model that treats TIPS yields as risk-free, indexation lag-adjusted real yields generates a poor fit of TIPS yields and BEI, as well as estimates of inflation expectations and inflation risk premiums with counterintuitive properties. In comparison, the two 4-factor models that allow for a TIPS spread generate notably smaller TIPS pricing errors, more reasonable estimates of inflation risk premiums, and estimated inflation expectations that are better aligned with survey inflation forecasts. Second, the estimated values of the TIPS spread were large ($\approx 1\%$) when TIPS were first issued, declined steadily thereafter through late 2003, and remained at relatively low levels until the recent financial crisis, consistent with the notion that TIPS market liquidity conditions had been improving over time. Those estimates jumped to close to 3% in July 2008 during the TIPS sell-off, but had largely returned to their pre-crisis levels by the beginning of 2010, a pattern similar to what has been documented for many other illiquid and/or risky asset prices. Finally, regression analysis shows that around 85% of the variations in our estimates of the TIPS spreads can be explained by observable measures of the liquidity conditions in the TIPS market. Other factors, including CPI seasonality and TIPS deflation floors, play a minor role.

The approach outlined above differs from that of the other two studies of TIPS liquidities in the literature, Shen (2006) and Pflueger and Viceira (2013), who based their results on regression analysis of either TIPS BEI itself or the difference between TIPS BEI and survey-based measures of inflation expectations. The usage of a pricing model allows us to bring in additional information from the cross section of nominal yields and TIPS BEI and from realized inflation. In addition, unlike Chen, Liu, and Cheng (2010), who study TIPS in a multivariate CIR framework, all models in this paper are from the Gaussian essentially-affine no-arbitrage term structure family that allows a flexible correlation structure between the factors and the market prices of risk. Such flexibilities are important for capturing the dynamics of bond risk premiums, as shown by Duffee (2002) and others, and for our purpose of accurately decomposing TIPS BEI into expected inflation, the inflation risk premium, and the TIPS spread.

To the best of our knowledge, this is the first paper examining TIPS liquidity in a no-arbitrage framework and is therefore related to the fast-growing literature on the link between liquidity and asset returns.[2] Previous studies have documented that assets with similar payoffs can carry significantly different prices due to their varying degrees of liquidity.[3] We add to the evidence by

[2] Vayanos and Wang (2013) and Amihud, Mendelson and Pedersen (2013) provide recent surveys of this literature.

[3] See, for example, Amihud and Mendelson (1986) and Brennan, Chordia, and Subrahmanyam (1998), and Brennan and Subrahmanyam (1996) for equities, Amihud and Mendelson (1991) and Longstaff (2004) for nominal Treasury securities, Longstaff, Mithal, and Neis (2005), Chen, Lesmond, and Wei (2007), Bao, Pan, and Wang

showing that this is also the case for TIPS and nominal Treasury securities. Our paper is also related to studies of indexed bond pricing, most of which are conducted using data from countries with longer histories of such bonds.[4] Studies using TIPS and other U.S. inflation-linked assets are fairly recent and relatively few,[5] and most of those using TIPS yields take them at their face value. In contrast, this paper shows that there is a persistent liquidity premium component in TIPS yields that, when ignored, will significantly bias the results. Finally, this paper is also related to the vast literature studying the behavior of real interest rates, inflation expectations, and inflation risk premiums with or without incorporating information from indexed bonds.[6] As discussed in Section VII, our estimates of inflation expectations and inflation risk premiums are consistent with those obtained in other studies using sample periods similar to ours.

The remainder of this paper is organized as follows. In Section II, we provide evidence that portions of TIPS yields and BEI are not spanned by nominal interest rates and are likely linked to the relative illiquidity of TIPS. Section III spells out the details of our no-arbitrage term structure models, including the specification of the TIPS-specific factor and the treatment of the indexation lag. Section IV describes the data and our estimation methodology, and Section V presents the main empirical results. Section VI examines the properties of the estimated TIPS spreads,

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(2011), and Dick-Nielsen, Feldhütter, and Lando (2012) for corporate bonds, Bongaerts, De Jong, and Driessen (2011) for credit default swaps, and Mancini, Ranaldo, and Wrampelmeyer (forthcoming) for the foreign exchange market.

[4] See Woodward (1990), Barr and Campbell (1997), Evans (1998), Remolona, Wickens, and Gong (1998), Risa (2001), and Joyce, Lildholdt, and Sorensen (2010) for the UK, Kandel, Ofer, and Sarig (1996) for Isreal, and Hördahl and Tristani (2012) for the Euro area.

[5] This paper and a contemporaneous study by Chen et al. (2010) are the first two to study TIPS in a no-arbitrage framework. More recent papers analyzing TIPS or inflation swaps include Chernov and Mueller (2012), Adrian and Wu (2008), Haubrich, Pennacchi, and Ritchken (2012), Christensen, Lopez, and Rudebusch (2010), Pflueger and Viceira (2013), Fleckenstein, Longstaff, and Lustig (2014b), Christensen and Gillan (2012), Fleming and Krishnan (2012), Grishchenko and Huang (2013), Abrahams, Adrian, Crump, and Moench (2013).

[6] For studies not using indexed bonds, see, among others, Pennacchi (1991), Foresi, Penati, and Pennacchi (1997), and Brennan, Wang, and Xia (2004), Buraschi and Jiltsov (2005), and Ang, Bekaert, and Wei (2008).

showing that they account for a significant portion of the time series variations in TIPS BEI and are mostly driven by the perspective and relative liquidity characteristics of TIPS versus nominal Treasury securities. Section VII compares our estimates of expected inflation and inflation risk premiums to those from other studies. Finally, Section VIII concludes.

II. A TIPS-Specific Factor: Simple Analysis

In this section we present evidence that there is a component of TIPS yields that is not reflected in nominal Treasury yields and is likely related to the relative illiquidity of TIPS. This serves as the motivation for introducing a TIPS-specific factor when we model nominal and TIPS yields jointly.

Simple Regression Analysis

As a starting point, we regress the 10-year TIPS BEI, defined as the spread between the 10-year nominal yield and the 10-year TIPS yield, on 3-month, 2-year and 10-year nominal yields.[7] Standard finance theory suggests that nominal yields of any maturity, $y^N_{t,\tau}$, can be decomposed into the real yield, $y^R_{t,\tau}$, inflation expectation, $I_{t,\tau}$, and the inflation risk premium, $\wp_{t,\tau}$:

$$y^N_{t,\tau} = y^R_{t,\tau} + I_{t,\tau} + \wp_{t,\tau}. \tag{1}$$

If TIPS yields accurately capture the underlying real yields, the TIPS BEI is the sum of expected inflation and the inflation risk premium, both parts of the nominal yields. A regression of TIPS BEI onto nominal yield curve factors can then be expected to generate a high R^2. On the other hand, variations in TIPS yields that are orthogonal to those in real yields could lead to a low R^2.

We estimate the regression both in levels and in weekly differences for three samples periods:

[7] We thank Greg Duffee for this suggestion. Results using three different nominal yields or using the first principal components of nominal yields are similar. See Section A for data details.

the full sample of January 6, 1999 to March 27, 2013, the pre-crisis period from January 6, 1999 to July 25, 2007, and the post-crisis period from August 1, 2008 to March 27, 2013. As shown in Panel A of Table 1, the R^2 from the full-sample regression is merely 6% in levels and 39% in weekly differences, well below the R^2 in excess of 95% typically found when regressing one nominal yield onto other nominal yields. In the pre-crisis sample, the R^2 is higher for both specifications, 30% in levels and 59% in differences, indicating that the very low R^2 is mostly due to the post-crisis period, during which the adjusted R^2 declines to 5% in levels and 26% in differences, respectively. This evidence suggests that a large portion of variations in the 10-year BEI cannot be explained by factors driving the nominal yields, and even more so when the most recent financial crisis is included in the analysis.

Principal Components Analysis

We next turn to a principal component analysis (PCA) of the cross section of nominal and TIPS yields over the full sample. It is well known that three factors explain most of the movement in nominal yields. This is confirmed by Panel B of Table 1: Over 97% of variations in the weekly changes of nominal yields can be explained by the first three principal components. However, once we add TIPS yields, at least four factors are needed to explain the same portion of the total variance. Panel C of Table 1 reports the correlations between the first four PCA factors extracted from nominal yields alone and those from a combination of nominal and TIPS yields. The first, second, and fourth factors constructed from all yields largely retain their interpretations as the level, slope and curvature of the nominal yield curve, as attested by their high correlations with the first three nominal factors, respectively. However, the third PCA factor extracted from nominal and TIPS yields combined is not highly correlated with any of the nominal PCA factors.

One Potential Explanation: TIPS Liquidity Premium

One interpretation of the TIPS-specific factor identified above is that it may reflect the lack of liquidity in TIPS relative to nominal Treasury securities. TIPS were introduced fairly recently and remain much less liquid than their nominal counterparts, against which the TIPS BEI is computed.[8] Investors are therefore likely to demand extra compensation for holding TIPS, especially in their early years and during the recent financial crisis when overall liquidity deteriorated, thereby pushing down TIPS prices and up TIPS yields. Indeed, TIPS outstanding, shown in the Graph A of Figure 1, did not begin to rise substantially until 2004, around which time both TIPS transaction volumes and TIPS mutual funds, shown in the Graphs B and C, also experienced significant growth.[9] Transaction volumes declined notably at the end of 2008 and remained low through 2011, but rose again in recent years. In view of this history, it is plausible that TIPS yields contain a significant liquidity premium in their early years and again during the financial crisis.

[Insert Figure 1 about here.]

A positive liquidity premium in TIPS yields can also help resolve an apparent inconsistency between long-term survey inflation forecasts and the 10-year TIPS BEI, plotted in Figure 2.[10] The true BEI can be expected to be higher than survey inflation expectations if the inflation risk premium is on average positive, and they can be considered a good measure of true expected inflation if such premium is relatively small and constant over time.

However, Figure 2 shows that this is not the case: the TIPS BEI lied below survey inflation forecasts almost the entire time before 2004.[11] In addition, TIPS yields surged while nominal Treasury yields plummeted shortly after Lehman failed, causing 10-year TIPS BEI to collapse by

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[8] Sack and Elsasser (2004) discusses liquidity conditions in the TIPS market in the early years.

[9] Data on TIPS mutual fund is from the Investment Company Institute (http://www.ici.org).

[10] We use the 10-year inflation forecasts from the Survey of Professional Forecasters (SPF) or the long-term inflation forecast from the Michigan Survey of Consumers. Mehra (2002) shows that these forecasts are unbiased, efficient, and have predictive content for future inflation. Ang, Bekaert, and Wei (2007) and Chun (2012) show that SPF, Blue Chip, and other surveys forecast inflation better than many types of models estimated with yields only. Finally, Chernov and Mueller (2012) show that all above-mentioned survey forecasts are consistent with inflation expectations embedded in yields.

[11] Other measures of inflation expectation based on time-series models also tend to be above the TIPS BEI in early years. Similar points are made by Shen and Corning (2001) and Shen (2006).

200 basis points to below 0.5% by the end of 2008 before returning to the pre-Lehman level over the following year; throughout the same period, survey inflation expectations were practically unchanged. For inflation risk premiums to be the sole source of such notable disparities between TIPS BEI and survey forecasts, they would need to be deeply negative during both episodes and highly volatile. Economic theory does no rule out negative inflation risk premiums per se, as noted by Piazzesi and Schneider (2007) and Campbell, Sunderam, and Viceira (2013). However, as summarized by Bekaert and Wang (2010, Table 11), most studies in the literature do find inflation risk premiums to be positive on average and relatively smooth during our sample period.[12] These findings therefore cast doubt on the ability of inflation risk premiums alone to account for these significant and sometimes volatile discrepancies.

A positive TIPS liquidity premium, on the other hand, could push up TIPS yields and push down TIPS BEI below the true BEI; a large enough TIPS liquidity premium could even outweight a positive inflation risk premium and depress the TIPS BEI to levels below survey inflation forecasts. Part of the volatility of the TIPS BEI may also reflect fluctuations in liquidity premiums. Indeed, as shown in Panel D of Table 1, three proxies of the TIPS-market liquidity conditions explain about 60% of the difference between the quarterly 10-year SPF inflation forecast and the 10-year TIPS BEI over the full sample. To formally test this hypothesis, we need a framework for identifying and measuring the relevant components, including inflation expectations, inflation risk premiums, and the potential TIPS liquidity premiums. For this purpose, we now switch to the no-arbitrage term structure modeling framework and re-examine the TIPS liquidity hypothesis in Section VI.

[Insert Table 1 about here.]

[Insert Figure 2 about here.]

[12] See, for example, Campbell and Shiller (1996), Foresi et al. (1997), Veronesi and Yared (1999), Buraschi and Jiltsov (2005), Ang et al. (2008), Haubrich et al. (2012), Chernov and Mueller (2012), and Hördahl and Tristani (2014).

III. A Joint Model of Nominal Yields, TIPS yields, and Inflation

We use a no-arbitrage pricing framework that models nominal yields, TIPS yields and inflation jointly. This approach avoids the tight assumptions that go into structural, utility-based models, while still requiring the cross section of yields and inflation to be priced in an internally consistent manner that is free of arbitrage opportunities.

A. State Variable Dynamics and the Nominal Pricing Kernel

We assume that real yields, expected inflation, and nominal yields are all driven by a vector of three latent variables, $x_t = [x_{1t}, x_{2t}, x_{3t}]'$, that follows a multivariate Gaussian process,

$$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t, \tag{2}$$

where B_t is an 3×1 vector of standard Brownian motion. The nominal pricing kernel takes the form

$$dM_t^N / M_t^N = -r^N(x_t)dt - \lambda^N(x_t)' dB_t, \tag{3}$$

with the nominal short rate and nominal prices of risk given by

$$r^N(x_t) = \rho_0^N + \rho_1^{N\prime} x_t, \tag{4}$$

$$\lambda^N(x_t) = \lambda_0^N + \Lambda^N x_t. \tag{5}$$

Note that the nominal term structure in this paper is fairly standard and falls into the "essentially affine" $A_0(3)$ category developed by Duffee (2002).

B. Inflation and the Real Pricing Kernel

We assume that the price level process takes the form:

$$d \log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^{\perp} dB_t^{\perp}. \tag{6}$$

where the instantaneous expected log inflation, $\pi(x_t)$, is an affine function of the state variables:

$$\pi(x_t) = \rho_0^{\pi} + \rho_1^{\pi'} x_t. \tag{7}$$

Unexpected inflation consists of a component, $\sigma_q' dB_t$, that loads on shocks that move the nominal interest rates and expected inflation, dB_t, and a component, $\sigma_q^{\perp} dB_t^{\perp}$, that loads on an orthogonal shock $dB_t^{\perp}$, with $dB_t dB_t^{\perp} = 0_{3\times 1}$. The orthogonal shock is included to capture short-run inflation variations that may not be spanned by yield curve movements.[13]

A real bond can be thought of as a nominal asset paying realized inflation upon maturity. Therefore, the real and the nominal pricing kernels are linked by the no-arbitrage relation

$$M_t^R = M_t^N Q_t. \tag{8}$$

As detailed in Appendix A, the real pricing kernel follows the dynamics

$$dM_t^R / M_t^R = -r^R(x_t)dt - \lambda^R(x_t)' dB_t - (\cdot) dB_t^{\perp} \tag{9}$$

C. Nominal and Real Bond Yields

The time-t prices of τ-period nominal and real bonds, $P_{t,\tau}^N$ and $P_{t,\tau}^R$, are given by

$$P_{t,\tau}^i = E_t(M_{t+\tau}^i)/M_t^i, \quad i = N, R. \tag{10}$$

[13] See Kim (2009) for more discussions about such variations.

They can also be expressed in terms of expectations taken under the risk-neutral measure Q

$$P_{t,\tau}^{i} = E_t^{Q}\left(\exp\left(-\int_t^{t+\tau} r_s^i\, ds\right)\right), \quad i = N, R. \tag{11}$$

Their closed-form solution can be derived following the standard literature:[14]

$$P_{t,\tau}^{i} = \exp\left(A_\tau^i + B_\tau^{i\prime} x_t\right), \quad i = N, R, \tag{12}$$

where

$$\frac{dA_\tau^i}{d\tau} = -\rho_0^i + B_\tau^{i\prime}\left(\mathcal{K}\mu - \Sigma\lambda_0^i\right) + \frac{1}{2} B_\tau^{i\prime}\Sigma\Sigma' B_\tau^i \tag{13}$$

$$\frac{dB_\tau^i}{d\tau} = -\rho_1^i - \left(\mathcal{K} + \Sigma\Lambda^i\right)' B_\tau^i \tag{14}$$

with initial conditions $A_0^i = 0$ and $B_0^i = 0_{3\times 1}$.

Nominal and real yields therefore both take the affine form,

$$y_{t,\tau}^i = a_\tau^i + b_\tau^{i\prime} x_t, \quad i = N, R, \tag{15}$$

with factor loadings $a_\tau^i = -A_\tau^i/\tau$ and $b_\tau^i = -B_\tau^i/\tau$.

D. Inflation Expectations and Inflation Risk Premiums

In this model, inflation expectations also take an affine form,

$$I_{t,\tau} \triangleq E_t(\log(Q_{t+\tau}/Q_t))/\tau = a_\tau^I + b_\tau^{I\prime} x_t, \tag{16}$$

[14] See Duffie and Kan (1996) and Dai and Singleton (2000), among others.

where the factor loadings a^I and b^I are given by

$$a^I_\tau = \rho^\pi_0 + (1/\tau)\rho^{\pi\prime}_1 \int_0^\tau ds(I - e^{-\mathcal{K}s})\mu$$
$$b^I_\tau = (1/\tau)\int_0^\tau ds\, e^{-\mathcal{K}'s}\rho^\pi_1,$$

From equations (15)-(16), it can be seen that the BEI, defined as before, and the inflation risk premium, defined as the difference between the BEI and the expected log inflation over the same horizon and denoted by $\wp_{t,\tau}$, are both affine in the state variables:

$$\text{BEI}_{t,\tau} \triangleq y^N_{t,\tau} - y^R_{t,\tau} = a^N_\tau - a^R_\tau + (b^N_\tau - b^R_\tau)' x_t. \tag{17}$$

$$\wp_{t,\tau} \triangleq y^N_{t,\tau} - y^R_{t,\tau} - I_{t,\tau} = a^N_\tau - a^R_\tau - a^I_\tau + (b^N_\tau - b^R_\tau - b^I_\tau)' x_t. \tag{18}$$

Using equation (8) we can write the price of a τ-period nominal bond as

$$P^N_{t,\tau} = \frac{E_t[M^R_{t+\tau} Q^{-1}_{t+\tau}]}{M^R_t Q^{-1}_t}. \tag{19}$$

It is then straightforward to show that the inflation risk premium $\wp_{t,\tau}$ consists of a covariance term, $c_{t,\tau}$, and a Jensen's inequality term, $J_{t,\tau}$:

$$\wp_{t,\tau} = c_{t,\tau} + J_{t,\tau}, \tag{20}$$

where

$$c_{t,\tau} \equiv -(1/\tau)\log[1 + cov_t(M^R_{t+\tau}/M^R_t, Q_t/Q_{t+\tau})/(E_t(M^R_{t+\tau}/M^R_t)E_t(Q_t/Q_{t+\tau}))].$$

In practice, the Jensen's inequality term is fairly small, and the inflation risk premium is mainly determined by the covariance between the real pricing kernel and inflation, and can assume either a positive or a negative sign depending on how the two terms covaries over time.

E. A TIPS-Specific Factor

Given the evidence presented in Section II on the existence of a TIPS-specific factor, we allow the TIPS yield to deviate from the underlying real yield. The resulting yield spread, $L_{t,\tau} = y^{\mathcal{T}}_{t,\tau} - y^{R}_{t,\tau}$, should primarily capture the liquidity premium TIPS investors demand for holding an instrument that is less liquid than nominal Treasury securities, but may also reflect other technical factors, such as seasonal variations in headline CPI and the embedded deflation protection in TIPS. We examine the relative importance of each of these factors in Section VI. Since the relative illiquidity of TIPS would raise TIPS yields, we would in general expect $L_{t,\tau}$ to be positive.

To model $L_{t,\tau}$, we assume that investors discount TIPS cash flows by adding a positive spread, l_s, to the instantaneous real short rate, resulting in a TIPS yield that exceeds the real yield by

(21) $$L_{t,\tau} = -(1/\tau)\log E^{Q}_{t}\left(\exp\left(-\int_{t}^{t+\tau}(r^{R}_{s} + l_s)ds\right)\right) - y^{R}_{t}.$$

This is analogous to the corporate bond pricing literature, where defaultable bonds are priced by discounting future cash flows using a default- and liquidity-adjusted short rate.[15] Without the instantaneous spread l_t in equation (21), the TIPS and the real yields y^R coincide, and $L_{t,\tau}$ becomes zero (see equation (11)).

We assume that l_t has both a systematic component and a TIPS-specific component:

(22) $$l_t = \gamma' x_t + \tilde{\gamma}\tilde{x}_t,$$

where the TIPS-specific factor, $\tilde{x}_t$, follows the Vasicek (1977) process and is independent of all other state variables contained in x_t:

(23) $$d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma}dW_t,$$

[15] See Duffie and Singleton (1999), Longstaff et al. (2005), and Driessen (2005).

with $dW_t dB_t = 0_{3\times 1}$. A non-zero γ allows the TIPS-real yield spread to be correlated with the other state variables in the economy. Finally, we assume that the idiosyncratic factor $\tilde{x}_t$ carries a market price of risk of

$$\tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{x}_t. \tag{24}$$

Appendix B shows that the spread between TIPS and real yields takes the affine form

$$L_{t,\tau} = \left[\tilde{a}_\tau + (a_\tau^{\mathcal{T}} - a_\tau^R)\right] + \left[\begin{array}{cc} (b_\tau^{\mathcal{T}} - b_\tau^R)' & \tilde{b}_\tau \end{array}\right] \left[\begin{array}{c} x_t \\ \tilde{x}_t \end{array}\right]. \tag{25}$$

Ignoring the indexation lag for now, the TIPS yield in this model is given by

$$y_{t,\tau}^{\mathcal{T}} = y_{t,\tau}^R + L_{t,\tau}. \tag{26}$$

Appendix C shows that this model can be restated in a four-factor Gaussian framework, in which the expanded state variables include x_t and the demeaned TIPS-specific factor, $\tilde{x}_t - \tilde{\mu}$.

F. Indexation Lag

An indexation lag of about 2.5 months introduces an additional complication in the pricing of TIPS. This implies that TIPS holders receive compensation for inflation over the 2.5 months prior to the purchase date but are still exposed to inflation risks during the final 2.5 months before the maturity or sale of the bond. In general, the yield on a τ-year indexed bond with an indexation lag of l years differs from the yield on a fully indexed real bond for two reasons: first, inflation between the two l-year periods could diverge, and second, investor would demand a risk premium for bearing inflation risks over the final l-year period:

$$y_{t,\tau,l}^I = y_{t,\tau}^R + \frac{1}{\tau}\left[E_t \log \frac{Q_{t+\tau}}{Q_{t+\tau-l}} - \log \frac{Q_t}{Q_{t-l}}\right] + \wp_{t,\tau,l}^{IL}, \tag{27}$$

where $\wp^{IL}_{t,\tau,l}$ denotes the indexation lag premium.

Evans (1998) and Risa (2001) have found the indexation lag premium to be small, between 1.5 and 6 basis points, for U.K. inflation-linked gilts with an even longer indexation lag of 8 months. Nonetheless, the indexation-lag effect on yields could be large during periods with significantly above- or below-trend inflation. For example, annualized CPI inflation was running below -9% in each of the last three months of 2008; over the same period, TIPS liquidity also reportedly deteriorated rapidly. It is therefore important to explicitly account for the indexation lag.

To do so, we follow Risa (2001) and note that at time t, a τ-year indexed bond with a l-year indexation lag is a claim to a nominal payoff of $\frac{Q_{T-l}}{Q_{t-l}}$, to be received at time $T = t + \tau$. Assuming that the current price level Q_t is observed without error at time t, we first consider an artificial indexed bond that pays $\frac{Q_{T-l}}{Q_t}$ at time T. Let $y^I_{t,\tau,l}$ and $\widetilde{y}^I_{t,\tau,l}$ represent the yields on the actual and the artificial indexed bond, respectively. The relationship between the two yields is

$$
\begin{aligned}
(28) \qquad y^I_{t,\tau,l} &= -\frac{1}{\tau}\log E_t\left[\frac{M^N_T}{M^N_t}\frac{Q_{T-l}}{Q_{t-l}}\right] = -\frac{1}{\tau}\log E_t\left[\frac{M^N_T}{M^N_t}\frac{Q_{T-l}}{Q_t}\right] - \frac{1}{\tau}\log\frac{Q_t}{Q_{t-l}} \\
&= \widetilde{y}^I_{t,\tau,l} - \frac{1}{\tau}\log\frac{Q_t}{Q_{t-l}}.
\end{aligned}
$$

This implies that to derive the yield on the *actual* indexed bond, we can first calculate the yield on the *artificial* indexed bond, $\widetilde{y}^I_{t,\tau,l}$, similarly to how we price the real bond, and then add back the realized inflation over the previous 2.5 months. Finally, to incorporate the TIPS-specific factor, the cash flows of both indexed bonds are discounted taking into account the instantaneous TIPS spread, l_t, specified in equation (22). Appendix D describes these steps in more details.

G. A Comparison to Previous Studies

Some of the models studied in the earlier literature, such as Pennacchi (1991) and Campbell and Viceira (2001), can be viewed as special cases of this model. For example, Pennacchi (1991)'s model corresponds to a two-factor version of our model with constant market price of risk. Campbell and Viceira (2001) is also a special case of this model, as their real term structure has a lower dimension than the nominal term structure. In this paper, we let the data decide the dimensionality of the real term structure.

Overall, compared with previous studies, two main features of this model help us better distinguish the inflation risk premium and the liquidity premium components of the TIPS BEI. On the one hand, the use of price level data Q_t in the estimation and the unrestricted correlation structure between factor innovations help us better pin down expected inflation and the inflation risk premium. On the other hand, the higher-dimensionality of the real term structure, the estimation of which is assisted by the additional information from TIPS yields, allows us to better identify the parameters governing the real yield dynamics. As a result, the spread between TIPS and indexation lag-adjusted real yields is pinned down, and can be estimated separately from the inflation risk premium. These features can only be fully appreciated when considered within the context of the empirical methodology used to estimate the model, to which we now turn.

IV. Data and Empirical Methodology

A. Data

We use 3- and 6-month, 1-, 2-, 4-, 7-, and 10-year nominal yields and CPI-U data from January 1990 to March 2013. In contrast, our TIPS yields are restricted by data availability and cover a shorter period from January 1999 to March 2013, with the earlier period without TIPS data (1990-1998) treated as missing observations.[16] We sample yields at the weekly frequency and assume that the monthly CPI-U data is observed in the last week of the current month.[17]

[16] 3- and 6-month T-bill yields are from the Federal Reserve Board's H.15 release and converted to continuously compounded basis. Longer-term nominal yields and TIPS yields are based on zero-coupon yield curves fitted at the Federal Reserve Board. In particular, nominal yields are based on the Svensson (1995) curve specification for the entire sample; TIPS yields are based on the Nelson and Siegel (1987) curve specification prior to January 2004 and the Svensson (1995) curve specification thereafter. See Gürkaynak, Sack, and Wright (2007, 2010) for details.

[17] Here we abstract from the real-time data issue by assuming that investors correctly infer the current inflation rate in a timely fashion.

Shorter-maturity TIPS yields cannot be estimated reliably before 2002, as there was only one TIPS with maturity below 5 years. We therefore only use 5-, 7-, and 10-year TIPS yields in our estimation. All nominal and TIPS yields used in the estimation are plotted in Figure 3. Although TIPS are indexed to non-seasonally-adjusted CPI, we use seasonally-adjusted CPI inflation because the models we estimate do not accommodate seasonality; this is not expected to have a big effect due to the relatively long maturities of our TIPS yields.

The sample period 1990-2013 was chosen as a compromise between utilizing more data to improve the efficiency of estimation and having a more homogeneous sample with no structural breaks in the relation between term structure variables and inflation.[18] Further, to avoid running into small sample problems, we follow Kim and Orphanides (2012) and supplement the data with survey forecasts of short-term rates to help stabilize the estimation and better pin down some of the model parameters. Specifically, we use the 6- and 12-month-ahead forecasts of the 3-month T-bill rate from Blue Chip Financial Forecasts, which are available monthly, and allow the size of the measurement errors to be determined within the estimation. We also use the semiannual long-range (5 to 10 years ahead) forecast of the same rate, with the standard deviation of its measurement error fixed at a fairly large value of 75 basis points at an annual rate. This is done to prevent the long-horizon survey forecasts from having unduly strong influence on the estimation, similarly to a quasi-informative prior in a Bayesian estimation.

Finally, in most cases, we include median SPF forecasts of average inflation over the following year and over the next ten years as additional data inputs to model estimation. When included, those survey forecasts are also treated as noisy measures of their model counterparts.

B. Identification and Summary of Models

We only impose restrictions that are necessary for achieving identification to allow a maximally flexible correlation structure between the factors, which has shown to be critical in

[18] The 1979-83 episode of Fed's experiment with reserve targeting is a well known example.

fitting the rich behavior of risk premiums observed in the data.[19] In particular, we employ the following normalization:

$$\mu = 0_{3\times 1}, \quad \Sigma = \begin{bmatrix} 0.01 & 0 & 0 \\ \Sigma_{21} & 0.01 & 0 \\ \Sigma_{31} & \Sigma_{32} & 0.01 \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} \mathcal{K}_{11} & 0 & 0 \\ 0 & \mathcal{K}_{22} & 0 \\ 0 & 0 & \mathcal{K}_{33} \end{bmatrix}, \quad \tilde{\sigma} = 0.01. \tag{29}$$

and leave all other parameters unrestricted. It can be shown that any specification of the affine Gaussian model that has a $\mathcal{K}$ matrix with all-real eigenvalues can be transformed to this form.[20]

To summarize, we consider three model specifications that differ in how TIPS yields are modeled, including one model that equates TIPS yields with indexation lag-adjusted real yields (Model NL)[21] , one model that allows TIPS yields to differ from indexation lag-adjusted real yields by a spread that is driven only by the TIPS-specific factor (Model LI), and the full model allowing the TIPS spread to be also correlated with other state variables in the economy (Model LII). Table 2 summarizes those model specifications and the associated parameter restrictions. Among these models, Models NL and LI can both be considered special cases of Model LII. In addition, Model NL has a 3-factor representation of TIPS yields, while in the other two models TIPS yields have a 4-factor specification. We estimate all three models using survey forecasts of inflation as additional data inputs. For comparison purposes, we also estimate Model NL without survey inflation forecasts and denote it with a "-noIE" suffix.

[Insert Table 2 about here.]

[19] See Duffie and Kan (1996) and Dai and Singleton (2000).

[20] With normalization (29), the specification we estimate in this paper can be shown to be equivalent to that of Sangvinatsos and Wachter (2005). The main difference between their paper and ours is empirical: they use a much longer sample, which would be desirable if the relationship between inflation and interest rates can be assumed to be stable.

[21] This is similar to the model studied by Chen et al. (2010), who also use a CIR-type model, which is known to have problems in fitting risk premiums. Their model also implies non-negative instantaneous inflation expectation and precludes the possibility of deflation.

C. Estimation Methodology

We rewrite the model in a state-space form and estimate it by maximum likelihood using the Kalman filter. More details are provided in Appendix E. Two aspects are worth noting here: first, the log price level q_t is nonstationary, so we use a diffuse prior for q_t when initializing the Kalman filter. Second, inflation, survey forecasts, and TIPS yields are not available for all dates, which introduces missing data in the observation equation and are handled in the standard way by allowing the dimensions of the matrices A and B in equation (E-4) to be time-dependent (see, for example, Harvey (1989, sec. 3.4.7)). To facilitate the estimation and also to make the results easily replicable, we follow a few easy steps in estimating all models:

1. We first perform a "pre"-estimation, where a set of preliminary estimates of the parameters governing the nominal term structure is obtained using nominal yields and survey forecasts of the 3-month T-bill rate alone.
2. Based on these estimates and data on nominal yields, we can obtain a preliminary estimate of the state variables, x_t.
3. A regression of the monthly inflation onto estimates of x_t obtained in the second step gives a preliminary set of estimates of the parameters governing the inflation dynamics.
4. Finally, these preliminary estimates are used as starting values in the full, one-step estimation of all model parameters.

V. Empirical Results

A. Parameter Estimates and Overall Fit

Parameter Estimates

Table 3 reports selected parameter estimates for all four models.[22] We draw four main conclusions: First, parameters governing the nominal term structure are fairly robustly estimated

[22] Complete parameter estimates can be found in the Supplementary Appendix.

and are almost identical across all models. All estimations uncover a factor that is fairly persistent with a half life of about 5 years. All four models also exhibit a similar fit to nominal yields and survey forecasts of nominal short-term interest rates, generating fitting errors at or below 6 basis points for most maturities and slightly larger at around 13 basis points for the 3-month yield.[23]

Second, there are notable differences in the estimates of parameters governing the expected inflation process. In particular, the loading of the instantaneous inflation on the second and the most persistent factor, $\rho^{\pi}_{1,2}$, is negligible in Model NL-noIE but becomes larger and statistically significant when survey inflation forecasts are used in the estimation. As a result, the monthly autocorrelation of one-year-ahead inflation expectation is about 0.9 in Model NL-noIE but above 0.99 in all other models. As we will see later, the lack of persistence in the inflation expectation process prevents Model NL-noIE from generating meaningful variations in longer-term inflation expectations as we observe in the data.

Third, the fit to TIPS yields is significantly better in models with a TIPS-specific factor. For example, the fitting errors on the 10-year TIPS yield is around 40 basis points in both Model NL-noIE and Model NL, but are much smaller at around 7 basis points in Models LI and LII. Some of the fitting errors are found to have substantial serial correlations. For example, in the case of the 7-year TIPS, we obtain a weekly AR(1) coefficient of 0.94 in all models. The finding of serial correlation in term structure fitting errors are however a fairly common phenomenon, and have been noted by Chen and Scott (1993) and others.

Finally, parameter estimates for the TIPS-specific factor process are significant in both Models LI and LII and assume similar values. The price of risk associated with this factor depends negatively on the factor itself, as can been from the negative $\tilde{\lambda}_1$. One possible interpretation is that the same amount of liquidity risk carries higher risk premiums when liquidity is poor, which is intuitive as one would generally expect any deterioration of liquidity conditions to occur during bad economic times. In Model LII, the loadings of the instantaneous TIPS spread on the other state variables, γ, are only significant for the first factor; however, a

[23] Selected nominal and TIPS yield fitting errors are shown in the Supplementary Appendix.

likelihood ratio test shows that they are jointly significant.

[Insert Table 3 about here.]

Overall Fit

Panel A of Table 4 reports some test statistics that compare the overall fit of the four models. We first report two information criteria commonly used for model selection, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Both information criteria are minimized by the most general model, Model LII.

We also report results from likelihood ratio (LR) tests of the two restricted models, Models NL and LI, against their more general counterparts, Model LI and LII, respectively. Here we follow Garcia and Perron (1996) and calculate a conservative upper bound for the significance of the LR test statistic as suggested by Davies (1987), outlined in more details in the Supplementary Appendix.[24] This procedure overwhelmingly rejected Model NL in favor of Model LI (with an LR p value of essentially zero). The LR test of Model LI against Model LII, on the other hand, is fairly standard. We obtain a LR statistic of 143.13 and are able to reject Model LI in favor of Model LII at the 1% level based on a χ_3^2 distribution.

[Insert Table 4 about here.]

B. Fitting TIPS Yields and TIPS BEI

The estimated standard deviations of TIPS measurement errors reported in Table 3 suggest that Model NL, estimated either with or without survey inflation forecasts, has trouble fitting TIPS yields. A better understanding of the problem can be gained by comparing the three rows of Figure 4, which plot the actual and model-implied TIPS yields and TIPS BEI, as well as

[24] The standard LR test does not apply here because the nuisance parameters, $\tilde{\kappa}$, $\tilde{\mu}$, $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$, are not identified under the null (Model NL) and appear only under the alternative (Model LI). For discussions on testing with nuisance parameters, see, for example, Davies (1977, 1987, 2002) and Andrews and Ploberger (1994, 1995).

model-implied risk-free real yields and BEI, for Models NL-noIE, NL, and LII, respectively.[25] By construction, the model-implied TIPS yields (TIPS BEI) and the model-implied risk-free real yields (BEI) coincide under Model NL(-noIE), after adjusting for the indexation lag.

The top and middle left panels of Figure 4 show that both versions of Model NL interpret the decline in 10-year TIPS yields from 1999 to 2004 as part of a broad downward shift in real yields from 7% in early 1990s to about 2% around 2003. The less than 5% decline in the 10-year nominal yield over the same period is therefore attributed almost entirely to a lower real yield, leaving little room for lower inflation expectations or risk premiums. However, the literature generally finds that long-term inflation expectations likely have edged down over this period,[26] and it is hard to imagine economic mechanisms that would generate such a large decline in long-term real yields. Furthermore, although the two NL models match the general trend of TIPS yields, they both have problems fitting the time variations, frequently resulting in large fitting errors, especially in the early part of the sample and again during the recent financial crisis. In contrast, the bottom left panel of Figure 4 shows a less pronounced and more gradual decline in real yields based on Model LII, which is able to fit TIPS yields almost perfectly, as shown by the juxtaposition of the red and black lines.

In addition, the top and middle right panels of Figure 4 show that the 10-year BEI implied by the two NL models, which by construction should equal the 10-year TIPS BEI after adjusting for the indexation lag, appears too smooth compared to the actual data and misses most of its short-run variations. The poor fitting of the TIPS BEI highlights the difficulty that the 3-factor model has in fitting nominal and TIPS yields simultaneously. In contrast, the 10-year BEI implied by Model LII, shown in the bottom right panel of Figure 4, exhibits substantial variations that closely match those of the actual TIPS BEI. In particular, the model-implied and the TIPS-based

[25] Model-implied true breakevens are calculated as the difference between model-implied nominal yields and model-implied real yields of comparable maturities. Model-implied values are calculated using smoothed estimates of the state variables. Results for Model LI are broadly similar to those for Model LII and are reported in the Supplementary Appendix.

[26] See Kozicki and Tinsley (2006), for example.

BEI plunge toward the end of 2008 following the Lehman collapse, consistent with reports of substantial liquidation of TIPS holdings over this period.[27]

To quantify the improvement in terms of the model fit, Panels B and C of Table 4 provide three goodness-of-fit statistics for TIPS yields at the 5-, 7- and 10-year maturities and TIPS BEI at the 7- and 10-year maturities, respectively. The first statistic, CORR, is the simple sample correlation between the fitted series and its data counterpart. The next two statistics—the root mean squared prediction errors (RMSE) and the coefficient of determination (R^2)—are based on one-step-ahead model prediction errors from the Kalman Filter, and are designed to capture how well each model can explain the data without resorting to large exogenous shocks or measurement errors.[28] All three statistics suggest that including a TIPS-specific factor improves the model fit for raw TIPS yields and even more so for TIPS BEI.

[Insert Figure 4 about here.]

C. Matching Survey Inflation Forecasts

Next, we briefly examine how closely the model-implied inflation expectations mimic survey-based counterparts. Ang et al. (2007) provide evidence that survey inflation forecasts outperforms various other measures of inflation expectations in predicting future inflation. In addition, survey inflation forecast has the benefit of being a real-time, model-free measure, and hence not subject to model estimation errors or look-ahead biases.

Panel D of Table 4 reports the three goodness-of-fit statistics, CORR, RMSE and R^2, for 1- and 10-year ahead SPF inflation forecasts. When survey inflation forecasts are not used in the estimation, Model NL-noIE generates inflation expectations that departs significantly from survey inflation forecasts, as can be seen from the large RMSEs and small and even negative R^2 statistics.

[27] For a brief account of the episode, see Hu and Worah (2009).

[28] The R^2 is defined as the percentage of in-sample variations of each data series explained by the model. Unlike in a regression setting, a negative value of R^2 could arise because the model expectation and the prediction errors are not guaranteed to be orthogonal in a small sample.

A visual comparison between the model-implied inflation expectations and survey forecasts, plotted in the first two top panels of Figure 5, show that Model NL-noIE fails to capture the downward trend in survey inflation forecasts during much of the sample period, and implies that the 10-year inflation expectation barely moved. This is the flip side of the discussions in Section B, where Model NL-noIE implied a 10-year real rate that was too variable and explained the entire decline in nominal yields during the 1990s. Adding information from survey inflation forecasts brought Model NL-implied inflation expectations more in line with survey forecasts, as can be seen from the first two middle panels of Figure 5. Model NL then explains the smaller decline in nominal yields by generating a sustained increase in inflation risk premiums from around -1.5% in the early 1990s to the current level of slight above zero, in contrast to previous findings of an overall positive and generally declining inflation risk premiums over the past three decades (see Section VII). By comparison, Model LII, which allows for a TIPS-specific factor and is shown in the bottom right panel of Figure 5, generates 10-year inflation risk premiums that are mostly positive and fluctuate in the 0 to 0.5% range, and short-term inflation risk premiums that are fairly small and became persistently negative during the recent financial crisis.

D. Out-of-Sample Forecasting

It is conceivable that a model with more parameters like Model LII could generate smaller in-sample fitting errors for variables whose fit is explicitly optimized, but preforms worse out of sample. To check this possibility, we run an out-of-sample forecasting horse race between the four term structure models estimated above, a random walk model, the monthly Michigan survey, the monthly Blue Chip Financial Forecasts (BCFF) survey, and the quarterly SPF, and report the root mean squared errors (RMSEs) in Table 5. Three conclusions emerge: First, the term structure models perform as well as the two professional surveys—SPF and BCFF—over a common sample period, and much better than the Michigan survey and the random walk model. Second, within the term structure models, adding survey inflation forecasts help improve the forecasting performance of Model NL. Finally, Model LII outperforms Model NL, with the improvement more pronounced at the longer 2-year horizon.

[Insert Table 5 about here.]

In addition, the robustness checks reported in the Supplementary Appendix show that the parameter estimates and good performance of Model LII remain largely intact when re-estimated over a pre-crisis sample. Therefore, we will mainly focus on this model in the remainder of our analysis.

E. The Effect of Indexation Lags

Graph A of Figure 6 shows the Model LII-implied differences between the indexed bond yields $y^I_{t,\tau,l}$, specified in equation (28), and the real yields $y^R_{t,\tau}$, specified in equation (15), for maturities of 5, 7, and 10 years. Those estimates are generally small but rose to 30 basis points at the 10-year maturity and 70 basis points at the 5-year maturity in December 2008, when 3-month CPI inflation was running at an annual rate of about -13%. The differences between indexed bond yields and fully-indexed real yields are estimated to be almost entirely due to the expected divergence between inflation over the 2.5 months prior to time t and inflation over the 2.5 months before the maturity of the bond, the second term on the left hand side of equation (27). The last term, the indexation lag premium, is estimated to be generally small, varying between -5 and 3 basis points at the 10-year maturity, consistent with Risa (2001), and slightly larger at shorter maturities. Similar patterns are observed in all models we estimate.

VI. What Drives the TIPS Spread

In this section, we examine estimates of the TIPS spread from Model LII and show them to be mostly explained by proxies of TIPS liquidity. Other factors, such as CPI seasonality, TIPS deflation floors and special demand for nominal Treasuries, also account for a small portion of their variations, though the effects of those factors appear to be much less significant.

A. Model Estimates of the TIPS Spread

The TIPS spread implied by Model LII is plotted in Graph B of Figure 6 for maturities of 5, 7 and 10 years. Three main features emerge: First, this spread exhibits substantial time variations at all maturities. Such variability at maturities as long as 10 years is in part due to the estimated high persistence of the TIPS-specific factor under the risk-neutral measure.[29] Second, the term structure of the estimated TIPS spread is downward sloping in 2001-2004 and 2008-2011. A market price of risk on the independent TIPS-specific factor that is on average positive, as is the case here, would contribute to a downward-sloping term structure of the TIPS spread.

Finally, the TIPS spreads were fairly high (1.5-2% range) when TIPS were first introduced but had been steadily declining until around 2004, likely reflecting the maturing process of a relatively new financial instrument. The spread surged to record-high levels ($\sim$3%) after the Lehman bankruptcy, reflecting a sharp increase in transaction and funding costs for TIPS as well as heightened risk aversion.[30] The heavy flight-to-safety flows into the nominal Treasury market likely also contributed to larger demand imbalances between nominal Treasuries and TIPS, which in turn should lead to a larger liquidity premium in TIPS yields relative to their nominal counterparts. Similarly sharp rises in liquidity premiums and/or illiquidity measures were seen in other key markets during the recent financial crisis, including equity and corporate bond markets.[31]

B. Link to Observable TIPS Liquidity Measures

Given the unobserved nature of the TIPS-specific factor in our model, one may question whether it is indeed capturing TIPS liquidity rather than other components of TIPS yields that are

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[29] As can be seen from Table 3, its risk-neutral persistence, $\tilde{\kappa}^* = \tilde{\kappa} + \tilde{\sigma}\tilde{\lambda_1}$, is estimated to be very small at around 0.1 in all models and is tightly estimated, with a standard error of about 0.006.

[30] These estimates are somewhat larger than those in Pflueger and Viceira (2013) but broadly in line with those in Abrahams et al. (2013).

[31] See, for example, Bao et al. (2011), Dick-Nielsen et al. (2012), and Brennan, Huh, and Subrahmanyam (2012).

orthogonal to nominal Treasury yields. In this section, we provide strong evidence in favor of a liquidity premium interpretation of the TIPS spreads by linking them to various observable measures of TIPS liquidity, while controlling for other factors that might have contributed to a wedge between TIPS yields and indexed bond yields.

One immediate difficulty we face is the lack of real-time, forward-looking measures of liquidity conditions in the TIPS market that are available over a reasonably long sample period. For example, as shown in the Graph A of Figure 7, one widely used measure of illiquidity, **the bid-ask spread**, only became available for TIPS in 2003 from TradeWeb.[32] A measure that is available over a longer sample is the **relative trading volumes** of TIPS versus nominal Treasury coupon securities, plotted in Graph B.[33] This measure remained low up to mid 2004 and then rose substantially, suggesting steady improvement in TIPS liquidity during the pre-crisis sample period. The rise in relative trading volumes coincides roughly with the decline in our estimated TIPS spread over the same time window, with the two series showing a highly negative correlation of about -80% over the period of 1999 to 2007.

[Insert Figure 7 about here]

Another observable measure of TIPS liquidity used in the literature is the **average absolute fitting errors** from the Svensson TIPS yield curve, plotted in Graph C of Figure 7. This measure captures funding constraints and limits to arbitrage that prevent investors from eliminating deviations of yields from their fundamental values as measured from a fitted yield curve.[34] It is plausible that during a financial crisis, capital becomes more scarce and risk aversion run higher, leaving significant arbitrage opportunities unexploited. According to this measure, liquidity

[32] TradeWeb data, ThomsonReuters.

[33] We construct the measure as 13-week averages of weekly dealer transaction volumes in TIPS divided by those in nominal Treasury coupon securities using data reported by primary dealers and collected by the Federal Reserve Bank of New York under Government Securities Dealers Reports (FR-2004).

[34] Similar measures have been used by Fontaine and Garcia (2012) and Hu, Pan, and Wang (2012) to measure the liquidity of nominal Treasury securities, and by Grishchenko and Huang (2013) for TIPS.

conditions in the TIPS market were fine until the inception of the recent financial crisis, when they suddenly deteriorated.

Three other considerations also affected our selection of liquidity proxies. First, as noted by Goldreich, Hanke, and Nath (2005), to capture liquidity premiums we need measures not only of current liquidity conditions, but more importantly of liquidity conditions expected to prevail over the life of the instruments. Second, unlike the bid-ask spread and the TIPS curve fitting errors, the liquidity proxies relevant to our study should capture not the absolute level of TIPS (il)liquidity, but rather their liquidity relative to nominal Treasury securities. Finally, the ideal proxies should also reflect variations in investors' risk attitude towards any given liquidity risk. To capture those additional dimensions of liquidity premiums, we examine two closely-related measures based on asset prices. The first such measure is the **difference between TIPS and off-the-run nominal asset swap (ASW) spreads**, obtained from Barclays and plotted in Graph D of Figure 7.[35] Because both nominal Treasury securities and TIPS are usually considered free of default risk, their ASW spreads can be regarded as a good market-based measure of the liquidity premiums in those assets, and the difference between TIPS and nominal Treasury ASW spreads would be an ideal measure of the relative illiquidity of TIPS.[36] Unfortunately TIPS ASWs only started trading in 2006; we therefore use the **difference between the off-the-run and on-the-run 10-year nominal Treasury ASW spreads**, obtained from JP Morgan, as an approximation when studying the full-sample. The correlation between the two measures of ASW spread differences is 0.93 since 2006 when both are available. As can be seen from Graphs D and E of Figure 7, both measures spiked during the crisis, reflecting general funding pressures leading to a dramatic divergence between prices of securities with only small liquidity differentials. The second forward-looking measure of TIPS liquidity premiums is the **difference between inflation swaps**

[35] In a fixed-income asset swap, one party exchanges the fixed-rate cash flows from the underlying security for a floating-rate cash flow, where the floating rate is typically quoted as 6-month LIBOR plus a spread—the asset swap spread.

[36] Such a measure is used in a recent study by Campbell, Shiller, and Viceira (2009), which focuses on a more recent sample of July 2007 to April 2009.

and TIPS BEI, available since late 2004 and plotted in Graph F of of Figure 7.[37] As noted by Campbell et al. (2009), in theory this measure is linked to the difference between the TIPS and nominal ASW spreads through a no-arbitrage relationship; empirically, the two measures have a high correlation of 88% in the post-2006 sample when both are available, although the swap-BEI difference exhibits a smaller spike during the recent crisis and also an earlier return to its normal levels afterwards.

[Insert Table 6 about here]

To quantify the effects of these factors, we run univariate and multivariate regressions of the 10-year TIPS spread from Model LII on various liquidity measures. Panel A of Table 6 examines the three measures that are available over the full sample period—the relative TIPS-nominal trading volumes, the nominal on- and off-the-run ASW spread difference, and the average TIPS curve fitting errors. The coefficients on the three variables all carry the expected signs and are statistically significant. In particular, one can expect TIPS liquidity premiums to be lower when TIPS transaction volumes rise relative to those of nominal Treasury securities, when nominal on- and off-the-run ASW spreads trade closer to each other, and/or when TIPS yields show smaller deviations from their fundamental values. Together these three variables explain 85% of the variations in the 10-year TIPS spread estimates.

Panel B of Table 6 expands the regressions to include all six liquidity measures over the sample period since September 20, 2006, when all measures became available. Again, most coefficients are of the expected sign and statistically significant, except for the TIPS bid-ask spread and the swaps-BEI difference, which are significant on their own but become insignificant when all TIPS liquidity measures are included. The magnitude of the coefficients on the relative

[37] Inflation swaps are over-the-counter swap contracts under which one party pays a fixed rate—the inflation swap rate—and the other party pays a floating rate that equals the realized inflation rate. Inflation swap rates are usually above the TIPS BEI at the same maturity due to liquidity differences between the TIPS and nominal Treasury markets as well as the lack of liquidity in the inflation swaps market. Fleckenstein et al. (2014b) and Christensen and Gillan (2012) examine these explanations in more details.

TIPS trading volumes is smaller in this more recent sample period, consistent with the intuition that the "growing pains" of the TIPS market is a more important story in the earlier part of the sample. In the univariate regression, the coefficient on the TIPS fitting errors nearly doubled and the regression now explains a much larger portion of TIPS spread variations, arguably reflecting the importance of funding constraints and limits to arbitrage during the recent crisis. The same three variables as in Panel A explain a very similar percentage (83%) of the TIPS spread variations, with this number rising to 86.3% when all observable liquidity measures (except the nominal on- and off-the run ASW spread difference) are included.

The results from Table 6 confirm that the model-implied TIPS spreads are indeed capturing current and expected future relative liquidity conditions in the TIPS market as well as the associated risk premiums. We caution, though, that the linear regression analysis may not capture the more complex relation that quantities like bid-ask spreads and trading volumes can be expected to have with liquidity premiums. In comparison, the latent-liquidity-factor approach used in this paper has the advantage of being more flexible without the need to assume a rigid link between the liquidity spreads and one or more observable measures. That said, our regression analysis suggests that the difference between the TIPS and nominal ASW spreads stands out as the most promising real-time, observable measure of TIPS liquidity premiums, at least based on data since 2006.

C. The Importance of Other Drivers

In addition to lower liquidity and the indexation lag, a few other aspects of TIPS might also affect their pricing relative to nominal Treasury securities. First, TIPS are indexed to non-seasonally-adjusted CPI; therefore, TIPS with a base reference month typically associated with higher (lower) inflation than the current reference month can be expected to trade at a slightly higher (lower) price, all else equal. To capture this effect, we estimate a seasonally-adjusted TIPS yield curve and compute the difference between the actual 10-year

TIPS yield and the seasonally-adjusted 10-year TIPS yield, plotted in Graph B of Figure 8.[38] This measure fluctuates between plus and minus 10 basis points and tends to be positive in the summer months and negative during the winter months, which, after taking into account the indexation lag, is consistent with the pattern in recent years that CPI tends to be higher from March to May and lower from October to December.

Second, TIPS contain an embedded put option—often called the “deflation floor”—as the principal repayment cannot fall below par.[39] This option is far out of the money and of little value in normal times, but could move into the money when deflation becomes a concern, especially for newly issued TIPS with reference CPIs close to the current level.[40] At the height of the recent financial crisis, valuable deflation protection would push up TIPS prices and down TIPS yields, partially offsetting the effect of higher TIPS liquidity premiums. The TIPS yields used in this study are based on a yield curve fitted to the entire universe of TIPS and are therefore less susceptible to the effect of the deflation floor than on-the-run TIPS yields. Nevertheless, to examine the remaining effect of the embedded deflation floor, we calculate the difference between the 10-year TIPS yield from the seasonally-adjusted curve fitted to all TIPS and the 10-year yield from a seasonally-adjusted curve estimated without the on-the-run and first-off-the-run 5- and 10-year TIPS. This difference, shown in Graph A of Figure 8, was less than 5 basis points in magnitude in normal times, but widened to -20 basis points during the recent crisis.[41]

[38] We adjust the quoted real prices and future real coupon and principal payments by the expected seasonal factor, which is proxied by the average ratio of non-seasonally-adjusted CPI to seasonally adjusted CPI over the previous five years. A detailed description of the methodology can be found in Grishchenko, Li, and Vollmer (2015).

[39] See Jacoby and Shiller (2008), Wright (2009), Christensen et al. (2010), Grishchenko, Vanden, and Zhang (2016) for more details.

[40] For example, Wright (2009) showed that two TIPS with similar maturity dates but different issue dates—the April 2013 TIPS issued in 2008 and the July 2013 TIPS issued in 2003—were traded at nearly identical real yields prior to September 2008, but the yield spread widened to as much as 200 basis points in December 2008, as the price of the more recently issued April 2013 TIPS rose, reflecting the higher value of the deflation floor, while the price of the July 2013 TIPS dropped.

[41] Alternatively, one could derive the value of the embedded deflation floor in any individual TIPS using the

Finally, any special demand for nominal Treasury securities would push down nominal Treasury yields as well as any estimates of real yields that are consistent with those nominal yields, leading to a wider gap between TIPS yields and model-implied real yields. This is most obvious during times of market panics with large flight-to-safety flows into the nominal Treasury market, but can also arise in normal times due to the special role nominal Treasury securities play in facilitating hedging and short covering activities and in satisfying regulation requirements. Such special demand is closely linked to the relative liquidity between nominal Treasuries and TIPS, but might merit a separate examination. To capture the safe-haven flows into nominal Treasuries, we use the VIX plotted in Graph C of Figure 8. To capture the more general concept of convenience yield in nominal Treasuries, we look at the difference between the overnight repo rates for the on-the-run 10-year nominal Treasury and the on-the-run 10-year TIPS, plotted in Graph D of Figure 8.[42] Motivated by Christensen and Gillan (2015), we also include a time dummy that equals one in the weeks during which the Federal Reserve purchased TIPS as part of its various asset purchase programs during the recent financial crisis.

The last column in Panel B of Table 6 adds those five measures to the joint regression. They all carry the expected signs but, with the exception of the repo difference, are not statistically significant at the 1% level and together add limited additional explanatory power for the TIPS spread. Indeed, as can be seen from Figure 9, the regression attributes almost all the variations in the TIPS spread to TIPS liquidity measures, whereas the contribution from the five other measures is essentially flat and close to zero. This does not imply risk aversion or the relative demand imbalances have no effects on the TIPS spread, but suggests those effects are largely subsumed in the price-based liquidity measures. Overall, these results confirm our conjecture that the TIPS

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risk-adjusted PDF of inflation constructed from inflation caps and floors as in Kitsul and Wright (2013) and Fleckenstein, Longstaff, and Lustig (2014a). However, quotes on inflation caps and floors have been available only since 2009. In addition, the effect of the embedded deflation floor on individual TIPS yields is likely to differ from that on fitted TIPS yields due to yield curve fitting errors mentioned above.

[42] Convenience yields on nominal Treasury securities are studied by Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2012), and Bansal, Coleman, and Lundblad (2010), among others.

spread is reflecting overwhelmingly a liquidity premium in TIPS yields. In the remainder of the paper, we'll use the terms "TIPS spread" and "TIPS liquidity premiums" interchangeably.

D. Economic Significance of TIPS Liquidity Premiums

We assess the economic significance of the TIPS liquidity premiums by examining the proportions of variations in TIPS yields and TIPS BEI that can be attributed to variations in those premiums. Using equations (1), (26) and (27), we can decompose TIPS yields, $y^{\mathcal{T}}_{t,\tau}$, and TIPS BEI, $BEI^{\mathcal{T}}_{t,\tau}$, into different components:

$$y^{\mathcal{T}}_{t,\tau} = y^{R}_{t,\tau} + (y^{I}_{t,\tau,l} - y^{R}_{t,\tau}) + L_{t,\tau}, \quad \mathrm{BEI}^{\mathcal{T}}_{t,\tau} = I_{t,\tau} + \wp_{t,\tau} - (y^{I}_{t,\tau,l} - y^{R}_{t,\tau}) - L_{t,\tau}, \tag{30}$$

where $y^{R}_{t,\tau}$ is the underlying real yield, $y^{I}_{t,\tau,l} - y^{R}_{t,\tau}$ represents the effect of the indexation lag, $L_{t,\tau}$ is the TIPS liquidity premium, $I_{t,\tau}$ is expected inflation over the next τ periods, and $\wp_{t,\tau}$ is the inflation risk premium. Table 7 reports an in-sample variance decomposition based on equation (30) and Model LII estimates. A time series plot of the decomposition is shown in Figure 10.

[Insert Table 7 about here.]

[Insert Figure 10 about here.]

Real yields dominate TIPS liquidity premiums in accounting for the time variations in TIPS yields. By contrast, the TIPS liquidity premium is a more important driver of TIPS BEI, explaining 36-49% of its movement across the three maturities, although expected inflation and inflation risk premiums jointly still account for the majority of time variations in TIPS BEI. Our results suggest that one should be especially cautious in interpreting fluctuations in TIPS BEI solely in terms of changes in inflation expectation or inflation risk premiums.

VII. Inflation Risk Premiums and Expected Inflation: Comparison Across Studies

In this section, we compare our estimates of inflation risk premiums and expected inflation to those obtained in previous and concurrent studies, collected in Table 8. We organize those studies based on the length of the sample periods, ordering last those that focus on more recent samples. For example, we start with Ang et al. (2008) (ABW) that cover the period from 1952 to 2004 and conclude with Fleckenstein et al. (2014a) (FLL) that focuse on the recent decade of 2004-2014. This organizing principle is motivated by Bekaert and Wang (2010), who find that the longer the sample period, the larger and more robustly positive the inflation risk premium estimates.

To understand the wide range of inflation risk premium estimates in Table 8, it is helpful to start from Kitsul and Wright ((2013), Fig. 12 and 13), who observe that the empirical pricing kernel appears to be U-shaped in inflation. This suggests that when the economy is close to deflation (hyperinflation), a pickup in inflation and the resulting losses to nominal bond holders are likely to coincide with lower (higher) marginal utility of wealth, as reflected in the downward-sloping (upward-sloping) arm of the U. As a result, inflation risk premiums are likely to be lower or even negative during times of deflation risks and higher and positive when hyperinflation is a concern.[43]

Historically, the 1970s and early 1980s saw counter-cyclical, high, and volatile inflation as well as unstable inflation expectations, which tend to be associated with hyperinflation risks and high inflation risk premiums. In addition, survey information shows that inflation uncertainty was elevated up to the early 1990s (D'Amico and Orphanides (2014)), likely further boosting inflation risk premiums. Subsequently, inflation expectations declined and stabilized at much lower levels after 1998, as evidenced in the SPF and other surveys, together with realized inflation, which can be expected to lead to a gradual decline in inflation risk premiums over this period. Finally, since the last crisis, realized inflation has been running persistently low, and both surveys and inflation

[43] Similar points are made by Campbell et al. (2013) and David and Veronesi (2013).

derivatives attached non-negligible odds to near- or below-zero inflation at horizons even above 5 years, which may result in very low or even negative inflation risk premiums.

It's perhaps not surprising then, that studies in Table 8 using data from the 1970s and early 1980s but not from the recent financial crisis—ABW, Buraschi and Jiltsov (2005) (BJ) and Chernov and Mueller (2012) (CM)—obtain estimates of inflation risk premiums that are mostly positive and large in magnitude. In contrast, models estimated over shorter sample periods that do include the financial crisis–Abrahams et al. (2013) (AACM), Grishchenko and Huang (2013) (GH) and FLL—produce lower inflation risk premium estimates that are often negative at shorter maturities. The evidence shown in our paper also suggests that any studies using TIPS but ignoring the liquidity mismatch between nominal Treasuries and TIPS are likely to underestimate the inflation risk premiums.[44]

Finally, studies like Haubrich et al. (2012) (HPR), Ajello, Benzoni, and Chyruk (2011) (ABC), and ours (DKW), that include both the period of the 1980s and/or early 1990s, when inflation gradually come down and stabilize, and the latest financial crisis, produce mostly positive estimates of inflation risk premiums, especially at longer maturities, that lie between those of the two groups of studies mentioned above. HPR obtained estimates for maturities up to 29 months that frequently turned negative during episodes of financial market turmoil, perhaps reflecting, as they note, the relative attractiveness of safe and liquid shorter-maturity Treasuries. This again suggests that the special features of nominal Treasuries, and the lack thereof in TIPS or inflation swaps, should be taken into account when estimating the inflation risk premiums.

Overall, our estimated inflation risk premiums fall in the middle of a rather wide range of estimates found in other studies. Similar to the broad pattern of lower values in more recent samples, our inflation risk premium estimates trended down over time and briefly turned negative at the 5-year maturity at the height of the recent financial crisis, likely reflecting the substantial

[44] Indeed, Grishchenko and Huang (2013) report negative inflation risk premiums even at longer maturities when TIPS are taken at their face value. Once a regression-based TIPS liquidity adjustment is made, the estimated 10-year inflation risk premium turns slightly positive.

risk of deflation at the time.

Similarly, the average inflation expectation estimates reported in Table 8 also depend strongly on the sample period: studies using a longer sample that spans the 1970s and 1980s but ends before the recent financial crisis—ABW and CM—obtain the highest inflation expectation estimates. Studies that focus on the post-1998 period of low and stable actual and expected inflation—AACM, GH, and FLL—obtain estimates that are much smaller in magnitude. Finally, studies that include both early 1990s and the latest financial crisis—HPR and ABC—report estimates that are roughly in line with our study (see also Figure 6, Panel B, in HPR).

VIII. Conclusion

This paper shows that a TIPS-specific factor is important for explaining TIPS yield and TIPS BEI variations but not for nominal yield changes, and provides evidence that this factor is linked to the relative illiquidity of TIPS versus nominal Treasury securities.

In particular, we develop a joint no-arbitrage term structure model of nominal yields, TIPS yields, and realized inflation, and show that ignoring the spread between TIPS and risk-free real yields leads to a poor model fit of TIPS yields, TIPS BEI, and survey inflation forecasts. In models allowing such a spread, its estimated values were fairly large ($\sim 1\%$) until about 2003 and fluctuated within narrow ranges between then and the onset of the crisis, consistent with the common perception that TIPS market liquidity had steadily improved over time. The estimated TIPS spread shot up to nearly 300 basis points after the Lehman collapse, amid reports of fire sales of TIPS and stringent funding liquidity conditions over that period. Consistent with these observations, regression analysis shows that about 85% of the variations in the estimated TIPS-indexed bond spread are explained by changes in observable measures of TIPS liquidity, while other factors such as TIPS deflation floor and CPI seasonality appear much less important.

TIPS BEI has increasingly gained attention as a measure of investors' inflation expectations that is available in real-time and at high frequencies. However, our results raise caution in interpreting movements in TIPS BEI solely in terms of changing inflation expectations, as substantial liquidity premiums and inflation risk premiums could drive a large wedge between the two, as demonstrated vividly during the recent financial crisis. A better understanding of the determinants of TIPS liquidity premiums and the sources of its variation remains an interesting topic for future research.

Appendix. Formulae and Notations

A. The Real Pricing Kernel

Applying Ito's lemma to equation (8) and using equations (4) to (7), the real pricing kernel can be derived as following the process

(A-1)
$$\begin{aligned} dM_t^R/M_t^R &= dM_t^N/M_t^N + dQ_t/Q_t + (dM_t^N/M_t^N) \times (dQ_t/Q_t) \\ &= -r^R(x_t)dt - \lambda^R(x_t)'dB_t - (\cdot)dB_t^{\perp}, \end{aligned}$$

where the real short rate and the real prices of risk are given by

(A-2)
$$r^R(x_t) = \rho_0^R + \rho_1^{R'} x_t,$$

(A-3)
$$\lambda^R(x_t) = \lambda_0^R + \Lambda^R x_t,$$

and the coefficients are linked to their nominal counterparts by

(A-4)
$$\rho_0^R = \rho_0^N - \rho_0^\pi - \frac{1}{2}(\sigma_q'\sigma_q + \sigma_q^{\perp 2}) + \lambda_0^{N'}\sigma_q$$

(A-5)
$$\rho_1^R = \rho_1^N - \rho_1^\pi + \Lambda^{N'}\sigma_q$$

(A-6)
$$\lambda_0^R = \lambda_0^N - \sigma_q, \quad \Lambda^R = \Lambda^N.$$

B. TIPS-Real Yield Spread

Since $\tilde{x}_t$ is independent of the other state variables in x_t, the first term on the right-hand side of equation (21) can be written as the sum of two components:

(B-1)
$$-(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}\tilde{\gamma}\tilde{x}_s ds}) - (1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}(\rho_0^R + (\rho_1^R + \gamma)'x_s)ds})$$

The first component can be solved to be

$$-(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}\tilde{\gamma}\tilde{x}_s ds}) = \tilde{a}_\tau + \tilde{b}_\tau \tilde{x}_t \tag{B-2}$$

where $\tilde{a}$ and $\tilde{b}$ has the familiar form of factor loadings in a one-factor Vasicek model:

$$\begin{aligned}
\tilde{a}_\tau &= \tilde{\gamma}\left[(\tilde{\mu}^* - \frac{\tilde{\sigma}^2}{2\tilde{\kappa}^*})(1-\tilde{b}_\tau) + \frac{\tilde{\sigma}^2}{4\tilde{\kappa}^*}\tau\tilde{b}_\tau^2\right] && \text{(B-3)}\\
\tilde{b}_\tau &= \tilde{\gamma}\frac{1-\exp(-\tilde{\kappa}^*\tau)}{\tilde{\kappa}^*\tau}, && \text{(B-4)}
\end{aligned}$$

in which $\tilde{\kappa}^* = \tilde{\kappa} + \tilde{\sigma}\tilde{\lambda}_1$ and $\tilde{\mu}^* = (\tilde{\kappa}\tilde{\mu} - \tilde{\sigma}\tilde{\lambda}_0)/\tilde{\kappa}^*$.

The second component can be shown to take the form

$$-(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}(\rho_0^R + (\rho_1^R+\gamma)' x_s)ds}) = a_\tau^{\mathcal{T}} + b_\tau^{\mathcal{T}\prime} x_t. \tag{B-5}$$

where $a_\tau^{\mathcal{T}} = -A_\tau^{\mathcal{T}}/\tau$ and $b_\tau^{\mathcal{T}} = -B_\tau^{\mathcal{T}}/\tau$ are given by

$$\begin{aligned}
\frac{dA_\tau^{\mathcal{T}}}{d\tau} &= -\rho_0^R + B_\tau^{\mathcal{T}\prime}\left(\mathcal{K}\mu - \Sigma\lambda_0^R\right) + \frac{1}{2}B_\tau^{\mathcal{T}\prime}\Sigma\Sigma' B_\tau^{\mathcal{T}} && \text{(B-6)}\\
\frac{dB_\tau^{\mathcal{T}}}{d\tau} &= -\left(\rho_1^R + \gamma\right) - \left(\mathcal{K} + \Sigma\Lambda^R\right)' B_\tau^{\mathcal{T}} && \text{(B-7)}
\end{aligned}$$

with initial conditions $A_0^{\mathcal{T}} = 0$ and $B_0^{\mathcal{T}} = 0_{3\times 1}$.

Taken together, we have that

$$\begin{aligned}
L_{t,\tau} &= \left(\tilde{a}_\tau + a_\tau^{\mathcal{T}}\right) + \begin{bmatrix} b_\tau^{\mathcal{T}\prime} & \tilde{b}_\tau \end{bmatrix}\begin{bmatrix} x_t \\ \tilde{x}_t \end{bmatrix} - y_t^R \\
&= \left[\tilde{a}_\tau + (a_\tau^{\mathcal{T}} - a_\tau^R)\right] + \begin{bmatrix} (b_\tau^{\mathcal{T}} - b_\tau^R)' & \tilde{b}_\tau \end{bmatrix}\begin{bmatrix} x_t \\ \tilde{x}_t \end{bmatrix} && \text{(B-8)}
\end{aligned}$$

where the second equality comes from equation (15).

C. A Four-Factor Framework

The models with a TIPS-specific factor can be restated in a 4-factor Gaussian framework, in which the expanded state variables $z_t = [x_t', \tilde{x}_t - \tilde{\mu}]$ follow the process

$$dz_t = \mathcal{K}_z(\mu_z - z_t)dt + \Sigma_z dB_t^z, \tag{C-1}$$

where

$$\mathcal{K}_z = \begin{bmatrix} \mathcal{K} & 0_{3\times 1} \\ 0_{1\times 3} & \tilde{\kappa} \end{bmatrix}, \ \mu_z = \begin{bmatrix} \mu \\ 0 \end{bmatrix}, \ \Sigma_z = \begin{bmatrix} \Sigma & 0_{3\times 1} \\ 0_{1\times 3} & \tilde{\sigma} \end{bmatrix}, \ B_t^z = \begin{bmatrix} B_t \\ W_t \end{bmatrix}.$$

We demean $\tilde{x}_t$ so that the new 4-factor model continues to satisfy the identification condition $\mu_z = 0$. Parameters governing the real and nominal short rates and prices of risks are redefined in obvious ways. In particular, the real and nominal prices or risks are given by

$$\lambda_0^i = \begin{bmatrix} \lambda_0^i \\ \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{\mu} \end{bmatrix}, \ \Lambda^i = \begin{bmatrix} \Lambda^i & 0 \\ 0_{1\times 3} & \tilde{\lambda}_1 \end{bmatrix}, \ i = N, R. \tag{C-2}$$

TIPS can then be priced as real bonds similar to equations (12) to (14) based on the adjusted real short rate

$$\begin{aligned} \tilde{r}_t^R &= r_t^R + l_t = \tilde{\rho}_0^R + \tilde{\rho}_1^{R'} z_t, \\ \tilde{\rho}_0^R &= \rho_0^R + \tilde{\gamma}\tilde{\mu}, \ \ \tilde{\rho}_1^R = \begin{bmatrix} \rho_1^R + \gamma & \tilde{\gamma} \end{bmatrix}. \end{aligned} \tag{C-3}$$

For reasons that will become clear in Appendix D, we also define the adjusted nominal short rate based on equations (A-4) through (A-5) as:

$$\begin{aligned} \tilde{r}_t^N &= r_t^N + l_t = \tilde{\rho}_0^N + \tilde{\rho}_1^{N'} z_t, \\ \tilde{\rho}_0^N &= \rho_0^N + \tilde{\gamma}\tilde{\mu}, \ \ \tilde{\rho}_1^N = \begin{bmatrix} \rho_1^N + \gamma & \tilde{\gamma} \end{bmatrix}. \end{aligned} \tag{C-4}$$

D. Indexed Bond Pricing

We adjust TIPS yields in equation (26) for the indexation lag in three steps. First, following Evans (1998) and Risa (2001) and as shown below in equation (D-1), the artificial indexed bond can be viewed as a claim to a real payoff that equals the time-$(T-l)$ price of a l-period nominal bond, $P^N_{T-l,l}$, to be received at time $T-l$:

$$\text{(D-1)} \qquad \begin{aligned} \widetilde{P}^I_{t,\tau,l} &= E_t\left[\frac{M^N_T}{M^N_t}\frac{Q_{T-l}}{Q_t}\right] = E_t\left[\frac{M^N_{T-l}}{M^N_t}E_{T-l}\left(\frac{M^N_T}{M^N_{T-l}}\right)\frac{Q_{T-l}}{Q_t}\right] \\ &= E_t\left[\frac{M^N_{T-l}}{M^N_t}\frac{Q_{T-l}}{Q_t}P^N_{T-l,l}\right] = E_t\left[\frac{M^R_{T-l}}{M^R_t}P^N_{T-l,l}\right] \end{aligned}$$

By the same argument as in deriving the real bond price formula, we can show that the time-t price of such an artificial indexed bond is given by

$$\text{(D-2)} \qquad \widetilde{P}^I_{t,\tau,l} = \exp\left(A_{\tau,l} + B'_{\tau,l}x_t\right)$$

where $A_{\tau,l}$ and $B_{\tau,l}$ satisfy the ODE:

$$\text{(D-3)} \qquad \frac{dA_{\tau,l}}{d\tau} = -\rho^R_0 + B'_{\tau,l}\left(\mathcal{K}\mu - \Sigma\lambda^R_0\right) + \frac{1}{2}B'_{\tau,l}\Sigma\Sigma'B_{\tau,l}$$

$$\text{(D-4)} \qquad \frac{dB_{\tau,l}}{d\tau} = -\rho^R_1 - \left(\mathcal{K} + \Sigma\Lambda^R\right)'B_{\tau,l}$$

with initial conditions $A_{l,l} = A^N_l$ and $B_{l,l} = B^N_l$. The yield on this artificial indexed bond is:

$$\text{(D-5)} \qquad \widetilde{y}^I_{t,\tau,l} = a_{\tau,l} + b'_{\tau,l}x_t$$

with factor loadings $a_{\tau,l} = -A_{\tau,l}/\tau$ and $b_{\tau,l} = -B_{\tau,l}/\tau$. In the second step, the yield on the actually marketed indexed bond can be obtained through equation (28) as:

$$\text{(D-6)} \qquad y^I_{t,\tau,l} = a_{\tau,l} + b'_{\tau,l}x_t - \frac{1}{\tau}\log\frac{Q_t}{Q_{t-l}}.$$

Finally, to incorporate the TIPS-specific factor, we make use of the four-factor framework outlined in Appendix C and price the indexed bond using the adjusted real and nominal short rates defined in equations (C-3) through (C-4). In particular, we now use the adjusted nominal short rate defined in equation (C-4) to calculate the time-$(T-l)$ price of a l-period nominal bond, $P^N_{\tau-l,l}$, in the first step. This gives us the yield on a τ-year TIPS with an indexation lag l:

$$y^{\mathcal{T}}_{t,\tau,l} = a^{\mathcal{T}}_{\tau,l} + b^{\mathcal{T}\prime}_{\tau,l} z_t - \frac{1}{\tau}\log\frac{Q_t}{Q_{t-l}}.$$

E. State-Space Form

Denote by $\mathrm{x}_t = [q_t, \ldots, q_{t-l}, x_t', \tilde{x}_t]'$ the augmented state vector including the log price level, $q_t \equiv \log(Q_t)$, its lags over the indexation period, and the TIPS-specific factor, $\tilde{x}_t$. The state equation is derived as Euler discretization of equations (2), (6), and (23) and can be written in a matrix form as

$$\mathrm{x_t} = \mathrm{G_h} + \Gamma_\mathrm{h}\mathrm{x_{t-h}} + \eta^\mathrm{x}_\mathrm{t}, \tag{E-1}$$

where

$$G_h = \begin{bmatrix} \rho_0^\pi h \\ 0 \\ \mathcal{K}\mu h \\ \tilde{\kappa}\tilde{\mu}h \end{bmatrix}, \ \Gamma_h = \begin{bmatrix} 1 & 0 & \rho_1^{\pi\prime} h & 0 \\ I_{(l-1)\times(l-1)} & 0 & 0 & 0 \\ 0 & 0 & I_{3\times3} - \mathcal{K}h & 0 \\ 0 & 0 & 0 & 1-\tilde{\kappa}h \end{bmatrix} \text{ and } \eta^\mathrm{x}_t = \begin{bmatrix} \sigma_q'\eta_t + \sigma_q^\perp \eta_t^\perp \\ 0 \\ \Sigma\eta_t \\ \tilde{\sigma}\tilde{\eta}_t \end{bmatrix}$$

in which η_t, $\eta_t^\perp$, and $\tilde{\eta}_t$ are independent of each other, and have the distribution

$$\eta_t \sim \mathcal{N}(0, hI_{n\times n}), \ \ \eta_t^\perp \sim \mathcal{N}(0,h), \ \ \tilde{\eta}_t \sim \mathcal{N}(0,h). \tag{E-2}$$

We collect in y_t all data used in the estimation, including current and lagged log price levels, $q_{t-i},\ i = 0\ldots l$, all nominal yields, $Y_t^N = \{y^N_{t,\tau_i}\}_{i=1}^7$, all TIPS yields, $Y^{\mathcal{T}}_{t,l} = \{y^{\mathcal{T}}_{t,\tau_i,l}\}_{i=1}^3$, short- and long-horizon survey forecasts of future 3-month TBill yield, $f_t = \{f_{t,6m}, f_{t,12m}, f_{t,long}\}$, and 1- and

10-year ahead survey inflation forecasts, $IE_t^{svy} = \{IE_{t,1y}^{svy}, IE_{t,10y}^{svy}\}$:

$$\text{(E-3)} \qquad \mathrm{y}_t = [q_t, \ldots, q_{t-l}, Y_t^N, Y_{t,l}^{\mathcal{T}}, f_t, IE_t^{svy}]'.$$

where l is set to 11 weeks. We assume that all nominal and TIPS yields and survey forecasts are observed with error. The observation equation therefore takes the form

$$\text{(E-4)} \qquad \mathrm{y}_t = A + B\mathrm{x}_t + \mathrm{e}_t$$

where

$$\text{(E-5)} \qquad A = \begin{bmatrix} 0 \\ a^N \\ a_l^{\mathcal{T}} \\ a^f \\ a^{IE} \end{bmatrix}, \quad B = \begin{bmatrix} I_{l\times l} & 0_{l\times 3} & 0_{l\times 1} \\ 0_{1\times l} & b^{N\prime} & 0 \\ 0_{1\times l} & \multicolumn{2}{c}{b_l^{\mathcal{T}\prime}} \\ 0_{1\times l} & b^{f\prime} & 0 \\ 0_{1\times l} & b^{IE\prime} & 0 \end{bmatrix}, \quad \mathrm{e}_t = \begin{bmatrix} 0 \\ e_t^N \\ e_{t,l}^{\mathcal{T}} \\ e_t^f \\ e_t^{IE} \end{bmatrix},$$

in which the various a vectors and b matrices collect the loadings of nominal and TIPS yields and survey forecasts in obvious ways. We assume a simple i.i.d. structure for the measurement errors:

$$\text{(E-6)} \qquad e_{t,\tau_i}^N \sim \mathcal{N}(0, \delta_{N,\tau_i}^2), \quad e_{t,\tau_i,l}^{\mathcal{T}} \sim \mathcal{N}(0, \delta_{\mathcal{T},\tau_i,l}^2), \quad e_{t,\tau_i}^f \sim \mathcal{N}(0, \delta_{f,\tau_i}^2), \quad e_{t,\tau_i}^{IE} \sim \mathcal{N}(0, \delta_{IE,\tau_i}^2).$$

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Table 1: Factors Driving Nominal and Inflation-Indexed Bonds

Panel A of Table 1 reports adjusted R^2's from regressions of 10-year TIPS breakeven inflation rates on 3-month, 2-year, and 10-year nominal Treasury yields over the full weekly sample of 1/6/99 to 3/27/13, a pre-crisis sub-sample of 1/6/99-7/25/07 and a post-crisis sub-sample of 8/1/07-3/27/13. Panel B reports cumulative percentages of variances of weekly changes in nominal yields only or nominal and indexed bond yields combined that are explained by their respective first four principal components. The in-sample pairwise correlations between the two sets of principal components are reported in Panel C. Panel D reports results from a quarterly regression of the difference between the 10-year inflation forecast from the Survey of Professional Forecasters (SPF) and the 10-year TIPS BEI on TIPS transaction volumes relative to those of nominal Treasuries, the difference between the asset swap (ASW) spreads on the on-the-run and the off-the-run nominal Treasury securities, and the average absolute TIPS curve fitting errors.

Panel A. Regressing 10-year Breakevens on Nominal Yields: Adjusted R^2 (in%)

Sample	in level	in weekly changes
Full Sample	6.0	39.2
Pre Crisis	30.1	58.7
Post Crisis	5.0	26.1

Panel B. Variances Explained by Principal Components (Full sample, in %)

PC	nominal yields only	nominal and TIPS yields
1st	70.6	65.4
2nd	92.7	86.2
3rd	97.6	94.1
4th	99.2	97.6

Panel C. Correlation of Principal Components (Full sample)

		nominal and TIPS yields			
		PC1	PC2	PC3	PC4
nominal	PC1	**0.95**	-0.28	-0.15	0.01
yields	PC2	0.18	**0.87**	-0.47	0.02
alone	PC3	0.03	0.06	0.16	**0.98**
	PC4	0.01	0.02	0.06	-0.05

Panel D. The Role of Liquidity (Full sample)

Constant	relative TIPS transaction volume	Nominal On/Off ASW Spread Diff	Average Absolute TIPS Curve fitting error	Adj. R^2
0.1766	-0.1633	0.0081	0.0534	60.2%
(0.1531)	(0.0618)	(0.0051)	(0.0126)	

Table 2: Summary of Models

Table 2 lists the parameter restrictions placed on the three models we estimate, including a model assuming zero TIPS liquidity premiums (Model NL), a model assuming TIPS liquidity premiums that are orthogonal to the other stare variables in the economy (Model LI), and a model allows correlation between TIPS liquidity premiums and the other state variables. Models estimated without using survey forecasts of inflation are denoted by an "-noIE" suffix.

Model	Restrictions and Identifications
Model NL / NL-noIE	$\gamma = 0_{3\times 1}, \quad \tilde{\gamma} = 0, \quad \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$ unidentified
Model LI	$\gamma = 0_{3\times 1}, \quad \tilde{\gamma}, \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$ unrestricted
Model LII	$\gamma, \tilde{\gamma}, \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$ unrestricted

Table 3: Selected Parameter Estimates

Table 3 reports selected parameter estimates and standard errors for all three models we estimate. Standard errors are calculated using the BHHH formula and are reported in parentheses. Complete parameter estimates can be found in the Supplementary Appendix.

	Model NL-noIE		Model NL		Model LI		Model LII	
State Variables Dynamics								
$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t$								
$\mathcal{K}_{11}$	0.8550	(0.3533)	0.6849	(0.4589)	0.8302	(0.6993)	0.4317	(0.1622)
$\mathcal{K}_{22}$	0.1343	(0.0562)	0.1309	(0.0471)	0.1004	(0.0425)	0.0961	(0.0499)
$\mathcal{K}_{33}$	1.4504	(0.3633)	1.4259	(0.7216)	1.2353	(0.9516)	1.8425	(0.4757)
$100 \times \Sigma_{21}$	-0.7526	(0.5524)	-1.8236	(1.1939)	-1.1547	(0.8448)	-1.6133	(0.9020)
$100 \times \Sigma_{31}$	-4.4450	(4.8007)	-4.8415	(8.4964)	-7.1258	(30.8741)	-1.7824	(0.9339)
$100 \times \Sigma_{32}$	-0.9597	(0.2356)	-1.0313	(0.2948)	-1.0456	(0.4755)	-0.7864	(0.1713)
Log Price Level								
$d\log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^\perp dB_t^\perp,\ \pi(x_t) = \rho_0^\pi + \rho_1^{\pi'} x_t$								
ρ_0^π	0.0262	(0.0016)	0.0285	(0.0015)	0.0294	(0.0021)	0.0288	(0.0026)
$\rho_{1,1}^\pi$	-0.0326	(0.5805)	-0.4711	(1.7446)	-0.5261	(4.3530)	0.1582	(0.3076)
$\rho_{1,2}^\pi$	0.0867	(0.0578)	0.2378	(0.0400)	0.3515	(0.0849)	0.2684	(0.0300)
$\rho_{1,3}^\pi$	-0.2213	(0.1859)	-0.2804	(0.1584)	-0.1999	(0.2596)	-0.1356	(0.1442)
$100 \times \sigma_{q,1}$	-0.0796	(0.0445)	0.0038	(0.0734)	0.0000	(0.1009)	-0.1495	(0.0409)
$100 \times \sigma_{q,2}$	0.0066	(0.0673)	0.0869	(0.0739)	0.1625	(0.0620)	0.0763	(0.0581)
$100 \times \sigma_{q,3}$	-0.0278	(0.0589)	-0.2586	(0.0459)	-0.1526	(0.0674)	0.0224	(0.0619)
$100 \times \sigma_q^\perp$	0.9229	(0.0268)	0.9461	(0.0300)	0.9508	(0.0346)	0.8975	(0.0264)
TIPS Liquidity Premium								
$l_t = \tilde{\gamma}\tilde{x}_t + \gamma' x_t,\ d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma} dW_t,\ \tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{x}_t.$								
$\tilde{\gamma}$					0.8376	(0.0224)	0.8393	(0.0225)
$\tilde{\kappa}$					0.5097	(0.2113)	0.4900	(0.2051)
$\tilde{\mu}$					0.0067	(0.0049)	0.0077	(0.0050)
$\tilde{\lambda}_0$					0.3754	(0.3571)	0.4136	(0.3413)
$\tilde{\sigma}\tilde{\lambda}_1$					-0.3981	(0.2114)	-0.3770	(0.2052)
γ_1							-0.8403	(0.2826)
γ_2							-0.0527	(0.1024)
γ_3							0.0121	(0.2293)
Measurement Errors: TIPS Yields								
$100 \times \delta_{\mathcal{T},5y}$	0.5374	(0.0801)	0.5400	(0.0785)	0.0806	(0.0033)	0.0812	(0.0033)
$100 \times \delta_{\mathcal{T},7y}$	0.4217	(0.0849)	0.4231	(0.0843)	-0.0000	(6302.1210)	-0.0000	(6307.8897)
$100 \times \delta_{\mathcal{T},10y}$	0.3879	(0.0632)	0.3874	(0.0605)	0.0653	(0.0033)	-0.0644	(0.0033)
Measurement Errors: Survey Forecasts of Nominal Short Rate								
$100 \times \delta_{f,6m}$	0.1890	(0.0146)	0.1893	(0.0146)	0.1872	(0.0141)	0.1891	(0.0146)
$100 \times \delta_{f,12m}$	0.2965	(0.0222)	0.2945	(0.0218)	0.2944	(0.0219)	0.2968	(0.0224)

Table 4: Specification Tests

Table 4 reports various diagnostic statistics for the three models estimated. Panel A reports the number of parameters, the log likelihood, the Akaike information criterion (AIC), the Bayesian information criterion (BIC) values, and the p-value from a Likelihood Ratio test of the current model against the more general Model to its right, where the p-values reported for Models NL is the Davies (1987) upper bound. Panels B to D report three goodness-of-fit statistics for the 5-, 7- and 10-year TIPS yields, 7- and 10-year TIPS breakeven inflation and 1- and 10-year survey inflation forecasts, respectively, including the correlation between the fitted series and the data counterpart (CORR), the root mean squared prediction errors (RMSE), and the coefficient of determination (R^2).

		Model NL-noIE	Model NL	Model LI	Model LII
Panel A. Overall Model Fit					
No. of parameters		42	44	49	52
Log likelihood		73,092.74	73,890.53	77,808.18	77,879.74
AIC		-146,101.48	-147,693.06	-155,518.35	-155,655.48
BIC		-145,887.25	-147,468.62	-155,268.41	-155,390.23
LR p-value			0.00*	0.00	
Panel B. Fitting TIPS yields					
5-year	CORR (in %)	94.86	94.81	99.89	99.89
	RMSE	0.54	0.55	0.16	0.16
	R^2 (in %)	87.51	87.42	98.90	98.91
7-year	CORR (in %)	95.71	95.70	100.00	100.00
	RMSE	0.44	0.44	0.13	0.13
	R^2 (in %)	90.53	90.48	99.16	99.19
10-year	CORR (in %)	96.11	96.13	99.87	99.88
	RMSE	0.41	0.41	0.14	0.14
	R^2 (in %)	89.20	89.18	98.80	98.83
Panel C. Fitting TIPS Breakeven Inflation					
7-year	CORR (in %)	57.13	56.93	100.00	100.00
	RMSE	0.43	0.43	0.11	0.11
	R^2 (in %)	26.21	25.62	95.24	95.42
10-year	CORR (in %)	39.99	40.05	98.68	98.71
	RMSE	0.37	0.37	0.12	0.12
	R^2 (in %)	-1.85	-1.84	89.94	90.25
Panel D. Matching survey inflation forecasts					
1-year	CORR (in %)	38.19	82.47	80.71	85.35
	RMSE	0.69	0.38	0.44	0.35
	R^2 (in %)	-8.30	67.96	56.20	72.29
10-year	CORR (in %)	59.96	82.72	83.11	83.01
	RMSE	0.53	0.31	0.30	0.30
	R^2 (in %)	2.26	66.30	68.80	67.73

Table 5: Out-of-Sample Inflation Forecasting

Table 5 reports out-of-sample root mean squared errors (RMSEs), expressed in annualized percentage terms, from forecasting inflation over the next one and two years using five term structure models and a random walk (RW) model, and over the next year from three surveys. The forecasting period is from January 5, 2005 to March 27, 2013. Term structure model forecasts are constructed based on parameters estimated using all data up to date. The RW model assumes that inflation over the next one and two years will be the same as inflation over the previous one and two years, respectively. We also compare the models to the monthly Michigan and Blue Chip Financial Forecasts (BCFF) surveys and the quarterly SPF survey on dates when those surveys are available. In particular, the monthly Michigan and BCFF surveys are assumed to be conducted on the third Wednesday of each month, and the quarter SPF survey is assumed to be conducted on the third Wednesday of the middle month of each quarter.

horizon	frequency	No. obs	NL-noIE	NL	LI	LII	RW	Mich	BCFF	SPF
1-year	Weekly	430	1.4827	1.4936	1.5052	1.4689	2.3724			
	Monthly	99	1.4787	1.4944	1.5062	1.4687	2.3691	2.1778	1.5798	
	Quarterly	33	1.4369	1.4595	1.4857	1.4566	2.3020			1.5144
2-year	Weekly	430	0.9414	0.9030	0.8956	0.8849	1.4685			
	Monthly	99	0.9416	0.9084	0.8995	0.8887	1.4683			
	Quarterly	33	0.9129	0.8680	0.8600	0.8552	1.4059			

Table 6: What Drives the TIPS Spreads

Panel A reports results from univariate and multivariate regressions of the model-implied 10-year TIPS spread on the relative TIPS trading volume, the nominal on/off-the-run ASW spread difference, and the average TIPS curve fitting error using weekly data from Jan. 6, 1999 to Mar. 27, 2013. Panel B reports results from univariate and multivariate regression of the model-implied 10-year TIPS spreadon all TIPS liquidity measures from Sep. 20, 2006 to Mar. 27, 2013. Finally, the last column adds to the joint regression other potential drivers of the TIPS BEI, including seasonality and deflation floor adjustment to 10-year TIPS yield, the VIX, and the difference in repo rates on 10-year on-the-run nominal Treasury notes and TIPS. OLS Standard errors are reported within the parentheses. * (**) denotes significance at the 1% (5%) level.

Panel A. Regression Analysis: Full Sample

	1	2	3	4
Constant	1.3550**	-0.0191	0.4022**	0.9109**
	(0.0342)	(0.0197)	(0.0220)	(0.0327)
Relative TIPS transaction volume	-0.4578**			-0.4215**
	(0.0170)			(0.0136)
Nominal On/off ASW spread diff		0.0332**		0.0183**
		(0.0010)		(0.0011)
Averge TIPS curve fitting error			0.0309**	0.0264**
			(0.0040)	(0.0028)
No. of observations	743	743	743	743
Adjusted R^2	49.4%	59.5%	7.4%	84.8%

Panel B. Regression Analysis: Since 2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	-0.0765	1.3680**	-0.0334	-0.1410**	-0.4097**	-0.4112**	0.7900**	0.4153**	0.3359**
	(0.0419)	(0.1161)	(0.0277)	(0.0214)	(0.0238)	(0.0423)	(0.0642)	(0.0604)	(0.0651)
Ten-year TIPS bid-ask spread	0.1601**							0.0030	0.0106
	(0.0143)							(0.0074)	(0.0085)
Relative TIPS transaction volume		-0.4402**					-0.3751**	-0.3014**	-0.2916**
		(0.0467)					(0.0249)	(0.0224)	(0.0239)
Average TIPS curve fitting error			0.0620**				0.0291**	0.0086*	0.0090*
			(0.0035)				(0.0038)	(0.0035)	(0.0037)
Nominal On/off ASW spread diff				0.0290**			0.0173**		
				(0.0010)			(0.0015)		
TIPS-nominal ASW spread diff					0.0187**			0.0177**	0.0180**
					(0.0005)			(0.0013)	(0.0017)
Inf swaps-BEI difference						0.0241**		-0.0036*	-0.0045**
						(0.0013)		(0.0015)	(0.0017)
Deflation floor adjustment									0.0045
									(0.0039)
Seasonality adjustment									0.0057*
									(0.0028)
VIX									0.0016
									(0.0016)
Nominal-TIPS repo spread									0.0007**
									(0.0002)
TIPS purchase dummy									-0.0118
									(0.0309)
No. of observations	337	341	341	341	341	341	341	337	326
Adjusted R^2	27.0%	20.5%	47.6%	71.1%	78.8%	51.2%	82.7%	86.3%	88.6%

Table 7: In-Sample Variance decomposition of TIPS Yields and TIPS BEI (Percent)

Table 7 reports the in-sample variance decompositions of TIPS yields into real yields, indexation lag effects, and TIPS liquidity premiums, and of nominal yields into expected inflation, inflation risk premiums, indexation lag effects, and the negative of TIPS liquidity premiums, all based on Model LII estimates. The variance decompositions are calculated as the in-sample covariance between TIPS yields (BEI) and the individual components, divided by the in-sample variance of TIPS yields (BEI). Standard errors calculated using the delta method are reported in parentheses.

	TIPS yield			TIPS BEI			
Maturity	real yield	indexation lag	liq prem	inf exp	inf risk prem	indexation lag	liq prem
5-year	83.1	0.6	16.3	38.5	7.0	5.9	48.6
	(5.5)	(1.2)	(5.4)	(2.4)	(6.8)	(3.3)	(6.3)
7-year	83.2	0.3	16.5	44.5	8.5	4.6	42.4
	(5.6)	(1.2)	(5.5)	(2.8)	(9.0)	(2.8)	(7.6)
10-year	84.0	0.1	15.9	49.6	11.1	3.5	35.9
	(5.7)	(1.2)	(5.6)	(5.2)	(11.8)	(2.5)	(9.4)

Table 8: Estimates of Inflation Risk Premiums and Expected Inflation

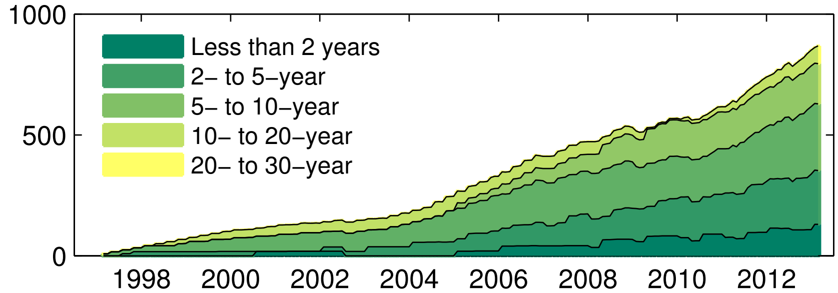
HPR estimates are obtained from the website of the Federal Reserve Bank of Cleveland. GH generate a range of estimates depending on the proxy used for expected inflation; the table reports the results using past average 5- and 10-year realized inflation as proxies for future 5- and 10-year expected inflation.

	ABW	BJ	CM	HPR	ABC	**DKW**	AACM	GH	FLL
Inf. Risk Prem. (basis points)									
5-year	114	42	NA	17	16	**19**	-17	-36	-4.5
10-year	NA	70	67	45	24	**29**	10	-12	2.8
Expected Inflation (percent)									
5-year	3.94	NA	NA	3.13	2.88	**2.73**	2.56	2.51	2.36
10-year	NA	NA	4.05	3.10	2.88	**2.77**	2.78	2.55	2.58
Data Used									
TIPS			Y			**Y**	Y	Y	
Inflation swaps				Y					Y
Survey inflation forecasts			Y	Y		**Y**			
Sample Period									
Start Year	1952	1960	1971	1982	1985	**1990**	1999	2000	2004
End Year	2004	2000	2008	2010	2011	**2013**	2013	2008	2014

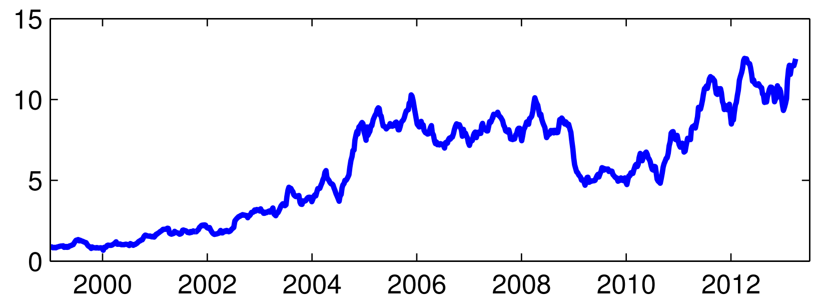
Figure 1: TIPS Outstanding, Transaction Volumes and Mutual Funds

Graph A of Figure 1 plots TIPS outstanding broken down by remaining maturities, based on data reported in the Treasury's Monthly Statement of the Public Debt (MSPD). Graph B plots the weekly TIPS transaction volumes, defined as 13-week moving average of weekly averages of daily TIPS transaction volumes reported by primary dealers in Government Securities Dealers Reports (FR-2004). Graph C plots number of TIPS mutual funds (right axis) and the total net assets under management (left axis) from the Investment Company Institute (http://www.ici.org).

Graph A. TIPS Outstanding (in billions of dollars)



Graph B. TIPS Transaction Volumes (in billions of dollars)



Graph C. Number of TIPS Mutual Funds and Assets under Management

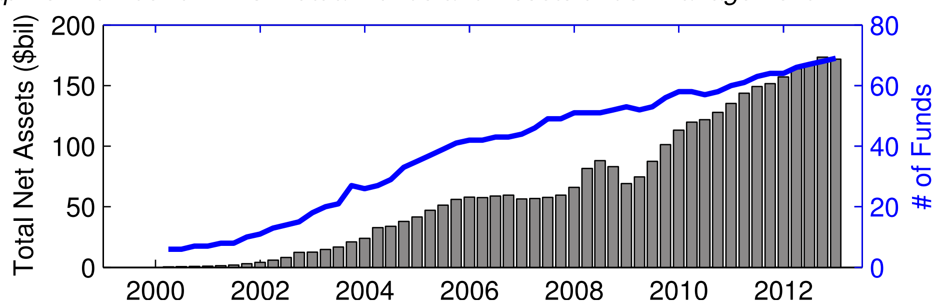


Figure 2: Survey Inflation Forecasts and TIPS Breakeven Inflation

Figure 2 shows the 10-year TIPS breakeven inflation (red line), long-horizon Michigan inflation forecast (blue line), and 10-year SPF inflation forecast (black pluses).

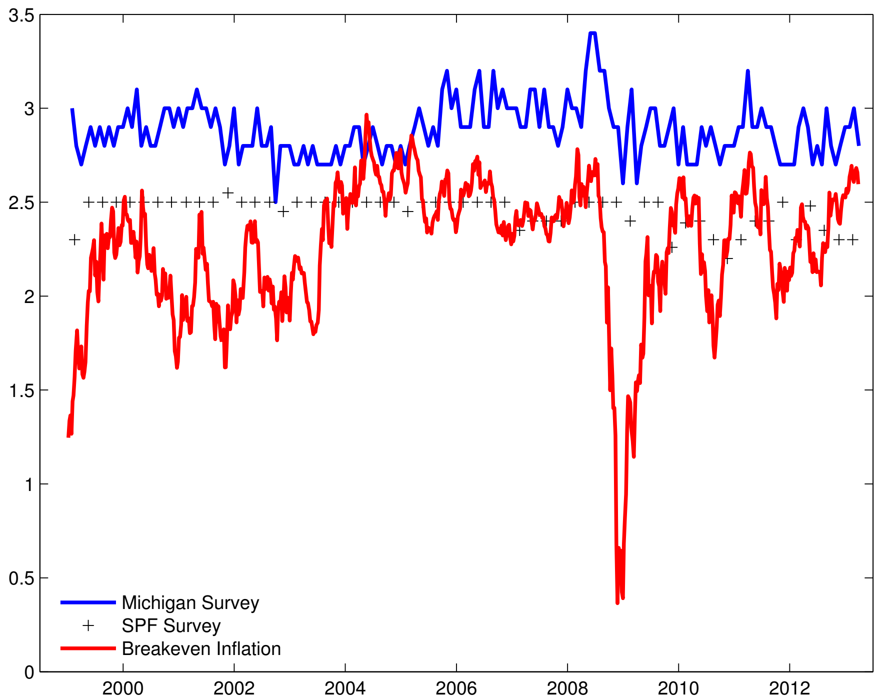
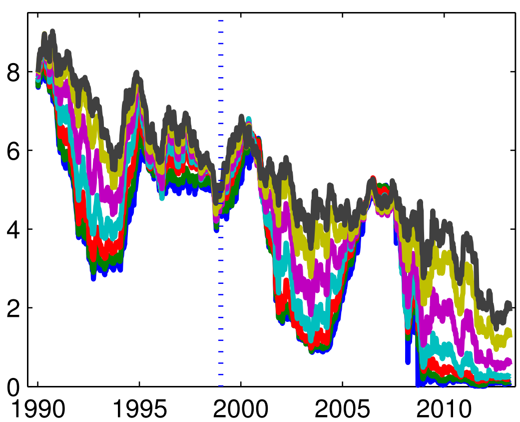


Figure 3: Nominal and TIPS Yields

Graph A of Table 3 plots the 3- and 6-month, 1-, 2-, 4-, 7- and 10-year nominal yields. Graph B plots the 5-, 7- and 10-year TIPS yields.

Graph A. Nominal Yields



Graph B. TIPS Yields

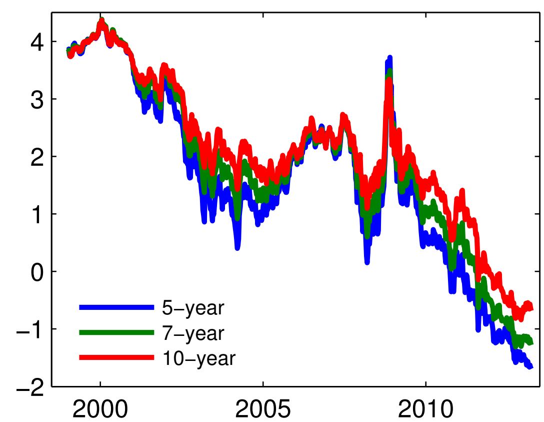


Figure 4: Model Fit of TIPS Yields and BEI

The three rows of Figure 4 plot results from Models NL-noIE, Model NL, and Model LII, respectively. The left graphs plot the 10-year actual TIPS yields (red), the 10-year model-implied TIPS yields (black) and the 10-year model-implied real yields (blue). The right graphs plots the 10-year actual TIPS breakevens (red), the 10-year model-implied TIPS breakevens (black) and the 10-year model-implied true breakevens (blue).

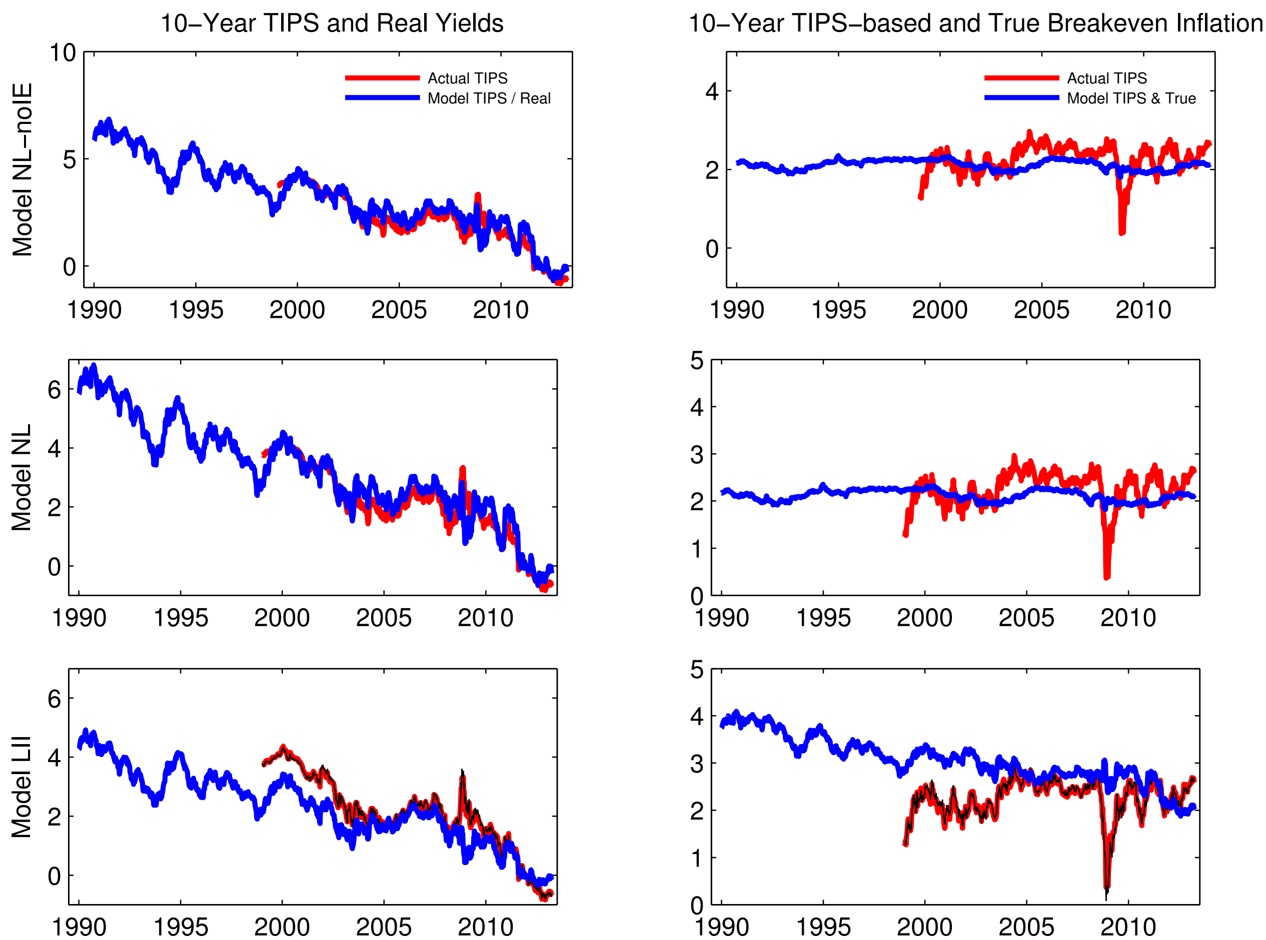


Figure 5: Model Implied Inflation Expectation and Inflation Risk Premiums

The three rows of Figure 5 plot results from Models NL-noIE, Model NL, and Model LII, respectively. The left and middle graphs plot 1- and 10-year model-implied inflation expectation together with the SPF counterpart, respectively. The right graphs plot the 1- and 10-year model-implied inflation risk premiums.

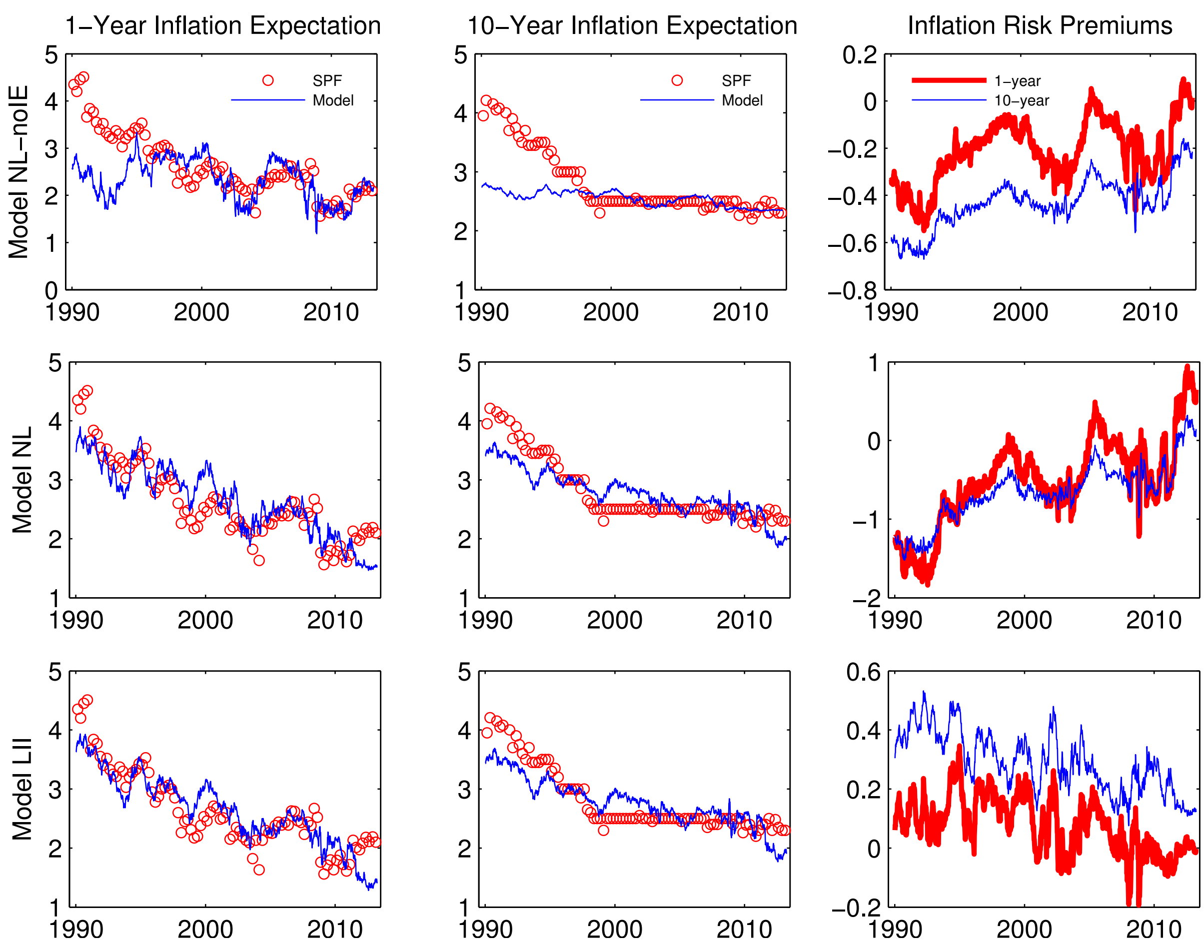


Figure 6: Model-Implied Yield Spreads

Graphs A and B of Figure 6 plot Model LII-implied differences between indexed bond yields and real yields and between TIPS yields and indexed bond yields, respectively, for maturities of 5, 7, and 10 years.

Graph A. Indexed Bond−Real Yield Differential *Graph B. TIPS−Indexed Bond Yield Differential*

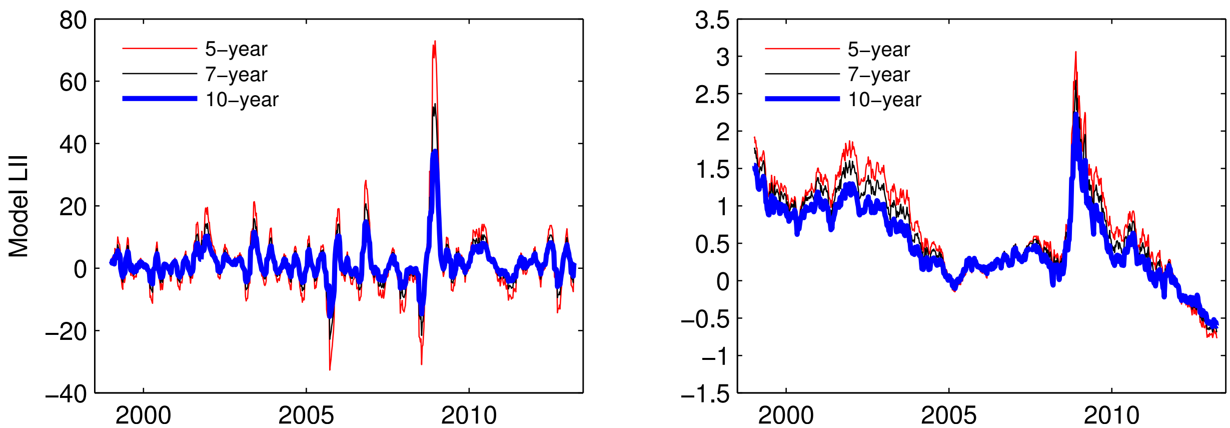


Figure 7: Observable Measures of TIPS Liquidity

Figure 7 plots various measures of liquidity conditions in the TIPS market, including the 10-year TIPS bid-ask spread (Graph A), the relative TIPS trading volumes relative to those in nominal Treasury coupon securities (Graph B), the average mean fitting errors from the Svensson TIPS yield curve (Graph C), the difference between the off-the-run and the on-the-run 10-year nominal Treasury par asset swap spreads (Graph D), the average difference between TIPS and nominal Treasury asset swap spreads (Graph E), and the difference between 10-year inflation swap rate and the 10-year BEI (Graph F).

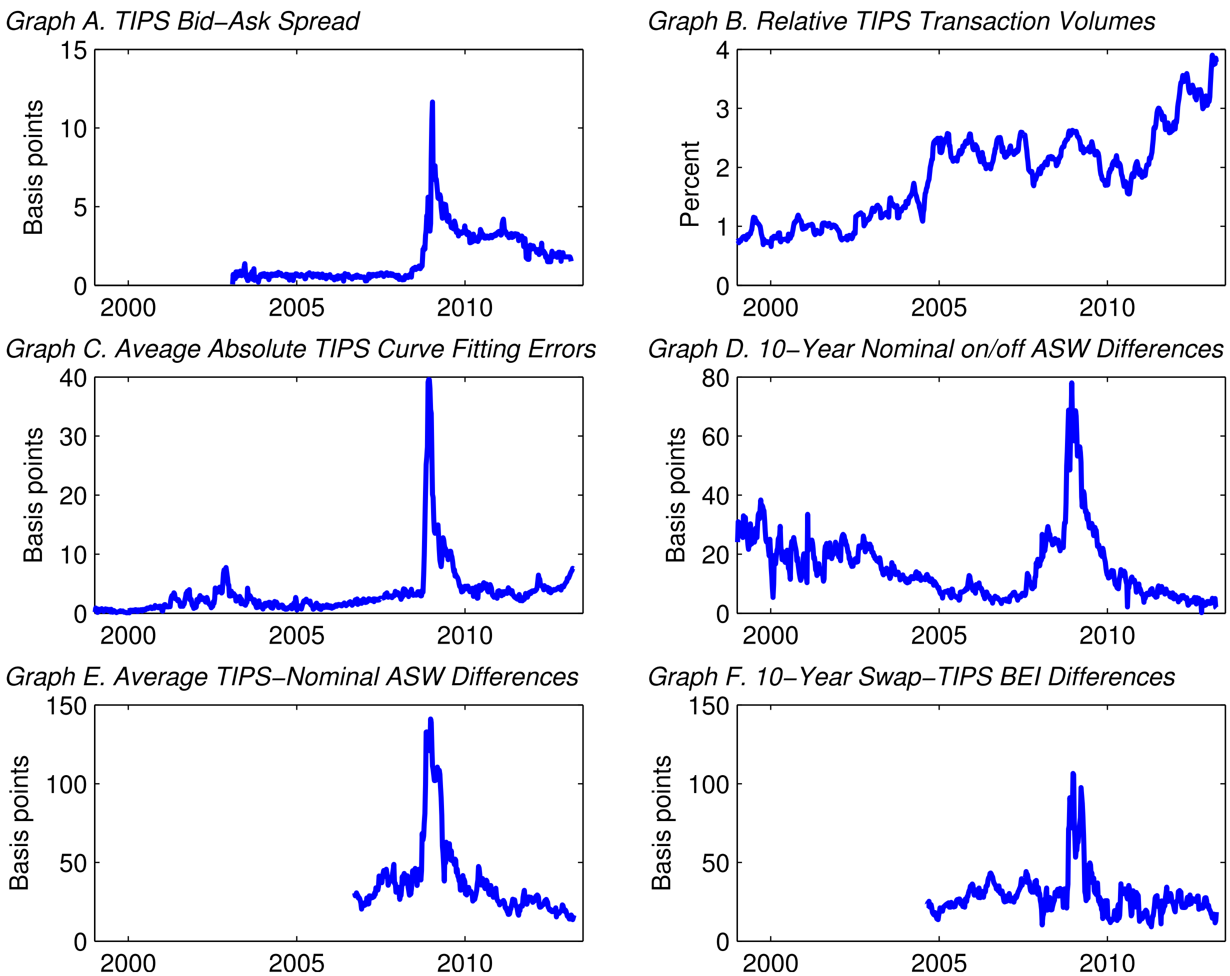


Figure 8: Other Potential Drivers of TIPS BEI

Figure 8 plots measures of other factors that might affect the TIPS BEI, including the incremental change in the 10-year TIPS yield after seasonality adjustment (Graph A) and adjustment for the deflation floor (Graph B), the probability of a flight-to-safety episode (Graph C), and the difference between the repo rates on 10-year on-the-run nominal Treasury notes and TIPS (Graph D).

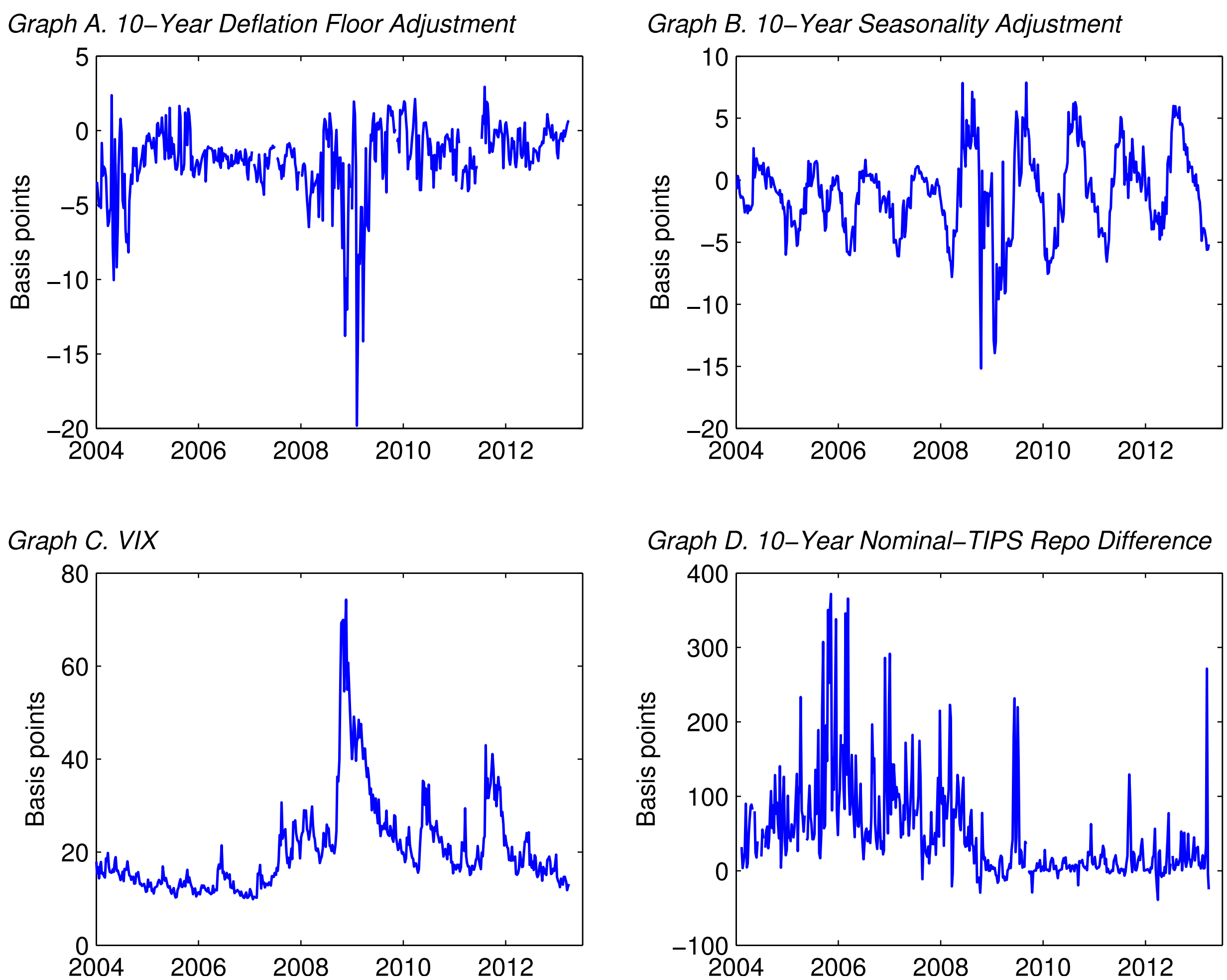


Figure 9: Contributions from TIPS Liquidity and Other Measures

Figure 9 shows the relative contributions from the liquidity and other measures based on Regression (9) in Table 6, Panel B. The blue line plots the Model LII-implied TIPS spread minus the regression constant, and the green and red lines plot the portion explained by TIPS liquidity measures and other measures, respectively.

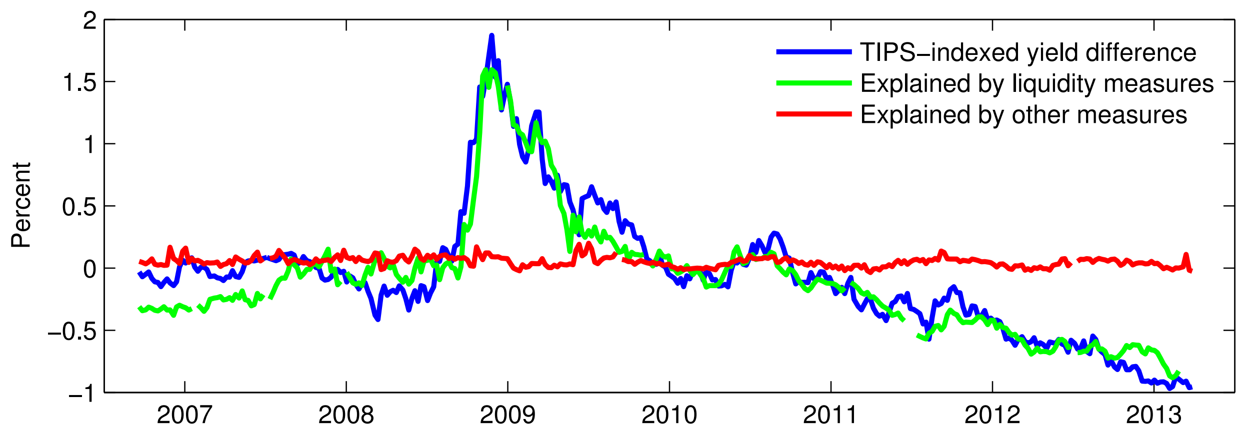
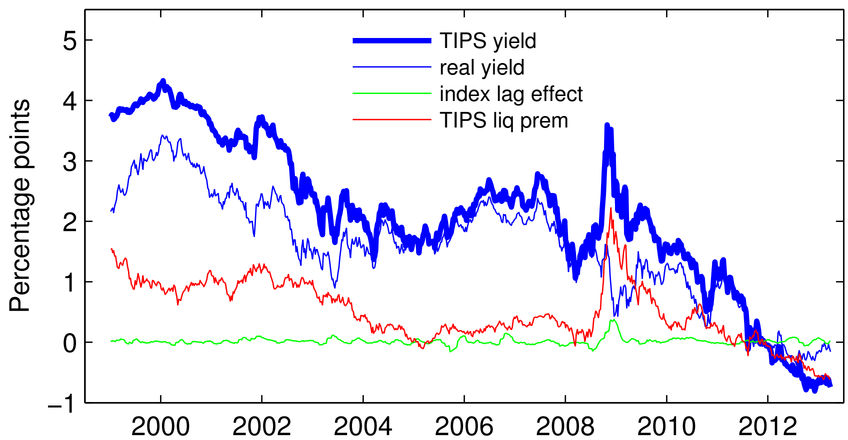


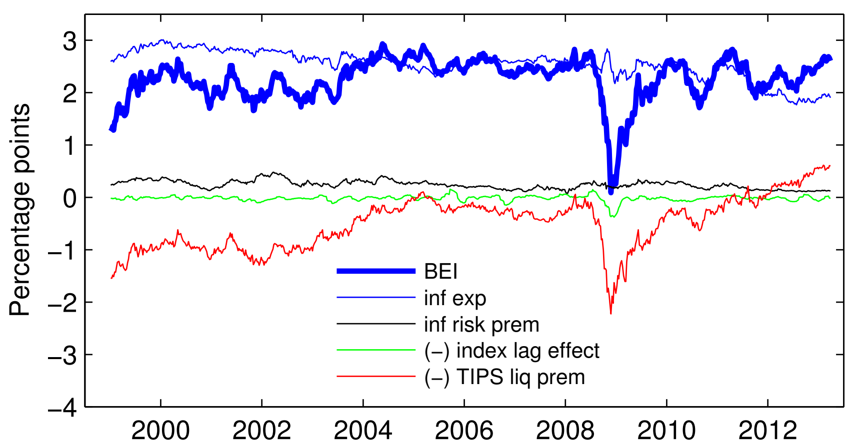
Figure 10: Decomposing TIPS Yields and TIPS Breakeven Inflation

Graph A of Figure 10 decomposes the 10-year TIPS yield into the real yield and the TIPS liquidity premium, while Graph B decomposes the 10-year TIPS BEI into expected inflation, the inflation risk premium and the TIPS liquidity premium, both according to equation (30).

Graph A. Decomposing 10–Year TIPS Yields



Graph B. Decomposing 10–Year Breakeven Inflation



Supplementary Appendix to

"Tips from TIPS: the Informational Content of Treasury Inflation-Protected Security Prices"

- Not intended for publication -

1 All Parameter estimates

Table 1 reports parameter estimates for all five models mentioned in the paper.

2 Additional Model Results

Figure 1 shows the results for Model LI, while Figure 2 shows the results of Model LII estimated over the pre-crisis period.

Figure 3 plots the yield fitting errors from Model LII. We see that the fit is generally worse for shorter-term nominal yields and during the crisis period.

3 Robustness Checks

3.1 Gaussian Assumption for Expected Inflation

It's now well known in the literature that, by allowing flexible correlations between the factors, the affine-Gaussian bond pricing model outperforms affine models with stochastic volatilities in matching term premium dynamics.[1] A similar argument can be made for using Gaussian models to study inflation risk premiums, which derive from the correlation between the real pricing kernel and inflation, even though such models by construction cannot capture *time-varying* inflation uncertainty and cannot decompose the inflation risk premium further into time-varying inflation risks and time-varying prices of inflation risk.

In addition to time-varying inflation uncertainties, recent studies of inflation caps and floors by Kitsul and Wright (2013) and Fleckenstein, Longstaff, and Lustig (2014a) find that investors appear to attach significantly more weight to extreme inflation outcomes (either deflation or high inflation) than a normal distribution would suggest. These observations raise some doubt on the appropriateness of modeling inflation as a conditional Gaussian process. Nevertheless, due to the short history of inflation caps and floors, both papers focus on a short sample dominated by the financial crisis and the zero lower bound period; it therefore remains to be seen whether the Gaussian assumption for inflation, both under the physical and the risk-neutral measures, works better over a longer time span as the one used in the current study.

We first examine the inflation distribution under the *physical* measure. Giordani and Söderlind (2003) analyze the probabilistic forecasts for inflation in the SPF over a long quarterly sample of 1969-2001. They find that for most years, the histograms are bell shaped, reasonably symmetric with most of the probability

[1] See Duffee (2002) and Dai and Singleton (2002), among others.

Table 1: Parameter Estimates

	Model NL-noIE		Model NL		Model LI		Model LII		Model LII-PC	
State Variables Dynamics										
$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t$										
$\mathcal{K}_{11}$	0.8550	(0.3533)	0.6849	(0.4589)	0.8302	(0.6993)	0.4317	(0.1622)	0.7358	(0.3542)
$\mathcal{K}_{22}$	0.1343	(0.0562)	0.1309	(0.0471)	0.1004	(0.0425)	0.0961	(0.0499)	0.0316	(0.0357)
$\mathcal{K}_{33}$	1.4504	(0.3633)	1.4259	(0.7216)	1.2353	(0.9516)	1.8425	(0.4757)	1.3386	(0.6951)
$100 \times \Sigma_{21}$	-0.7526	(0.5524)	-1.8236	(1.1939)	-1.1547	(0.8448)	-1.6133	(0.9020)	-0.6414	(0.2435)
$100 \times \Sigma_{31}$	-4.4450	(4.8007)	-4.8415	(8.4964)	-7.1258	(30.8741)	-1.7824	(0.9339)	-4.8511	(8.3985)
$100 \times \Sigma_{32}$	-0.9597	(0.2356)	-1.0313	(0.2948)	-1.0456	(0.4755)	-0.7864	(0.1713)	-0.5316	(0.2111)
Nominal Pricing Kernel										
$dM_t^N/M_t^N = -r^N(x_t)dt - \lambda(x_t)'dB_t$										,
$r^N(x_t) = \rho_0^N + \rho_1^{N'} x_t,\ \ \lambda(x_t) = \lambda_0^N + \Lambda^N x_t$										
ρ_0^N	0.0474	(0.0048)	0.0468	(0.0046)	0.0467	(0.0048)	0.0480	(0.0062)	0.0480	(0.0086)
$\rho_{1,1}^N$	3.6695	(3.0529)	4.9405	(5.2771)	6.2746	(22.3760)	2.4285	(0.7752)	3.2396	(5.1459)
$\rho_{1,2}^N$	0.8844	(0.1321)	0.9109	(0.1387)	0.8850	(0.2141)	0.7810	(0.0968)	0.4424	(0.0892)
$\rho_{1,3}^N$	0.7169	(0.0355)	0.7173	(0.0175)	0.7419	(0.0226)	0.7031	(0.0195)	0.6333	(0.0256)
$\lambda_{0,1}^N$	0.3241	(0.1606)	0.3270	(0.1484)	-0.0097	(0.2087)	0.1557	(0.1697)	0.2216	(0.2936)
$\lambda_{0,2}^N$	-0.4335	(0.1819)	-0.4019	(0.1533)	-0.3696	(0.2725)	-0.5355	(0.2110)	-0.4906	(0.4491)
$\lambda_{0,3}^N$	-1.2754	(0.3726)	-1.2417	(0.3888)	-1.1435	(0.3903)	-1.3591	(0.4438)	-1.5077	(2.1659)
$[\Sigma\Lambda^N]_{11}$	-0.6953	(0.9033)	-0.6529	(1.5192)	0.6238	(2.4162)	-0.0138	(0.1295)	-0.3677	(1.1468)
$[\Sigma\Lambda^N]_{21}$	2.1331	(2.6939)	2.4644	(4.7106)	0.5454	(3.2112)	0.2964	(0.5159)	1.1200	(2.8701)
$[\Sigma\Lambda^N]_{31}$	3.0734	(6.4541)	3.8734	(13.5898)	-1.0467	(22.6787)	0.1262	(0.5179)	3.6061	(12.9735)
$[\Sigma\Lambda^N]_{12}$	0.0339	(0.0409)	0.0650	(0.0583)	-0.0732	(0.0474)	-0.0223	(0.0650)	-0.0827	(0.1901)
$[\Sigma\Lambda^N]_{22}$	-0.1447	(0.0233)	-0.2128	(0.0531)	-0.0458	(0.0418)	-0.1151	(0.0731)	-0.1613	(0.1064)
$[\Sigma\Lambda^N]_{32}$	-0.3576	(0.3013)	-0.6065	(0.9297)	0.3068	(1.8944)	-0.1980	(0.1120)	-0.4788	(0.5115)
$[\Sigma\Lambda^N]_{13}$	-0.0809	(0.1141)	-0.1135	(0.1026)	0.1866	(0.4329)	0.0790	(0.1603)	-0.0369	(0.2512)
$[\Sigma\Lambda^N]_{23}$	0.6000	(0.1980)	0.7232	(0.3218)	0.0875	(0.1439)	0.5394	(0.2512)	0.4512	(0.2111)
$[\Sigma\Lambda^N]_{33}$	0.1553	(0.8626)	0.3736	(1.8229)	-1.0059	(2.1457)	-0.4945	(0.4766)	0.7245	(1.5411)
Log Price Level										
$d\log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^\perp dB_t^\perp,\ \pi(x_t) = \rho_0^\pi + \rho_1^{\pi'} x_t$										
ρ_0^π	0.0262	(0.0016)	0.0285	(0.0015)	0.0294	(0.0021)	0.0288	(0.0026)	0.0278	(0.0079)
$\rho_{1,1}^\pi$	-0.0326	(0.5805)	-0.4711	(1.7446)	-0.5261	(4.3530)	0.1582	(0.3076)	0.3895	(0.5149)
$\rho_{1,2}^\pi$	0.0867	(0.0578)	0.2378	(0.0400)	0.3515	(0.0849)	0.2684	(0.0300)	0.3883	(0.0376)
$\rho_{1,3}^\pi$	-0.2213	(0.1859)	-0.2804	(0.1584)	-0.1999	(0.2596)	-0.1356	(0.1442)	0.0485	(0.0845)
$100 \times \sigma_{q,1}$	-0.0796	(0.0445)	0.0038	(0.0734)	0.0000	(0.1009)	-0.1495	(0.0409)	-0.0815	(0.0585)
$100 \times \sigma_{q,2}$	0.0066	(0.0673)	0.0869	(0.0739)	0.1625	(0.0620)	0.0763	(0.0581)	0.0575	(0.0581)
$100 \times \sigma_{q,3}$	-0.0278	(0.0589)	-0.2586	(0.0459)	-0.1526	(0.0674)	0.0224	(0.0619)	0.0154	(0.0533)
$100 \times \sigma_q^\perp$	0.9229	(0.0268)	0.9461	(0.0300)	0.9508	(0.0346)	0.8975	(0.0264)	0.7018	(0.0213)

Table 1 Continued

	Model NL-noIE		Model NL		Model LI		Model LII		Model NL-PreCrisis	
TIPS Liquidity Premium										
$l_t = \tilde{\gamma}\tilde{x}_t + \gamma' x_t,\ d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma} dW_t,\ \tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{x}_t.$										
$\tilde{\gamma}$					0.8376	(0.0224)	0.8393	(0.0225)	0.5427	(0.0344)
$\tilde{\kappa}$					0.5097	(0.2113)	0.4900	(0.2051)	0.1936	(0.2416)
$\tilde{\mu}$					0.0067	(0.0049)	0.0077	(0.0050)	0.0167	(0.0122)
$\tilde{\lambda}_0$					0.3754	(0.3571)	0.4136	(0.3413)	0.2847	(0.5339)
$\tilde{\sigma}\tilde{\lambda}_1$					-0.3981	(0.2114)	-0.3770	(0.2052)	-0.1041	(0.2412)
γ_1							-0.8403	(0.2826)	-0.3915	(0.5743)
γ_2							-0.0527	(0.1024)	0.1032	(0.0802)
γ_3							0.0121	(0.2293)	-0.0000	(0.1607)
Measurement Errors: Nominal Yields										
$100 \times \delta_{N,3m}$	0.1314	(0.0020)	0.1314	(0.0020)	0.1311	(0.0021)	0.1312	(0.0021)	0.1028	(0.0027)
$100 \times \delta_{N,6m}$	0.0188	(0.0015)	0.0192	(0.0015)	0.0211	(0.0015)	-0.0212	(0.0014)	-0.0215	(0.0017)
$100 \times \delta_{N,1y}$	0.0655	(0.0022)	0.0655	(0.0022)	0.0653	(0.0022)	0.0653	(0.0022)	0.0529	(0.0018)
$100 \times \delta_{N,2y}$	0.0000	(51.7227)	0.0000	(9.0140)	0.0000	(3995.5010)	0.0000	(4062.7066)	-0.0000	(104.5475)
$100 \times \delta_{N,4y}$	0.0397	(0.0016)	0.0396	(0.0016)	0.0396	(0.0016)	0.0396	(0.0016)	0.0292	(0.0012)
$100 \times \delta_{N,7y}$	0.0000	(150.5043)	-0.0000	(100.1489)	0.0000	(4423.6406)	0.0000	(5024.8333)	-0.0000	(148.9753)
$100 \times \delta_{N,10y}$	0.0530	(0.0015)	0.0529	(0.0015)	0.0533	(0.0015)	0.0530	(0.0015)	0.0487	(0.0018)
Measurement Errors: TIPS Yields										
$100 \times \delta_{\mathcal{T},5y}$	0.5374	(0.0801)	0.5400	(0.0785)	0.0806	(0.0033)	0.0812	(0.0033)	0.0642	(0.0047)
$100 \times \delta_{\mathcal{T},7y}$	0.4217	(0.0849)	0.4231	(0.0843)	-0.0000	(6302.1210)	-0.0000	(6307.8897)	0.0000	(26.1627)
$100 \times \delta_{\mathcal{T},10y}$	0.3879	(0.0632)	0.3874	(0.0605)	0.0653	(0.0033)	-0.0644	(0.0033)	-0.0610	(0.0050)
Measurement Errors: Survey Forecasts of Nominal Short Rate										
$100 \times \delta_{f,6m}$	0.1890	(0.0146)	0.1893	(0.0146)	0.1872	(0.0141)	0.1891	(0.0146)	0.1654	(0.0137)
$100 \times \delta_{f,12m}$	0.2965	(0.0222)	0.2945	(0.0218)	0.2944	(0.0219)	0.2968	(0.0224)	0.2225	(0.0203)

This table reports parameter estimates and standard errors for all five models we estimate. Standard errors are calculated using the BHHH formula and are reported in parentheses.

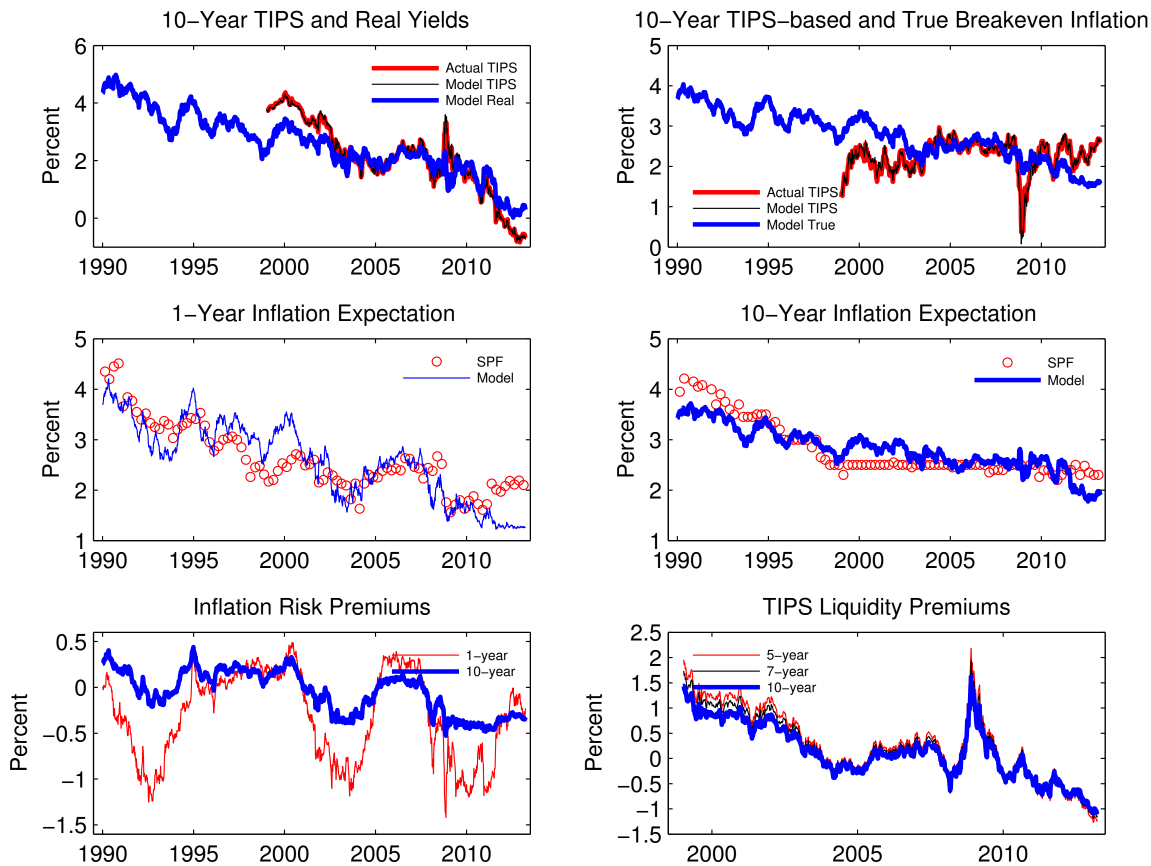


Figure 1: Model LI

The top left panel plot the 10-year actual TIPS yields (red), the 10-year model-implied TIPS yields (black) and the 10-year model-implied real yields (blue). The top right panel plots the 10-year actual TIPS breakevens (red), the 10-year model-implied TIPS breakevens (black) and the 10-year model-implied true breakevens (blue). The middle panels plot the 1- and 10-year model-implied inflation expectation, respectively, together with their survey counterparts from the SPF. The bottom left panel plot the 1- and 10-year model-implied inflation risk premiums. The bottom right panel plot the 5-, 7-, and 10-year model-implied TIPS-indexed bond yield differences.

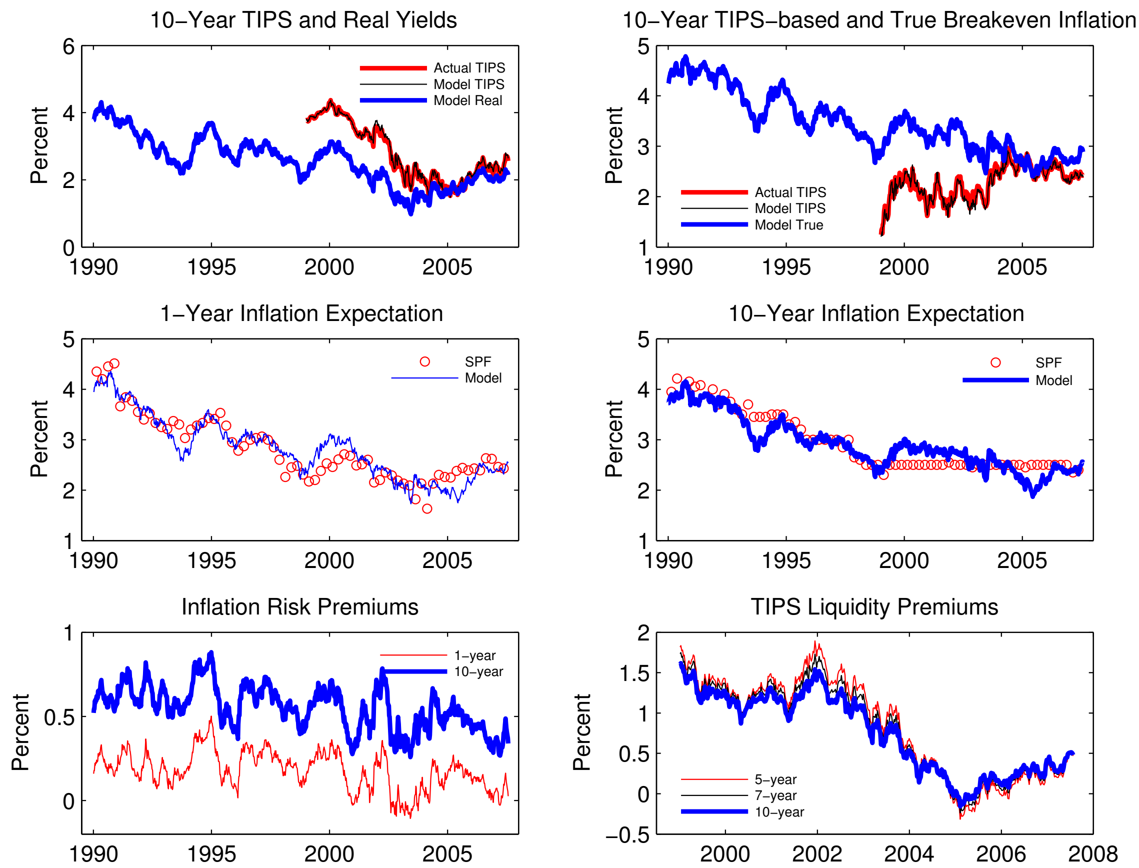
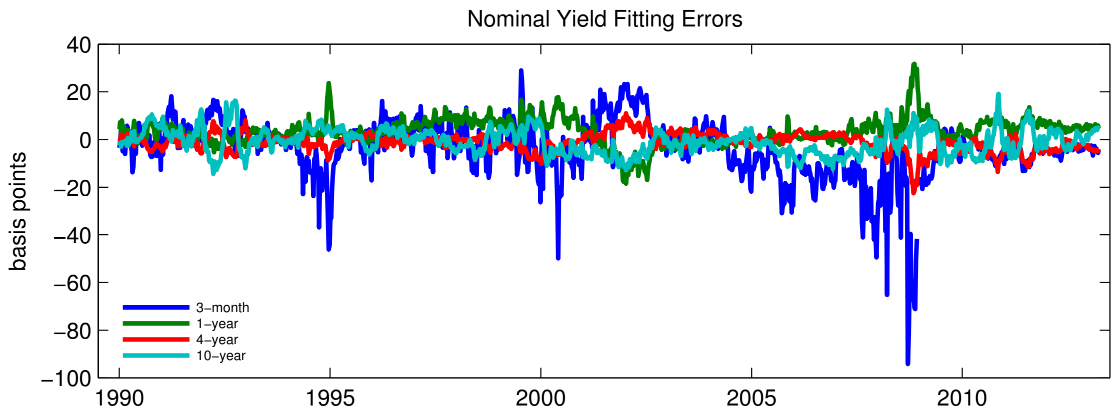


Figure 2: Model LII Estimated over the Pre-Crisis Period

The top left panel plot the 10-year actual TIPS yields (red), the 10-year model-implied TIPS yields (black) and the 10-year model-implied real yields (blue). The top right panel plots the 10-year actual TIPS breakevens (red), the 10-year model-implied TIPS breakevens (black) and the 10-year model-implied true breakevens (blue). The middle panels plot the 1- and 10-year model-implied inflation expectation, respectively, together with their survey counterparts from the SPF. The bottom left panel plot the 1- and 10-year model-implied inflation risk premiums. The bottom right panel plot the 5-, 7-, and 10-year model-implied TIPS-indexed bond yield differences.



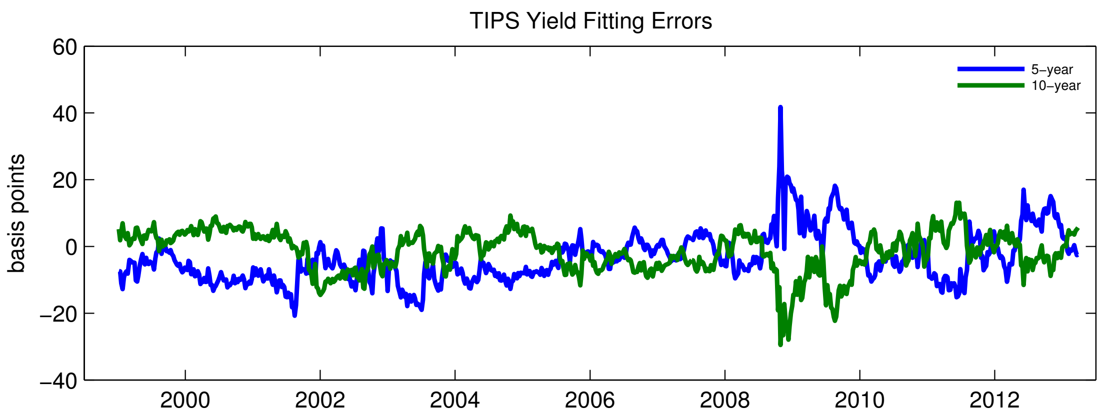


Figure 3: Time Series of Yield Fitting Errors from Model LII

This chart plots the time series of fitting errors on nominal yields (top panel) and TIPS yields (bottom panel) based on Model LII.

mass concentrated in interior intervals, suggesting that the normal distribution provides a good approximation to the physical distribution of inflation in those years. An update of their results using the same methodology for each year from 1992 to 2013, shown in Figures 4 and 5, demonstrates that the normal distribution continues to provide a reasonable approximation to the physical distribution of inflation forecasts over recent years. This is consistent with the findings in Kitsul and Wright (2013): Figures 8 and 9 of their paper show that even during the years of 2010-2012, a period that was dominated by deflation scares, the physical distribution of expected inflation remains reasonably symmetric and assign much lower odds to tail outcomes than the corresponding options-implied PDFs.[2]

To formally test the normality of each distribution shown in Figure 4 and 5, we use the χ^2 statistic described in Lahiri and Teigland (1987). The values of this statistic for one- and two-year ahead forecasts are reported in the third and fifth columns of Table 2, respectively. The associated levels of significance indicate that we reject the normality assumption for 13 out of 22 distributions (60% of the time) at the one-year horizon and for 9 out of 22 distributions (40% of time) at the two-year horizon. We interpret the results as suggesting that, despite the crude approximation of the true distribution using a few bins and the sensitivity of the test to the treatment of the open intervals, the normality distribution can be thought of as a reasonable approximation about half of the time over this period and more so for longer forecast horizons.

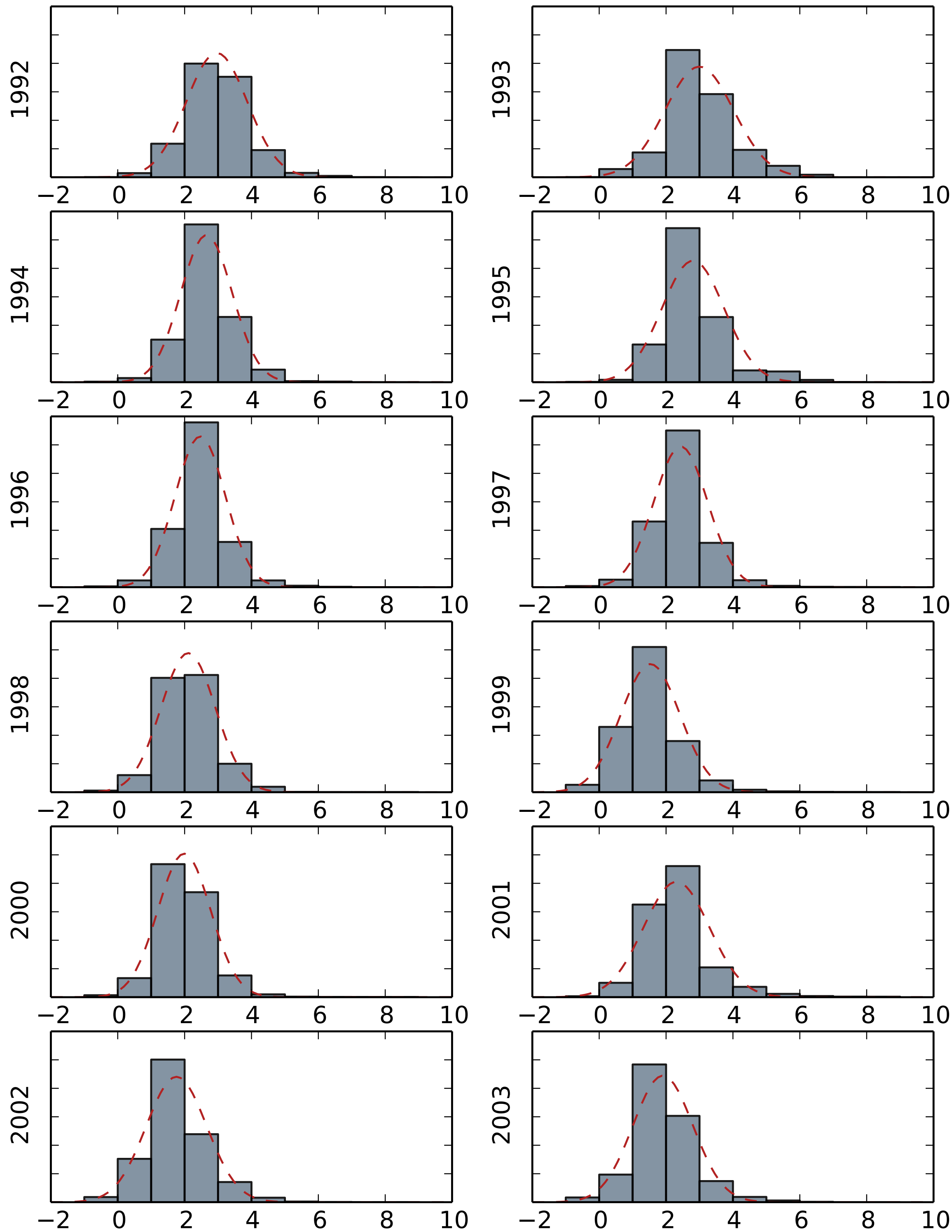
Turning to *risk-neutral* distributions, Figure 6 plot the skewness and excess kurtosis of risk-neutral distributions of inflation over the next one, five, and ten years, constructed from zero-coupon inflation caps using a similar model-free methodology as in Kitsul and Wright (2013). The caps-implied skewness was notably negative at 5- and 10-year horizons in the immediate aftermath of the crisis, but have hovered around zero since late 2010 despite lingering worries about deflation. Similarly, the excess kurtosis was significantly positive between late 2009 and late 2010, suggesting investors perceived higher risks of tail inflation outcomes than implied by a normal distribution. The excess kurtosis had also largely dissipated by late 2010, although more recently it has drifted up again for the 5-year horizon.

Overall, the Gaussian model seems to be a more reasonable approximation of inflation dynamics over a long sample period like ours, although its inability to capture time-varying volatilities, asymmetric distributions, or heavy tails can be more problematic for periods with heightened deflation concerns such as 2009-2010, which nonetheless constitutes only a small part of our sample period. We therefore view the general affine-Gaussian model as an important benchmark to investigate before exploring more sophisticated models.

3.2 Parameter Stability

The literature has documented significant market dislocations in the nominal Treasury/TIPS market during the 2008 financial crisis (see Campbell, Shiller, and Viceira (2009) and Fleckenstein, Longstaff, and Lustig (2014b), among others). We therefore re-estimate Model LII over a pre-crisis sample ending on July 25, 2007. As can be seen from Table 1, the parameter estimates are very similar to those from Model LII estimated over the full sample. A comparison between Figure 2 in this appendix and the bottom panels of

[2]Those PDFs are constructed using two different models: the unobserved component stochastic volatility model of Stock and Watson (2007) and the time-varying-parameter VAR model with stochastic volatility of Primiceri (2005).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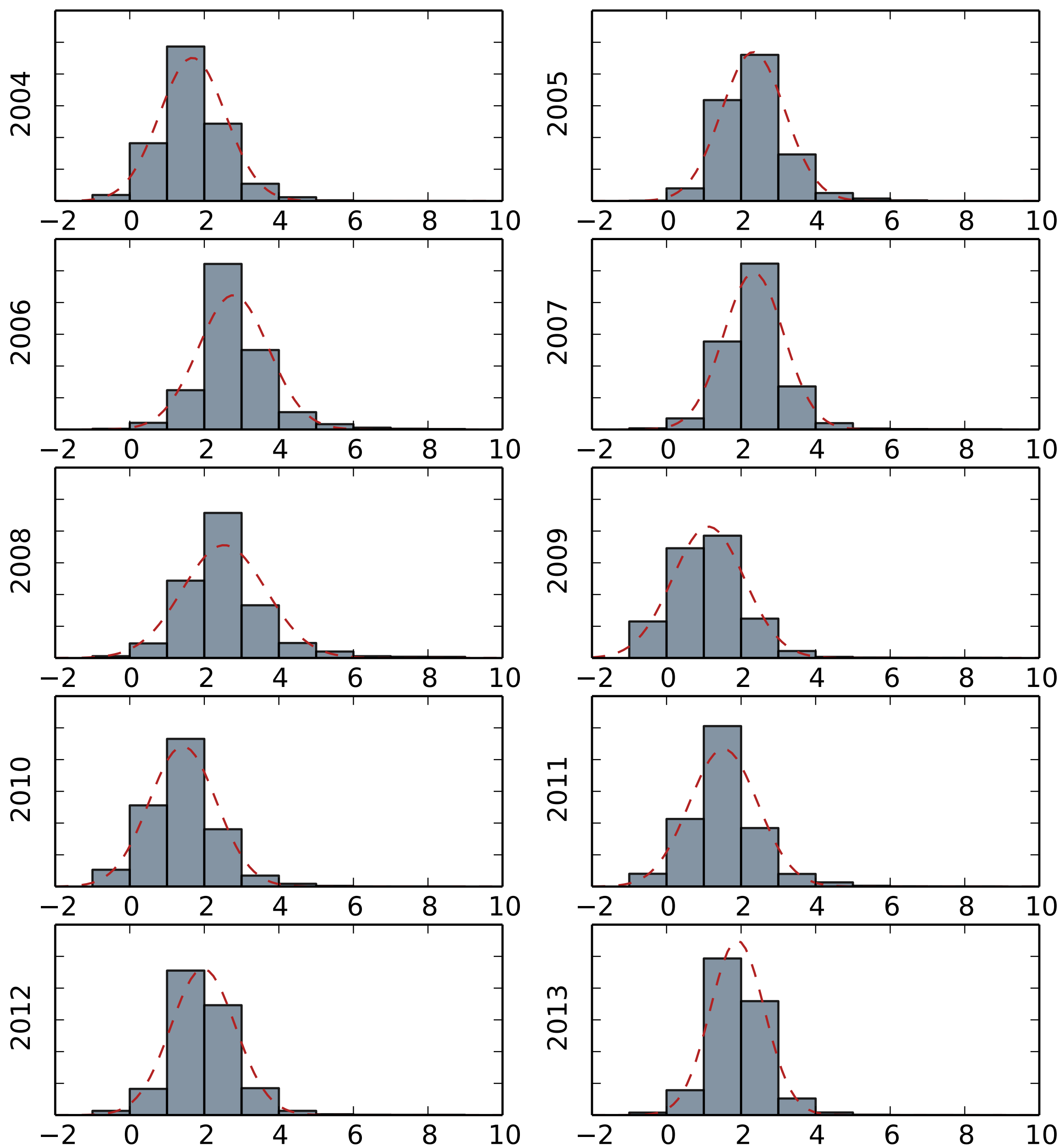
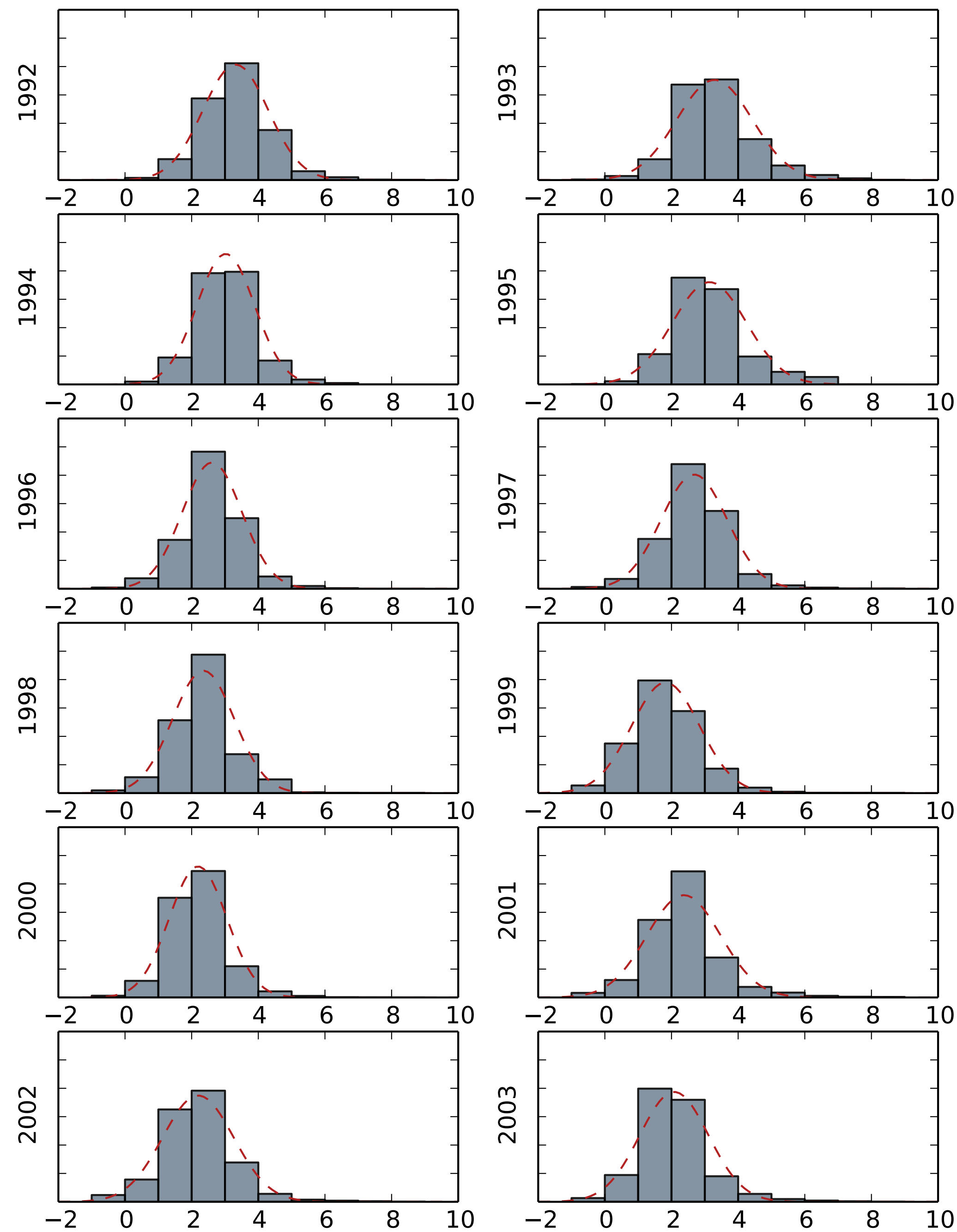


Figure 4: Distribution of 1-Year Ahead Expected Inflation

Histograms of 1-year ahead inflation forecasts from the SPF and the fitted distributions.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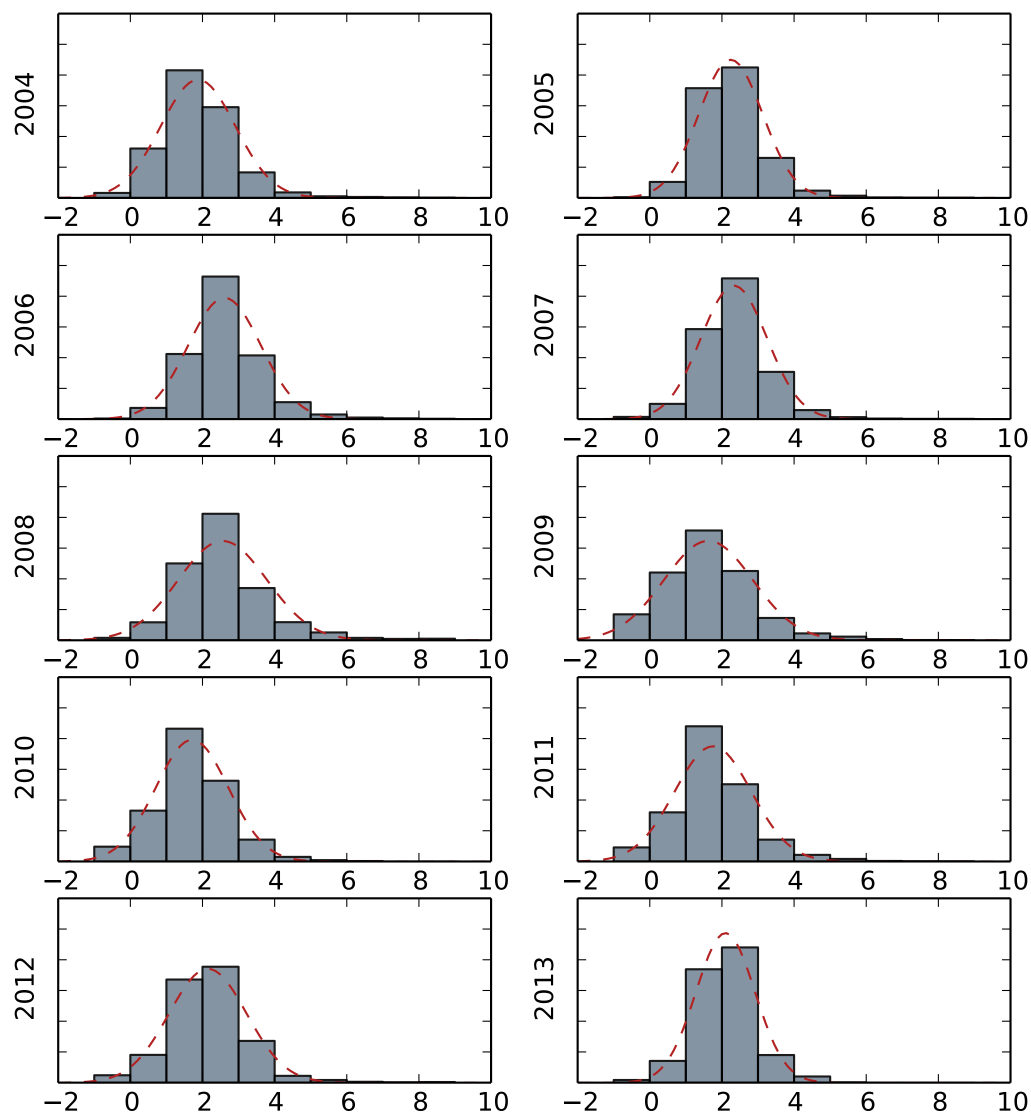

Figure 5: Distribution of 2-Year Ahead Expected Inflation
Histograms of 2-year ahead inflation forecasts from the SPF and the fitted distributions.

Table 2: Normality Test of SPF Forecasts

Year	No. of Forecasts	1-year test stat	1-year p-value	2-year test stat	2-year p-value
1992	34	0.04	1.00	0.08	1.00
1993	31	0.11	1.00	0.10	1.00
1994	26	0.26	1.00	0.08	1.00
1995	26	0.33	1.00	0.14	1.00
1996	36	0.45	1.00	0.06	1.00
1997	35	8419.37	0.00	10.46	0.16
1998	29	22185.89	0.00	43.15	0.00
1999	28	57274.68	0.00	38.78	0.00
2000	33	194851.32	0.00	1006.51	0.00
2001	29	194.97	0.00	7.96	0.34
2002	30	4377.78	0.00	7.60	0.37
2003	33	0.35	1.00	27.12	0.00
2004	27	26149.43	0.00	216.25	0.00
2005	31	7.80	0.35	400.02	0.00
2006	50	33.64	0.00	7.94	0.34
2007	42	21526.65	0.00	273.61	0.00
2008	41	11.61	0.11	3.19	0.87
2009	38	66105.14	0.00	1.33	0.99
2010	40	0.18	1.00	0.13	1.00
2011	41	15830.11	0.00	21.89	0.00
2012	41	16414.07	0.00	53.26	0.00
2013	41	20.79	0.00	0.13	1.00

This table reports the χ^2 test statistics and the associated p-values for one- and two-year ahead forecasts. The p-values are calculated as the probability of a χ^2-distributed variable with 7 degrees of freedom exceeding the actual test statistic under the null hypothesis of normality.

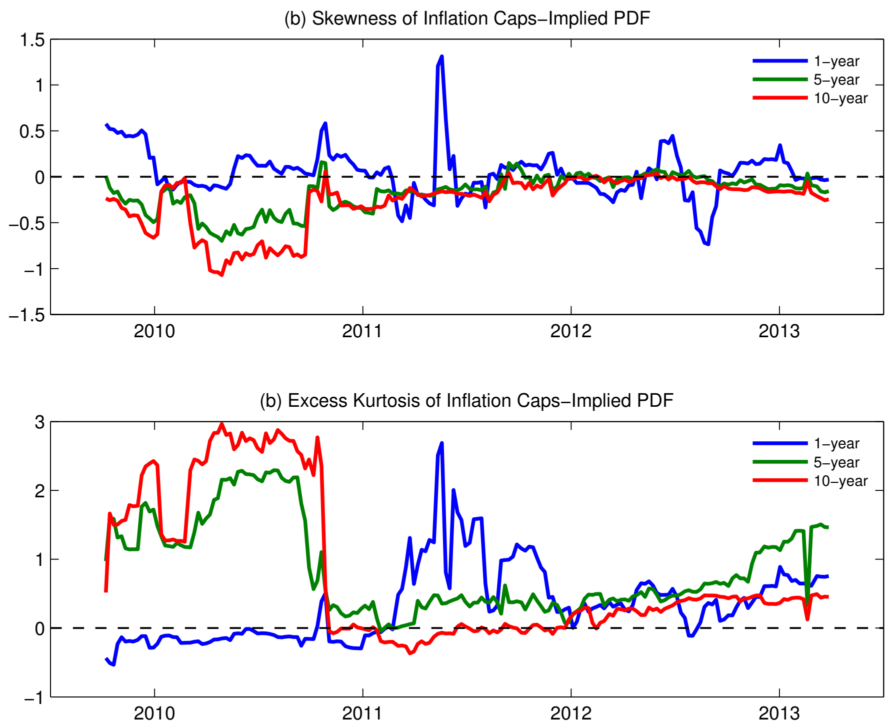

Figure 6: Skewness and Kurtosis from Inflation Caps

Panels (a) and (b) plot the skewness and excess kurtosis, respectively, of the 1-, 5-, and 10-year inflation probability distributions constructed from inflation caps.

Figures 4 and 5 in the paper shows that the model-implied real yields, inflation expectations, inflation risk premiums, and the difference between TIPS yields and indexed bond yields are almost identical to what the full-sample Model LII predicts for the same period.

4 Decomposing Nominal Yields

Although it is not the focus of the current paper, our models can also be used to separate nominal yields into real yields, expected inflation and inflation risk premiums:

$$y_{t,\tau}^{N} = y_{t,\tau}^{R} + I_{t,\tau} + \wp_{t,\tau}. \tag{1}$$

Figure 7 plots 1- and 10-year nominal yields and their constituents, whereas Table 3 reports the variance decomposition results.

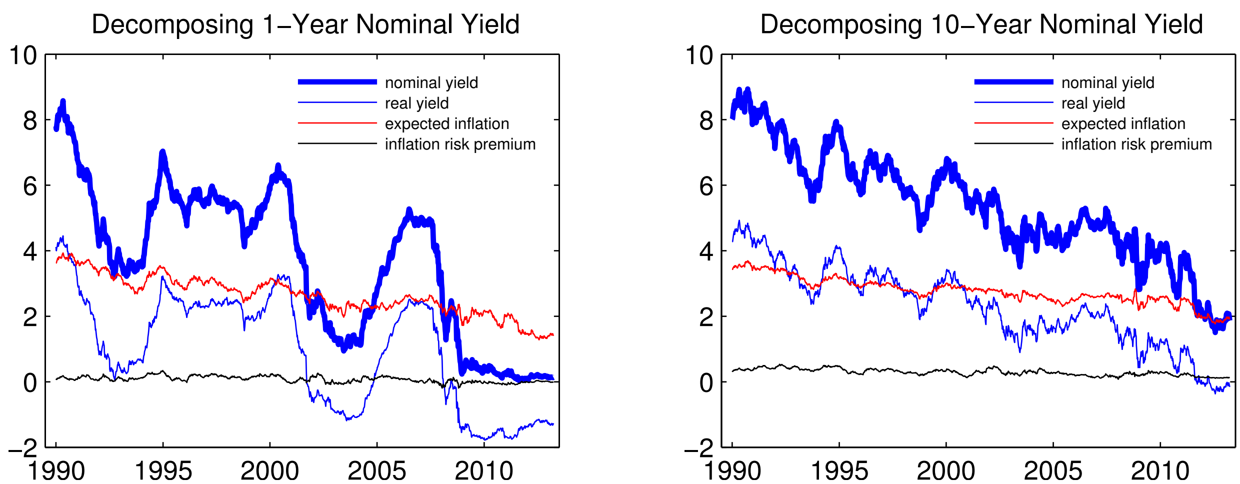

Figure 7: Decomposing Nominal Yields

The two panels decompose 1- and 10-year nominal yields into real yields, expected inflation and inflation risk premiums according to Equation (1).

These results indicate that, at least during our sample period, real yield changes explain more than three quarters of the variations in nominal yields at all maturities. Inflation expectation explains about 20% (10%) of the variations in the 1-quarter (10-year) nominal yield. Inflation risk premiums account for the remaining 2-10% of the nominal yield changes. This stands in contrast to previous studies using a longer sample period but not using TIPS yields, which typically find relatively smooth real yields but volatile inflation expectation or inflation risk premiums.[3] The limited evidence we have so far from TIPS seems to suggest that real yields may also vary considerably over time.

[3] See Ang, Bekaert, and Wei (2008, Figure 2) and Chernov and Mueller (2012, Figure 7) for example.

Table 3: In-Sample Variance decomposition of Nominal Yields

Maturity	real yield	inf exp	inf risk prem
1-quarter	0.7639	0.2214	0.0147
	(0.1078)	(0.1039)	(0.0193)
1-year	0.7743	0.2032	0.0224
	(0.1101)	(0.0987)	(0.0246)
5-year	0.7852	0.1716	0.0433
	(0.1262)	(0.0943)	(0.0579)
10-year	0.7850	0.1488	0.0663
	(0.1326)	(0.0884)	(0.0720)

Note: This table reports the in-sample variance decompositions of nominal yields into real yields, expected inflation, the inflation risk premiums, all based on Model LII estimates. The variance decomposition is calculated according to

$$1 = \frac{cov\left(y^N_{t,\tau}, y^R_{t,\tau}\right)}{var\left(y^N_{t,\tau}\right)} + \frac{cov\left(y^N_{t,\tau}, I_{t,\tau}\right)}{var\left(y^N_{t,\tau}\right)} + \frac{cov\left(y^N_{t,\tau}, \wp^I_{t,\tau}\right)}{var\left(y^N_{t,\tau}\right)}.$$

Standard errors calculated using the delta method are reported in parentheses.

5 Davies (1987) Likelihood Ration Test Statistic

This section describes the details in constructing the Davies (1987) Likelihood Ration Test Statistic mentioned in Section A. Denote by θ the vector of nuisance parameters of size s, and define the likelihood ratio statistic as a function of θ:

$$LR\left(\theta\right) = 2\left[\log L_1\left(\theta\right) - \log L_0\right],$$

where $L_1\left(\theta\right)$ is the likelihood value of the alternative model for any admissible values of the nuisance parameters $\theta \in \Omega$, and L_0 is the maximized likelihood value of the null model. For an estimated LR value of M, Davies (1987) derives an upper bound for its significance as

$$\Pr\left[\sup_{\theta\in\Omega} LR\left(\theta\right) > M\right] < \Pr\left[LR\left(\theta\right) > M\right] + VM^{\frac{1}{2}(s-1)} \exp^{-(M/2)} \frac{2^{-s/2}}{\Gamma\left(s/2\right)}$$

where $\Gamma\left(.\right)$ represents the Gamma function and V is defined as

$$V = \int_\Omega \left|\frac{\partial LR\left(\theta\right)}{\partial\theta}\right| d\theta.$$

Garcia and Perron (1996) further assumes that the likelihood ratio statistic has a single peak at $\widehat{\theta}$, which reduces V to $2M^{\frac{1}{2}}$.

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