Credit Spreads and Monetary Policy*

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Abstract

We consider the desirability of modifying a standard Taylor rule for a central bank's interest-rate policy to incorporate either an adjustment for changes in interest-rate spreads (as proposed by Taylor, 2008, and by McCulley and Toloui, 2008) or a response to variations in the aggregate volume of credit (as proposed by Christiano *et al.*, 2007). We consider the consequences of such adjustments for the way in which policy would respond to a variety of types of possible economic disturbances, including (but not limited to) disturbances originating in the financial sector that increase equilibrium spreads and contract the supply of credit. We conduct our analysis using the simple DSGE model with credit frictions developed in Cúrdia and Woodford (2009), and compare the equilibrium responses to a variety of disturbances under the modified Taylor rules to those under a policy that would maximize average expected utility. According to our model, either type of adjustment, if of an appropriate magnitude, can improve equilibrium responses to disturbances originating in the financial sector. However, neither simple rule of thumb is ideal even in this case, and the specific adjustments that would best improve the response to purely financial disturbances are less desirable in the case of other types of disturbances.

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The recent turmoil in financial markets has confronted the central banks of the world with a number of unusual challenges. To what extent do standard approaches to the conduct of monetary policy continue to provide reasonable guidelines under such circumstances? For example, the Federal Reserve aggressively reduced its operating target for the federal funds rate in late 2007 and January 2008, though official statistics did not yet indicate that real GDP was declining, and according to many indicators inflation was if anything increasing; a simple "Taylor rule" (Taylor, 1993) for monetary policy would thus not seem to have provided any ground for the Fed's actions at the time. Obviously, they were paying attention to other indicators than these ones alone, some of which showed that serious problems had developed in the financial sector.[1] But does a response to such additional variables make sense as a general policy? Should it be expected to lead to better responses of the aggregate economy to disturbances more generally?

Among the most obvious indicators of stress in the financial sector since August 2007 have been the unusual increases in (and volatility of) the spreads between the interest rates at which different classes of borrowers are able to fund their activities.[2] Indeed, McCulley and Toloui (2008) and Taylor (2008) have proposed that the intercept term in a "Taylor rule" for monetary policy should be adjusted downward in proportion to observed increases in spreads. Similarly, Meyer and Sack (2008) propose, as a possible account of recent U.S. Federal Reserve policy, a Taylor rule in which the intercept — representing the Fed's view of "the equilibrium real funds rate" — has been adjusted downward in response to credit market turmoil, and use the size of increases in spreads in early 2008 as a basis for a proposed magnitude of the appropriate adjustment. A central objective of this paper is to assess the degree to which a modification of the classic Taylor rule of this kind would generally improve the way in which the economy responds to disturbances of various sorts, including in particular to those originating in the financial sector. Our model also sheds light on the question whether it is correct to say that the "natural" or "neutral" rate of interest is lower when credit spreads increase (assuming unchanged fundamentals otherwise), and to the extent that it is, how the size of the change in the natural rate compares to the size of the change in credit spreads.

Other authors have argued that if financial disturbances are an important source

[1] For a discussion of the FOMC's decisions at that time by a member of the committee, see Mishkin (2008).

[2] See, for example, Taylor and Williams (2008a, 2008b).

of macroeconomic instability, a sound approach to monetary policy will have to pay attention to the balance sheets of financial intermediaries. It is sometimes suggested, for example, that a Taylor rule that is modified to include a response to variations in some measure of aggregate credit would be an improvement upon conventional policy advice (see, *e.g.,* Christiano *et al.,* 2007). We also consider the cyclical variations in aggregate credit that should be associated with both non-financial and financial disturbances, and the desirability of a modified Taylor rule that responds to credit variations in both of these cases.

Many of the models used both in theoretical analyses of optimal monetary policy and in numerical simulations of alternative policy rules are unsuitable for the analysis of these issues, because they abstract altogether from the economic role of financial intermediation. Thus it is common to analyze monetary policy in models with a single interest rate (of each maturity) — "the" interest rate — in which case we cannot analyze the consequences of responding to variations in spreads, and with a representative agent, so that there is no credit extended in equilibrium and hence no possibility of cyclical variations in credit. In order to address the questions that concern us here, we must have a model of the monetary transmission mechanism with both heterogeneity (so that there are both borrowers and savers at each point in time) and segmentation of the participation in different financial markets (so that there can exist non-zero credit spreads).

The model that we use is one developed in Cúrdia and Woodford (2009), as a relatively simple generalization of the basic New Keynesian model used for the analysis of optimal monetary policy in sources such as Goodfriend and King (1997), Clarida *et al.* (1999), and Woodford (2003). The model is still highly stylized in many respects; for example, we abstract from the distinction between the household and firm sectors of the economy, and instead treat all private expenditure as the expenditure of infinite-lived household-firms, and we similarly abstract from the consequences of investment spending for the evolution of the economy's productive capacity, instead treating all private expenditure as if it were all non-durable consumer expenditure (yielding immediate utility, at a diminishing marginal rate). The advantage of this very simple framework, in our view, is that it brings the implications of the credit frictions into very clear focus, by using a model that reduces, in the absence of those frictions, to a model that is both simple and already very well understood.

In section 1, we review the structure of the model, stressing the respects in which the introduction of heterogeneity and imperfect financial intermediation requires the equations of the basic New Keynesian model to be generalized, and discuss its numerical calibration. We then consider the economy's equilibrium responses to both non-financial and financial disturbances under the standard Taylor rule, according to this model. Section 2 then analyzes the consequences of modifying the Taylor rule, to incorporate an automatic response to either changes in credit spreads or in a measure of aggregate credit. We consider the welfare consequences of alternative policy rules, from the standpoint of the average level of expected utility of the heterogenous households in our model. Section 3 then summarizes our conclusions.

1 A New Keynesian Model with Financial Frictions

Here we briefly describe the model developed in Cúrdia and Woodford (2009). (The reader is referred to that paper for more details.) We stress the similarity between the model developed there and the basic New Keynesian [NK] model, and show how the standard model is recovered as a special case of the extended model. This sets the stage for a quantitative investigation of the degree to which credit frictions of an empirically realistic magnitude change the predictions of the model about the responses to shocks other than changes in the severity of financial frictions.

1.1 Sketch of the Model

We depart from the assumption of a representative household in the standard model, by supposing that households differ in their preferences. Each household i seeks to maximize a discounted intertemporal objective of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u^{\tau_t(i)} \left(c_t(i); \xi_t \right) - \int_0^1 v^{\tau_t(i)} \left(h_t \left(j; i \right); \xi_t \right) dj \right],$$

where $\tau_t(i) \in \{b, s\}$ indicates the household's "type" in period t. Here $u^b(c;\xi)$ and $u^s(c;\xi)$ are two different period utility functions, each of which may also be shifted by the vector of aggregate taste shocks ξ_t, and $v^b(h;\xi)$ and $v^s(h;\xi)$ are correspondingly two different functions indicating the period disutility from working. As in the basic NK model, there is assumed to be a continuum of differentiated goods, each produced by a monopolistically competitive supplier; $c_t(i)$ is a Dixit-Stiglitz aggegator of the household's purchases of these differentiated goods. The household similarly supplies a continuum of different types of specialized labor, indexed by j, that are hired by firms in different sectors of the economy; the additively separable disutility of work $v^\tau(h;\xi)$ is the same for each type of labor, though it depends on the household's type and the common taste shock.

Each agent's type $\tau_t(i)$ evolves as an independent two-state Markov chain. Specifically, we assume that each period, with probability $1-\delta$ (for some $0 \leq \delta < 1$) an event occurs which results in a new type for the household being drawn; otherwise it remains the same as in the previous period. When a new type is drawn, it is b with probability π_b and s with probability π_s, where $0 < \pi_b, \pi_s < 1, \pi_b + \pi_s = 1$. (Hence the population fractions of the two types are constant at all times, and equal to π_τ for each type τ.) We assume moreover that

$$u_c^b(c;\xi) > u_c^s(c;\xi)$$

for all levels of expenditure c in the range that occur in equilibrium. (See Figure 1, where these functions are graphed in the case of the calibration discussed below.) Hence a change in a household's type changes its relative impatience to consume, given the aggregate state ξ_t; in addition, the current impatience to consume of all households is changed by the aggregate disturbance ξ_t. We also assume that the marginal utility of additional expenditure diminishes at different rates for the two types, as is also illustrated in the figure; type b households (who are borrowers in equilibrium) have a marginal utility that varies less with the current level of expenditure, resulting in a greater degree of intertemporal substitution of their expenditures in response to interest-rate changes. Finally, the two types are also assumed to differ in the marginal disutility of working a given number of hours; this difference is calibrated so that the two types choose to work the same number of hours in steady state, despite their differing marginal utilities of income. For simplicity, the elasticities of labor supply of the two types are not assumed to differ.

The coexistence of the two types with differing impatience to consume creates a social function for financial intermediation. In the present model, as in the basic New Keynesian model, all output is consumed either by households or by the government;[3] hence intermediation serves an allocative function only to the extent that

[3]The "consumption" variable is therefore to be interpreted as representing all of private expendi-

there are reasons for the intertemporal marginal rates of substitution of households to differ in the absence of financial flows. The present model reduces to the standard representative-household model in the case that one assumes that $u^b(c;\xi) = u^s(c;\xi)$ and $v^b(h;\xi) = v^s(h;\xi)$.

We assume that most of the time, households are able to spend an amount different from their current income *only* by depositing funds with or borrowing from financial intermediaries, and that the same nominal interest rate i_t^d is available to all savers, and that a (possibly) different nominal interest i_t^b is available to all borrowers,[4] independent of the quantities that a given household chooses to save or to borrow. (For simplicity, we also assume that only one-period riskless nominal contracts with the intermediary are possible for either savers or borrowers.) The assumption that households cannot engage in financial contracting other than through the intermediary sector represents the key financial friction.

The analysis is simplified by allowing for an additional form of financial contracting. We assume that households are able to sign state-contingent contracts with one another, through which they may insure one another against both aggregate risk and the idiosyncratic risk associated with a household's random draw of its type, but that households are *only intermittently* able to receive transfers from the insurance agency; between the infrequent occasions when a household has access to the insurance agency,[5] it can only save or borrow through the financial intermediary sector mentioned in the previous paragraph. The assumption that households are *eventually* able to make transfers to one another in accordance with an insurance contract signed earlier means that they continue to have identical expectations regarding their marginal utilities of income far enough in the future, regardless of their differing type

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ture, not only consumer expenditure. In reality, one of the most important reasons for some economic units to wish to borrow from others is that the former currently have access to profitable *investment* opportunities. Here we treat these opportunities as if they were opportunities to *consume*, in the sense that we suppose that the expenditure opportunities are valuable to the household, but we abstract from any consequences of current expenditure for future productivity. For discussion of the interpretation of "consumption" in the basic New Keynesian model, see Woodford (2003, pp. 242-243).

[4]Here "savers" and "borrowers" identify households according to whether they choose to save or borrow, and not by their "type".

[5]For simplicity, these are assumed to coincide with the infrequent occasions when the household draws a new "type"; but the insurance payment is claimed before the new type is known, and cannot be contingent upon the new type.

histories.

As long as certain inequalities discussed in our previous paper are satisfied,[6] it turns out that in equilibrium, type b households choose always to borrow from the intermediaries, while type s households deposit their savings with them (and no one chooses to do both, given that $i_t^b \geq i_t^d$ at all times). Moreover, because of the asymptotic risk-sharing, one can show that all households of a given type at any point in time have a common marginal utility of real income (which we denote λ_t^τ for households of type τ) and choose a common level of real expenditure c_t^τ. Household optimization of the timing of expenditure requires that the marginal-utility processes $\{\lambda_t^\tau\}$ satisfy the two Euler equations

$$\lambda_t^b = \beta E_t \left[\frac{1+i_t^b}{\Pi_{t+1}} \left\{ \left[\delta + (1-\delta)\,\pi_b\right] \lambda_{t+1}^b + (1-\delta)\,\pi_s \lambda_{t+1}^s \right\} \right], \tag{1.1}$$

$$\lambda_t^s = \beta E_t \left[\frac{1+i_t^d}{\Pi_{t+1}} \left\{ (1-\delta)\,\pi_b \lambda_{t+1}^b + \left[\delta + (1-\delta)\,\pi_s\right] \lambda_{t+1}^s \right\} \right] \tag{1.2}$$

in each period. Here $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, where P_t is the Dixit-Stiglitz price index for the differentiated goods produced in period t. Note that each equation takes into account the probability of switching type from one period to the next.

Assuming an interior choice for consumption by households of each type, the expenditures of the two types must satisfy

$$\lambda_t^b = u^{b\prime}(c_t^b), \qquad \lambda_t^s = u^{s\prime}(c_t^s),$$

which relations can be inverted to yield demand functions

$$c_t^b = c^b(\lambda_t^b; \xi_t), \qquad c_t^s = c^s(\lambda_t^s; \xi_t).$$

Aggregate demand Y_t for the Dixit-Stiglitz composite good is then given by

$$Y_t = \pi_b c^b(\lambda_t^b; \xi_t) + \pi_s c^s(\lambda_t^s; \xi_t) + G_t + \Xi_t, \tag{1.3}$$

where G_t indicates the (exogenous) level of government purchases and Ξ_t indicates resources consumed by the intermediary sector (discussed further below). Equations

[6] We verify that in the case of the numerical parameterization of the model discussed below, these inequalities are satisfied at all times, in the case of small enough random disturbances of any of the kinds discussed.

(1.1)–(1.2) together with (1.3) indicate the way in which the two real interest rates of the model affect aggregate demand. This system directly generalizes the relation that exists in the basic NK model as a consequence of the Euler equation of the representative household.

It follows from the same assumptions that optimal labor supply in any given period will be the same for all households of a given type. Specifically, any household of type τ will supply hours $h^\tau(j)$ of labor of type j, so as to satisfy the first-order condition

$$\mu_t^w v_h^\tau(h_t^\tau(j);\xi_t) = \lambda_t^\tau W_t(j)/P_t, \tag{1.4}$$

where $W_t(j)$ is the wage for labor of type j, and the exogenous factor μ_t^w represents a possible "wage markup" (the sources of which are not further modeled). Aggregation of the labor supply behavior of the two types is facilitated if, as in Benigno and Woodford (2005), we assume the isoelastic functional form

$$v^\tau\left(h;\xi_t\right) \equiv \frac{\psi_\tau}{1+\nu} h^{1+\nu} \bar{H}_t^{-\nu}, \tag{1.5}$$

where $\{\bar{H}_t\}$ is an exogenous labor-supply disturbance process (assumed common to the two types, for simplicity); $\psi_b, \psi_s > 0$ are (possibly) different multiplicative coefficients for the two types; and the coefficient $\nu \geq 0$ (inverse of the Frisch elasticity of labor supply) is assumed to be the same for both types. Solving (1.4) for the competitive labor supply of each type and aggregating, we obtain

$$h_t(j) = \bar{H}_t \left[\frac{\tilde{\lambda}_t}{\psi \mu_t^w} \frac{W_t(j)}{P_t}\right]^{1/\nu}$$

for the aggregate supply of labor of type j, where

$$\tilde{\lambda}_t \equiv \psi \left[\pi_b (\frac{\lambda_t^b}{\psi_b})^{1/\nu} + \pi_s (\frac{\lambda_t^s}{\psi_s})^{1/\nu}\right]^\nu, \tag{1.6}$$

$$\psi \equiv \left[\pi_b \psi_b^{-1/\nu} + \pi_s \psi_s^{-1/\nu}\right]^{-\nu}.$$

We furthermore assume an isoelastic production function

$$y_t(i) = Z_t h_t(i)^{1/\phi}$$

for each differentiated good i, where $\phi \geq 1$ and Z_t is an exogenous, possibly time-varying productivity factor, common to all goods. We can then determine the demand for each differentiated good as a function of its relative price using the usual Dixit-Stiglitz demand theory, and determine the wage for each type of labor by equating supply and demand for that type. We finally obtain a total wage bill

$$\int W_t(j)h_t(j)dj = \psi\mu_t^w \frac{P_t}{\tilde{\lambda}_t \bar{H}_t^\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t, \tag{1.7}$$

where $\omega_y \equiv \phi(1+\nu) - 1 \geq 0$ and

$$\Delta_t \equiv \int \left(\frac{p_t(i)}{P_t}\right)^{-\theta(1+\omega_y)} di \geq 1$$

is a measure of the dispersion of goods prices (taking its minimum possible value, 1, if and only if all prices are identical), in which $\theta > 1$ is the elasticity of substitution among differentiated goods in the Dixit-Stiglitz aggregator. Note that in the Calvo model of price adjustment, this dispersion measure evolves according to a law of motion

$$\Delta_t = h(\Delta_{t-1}, \Pi_t), \tag{1.8}$$

where the function $h(\Delta, \Pi)$ is defined as in Benigno and Woodford. Finally, since households of type τ supply fraction

$$\pi_\tau \left(\frac{\lambda_t^\tau}{\tilde{\lambda}_t} \frac{\psi}{\psi_\tau}\right)^{\frac{1}{\nu}}$$

of total labor of each type j, they also receive this fraction of the total wage bill each period. This observation together with (1.7) allows us to determine the wage income of each household at each point in time.

These solutions for expenditure on the one hand and wage income on the other for each type allow us to solve for the evolution of the net saving or borrowing of households of each type. We let the credit spread $\omega_t \geq 0$ be defined as the factor such that

$$1 + i_t^b = (1 + i_t^d)(1 + \omega_t), \tag{1.9}$$

and observe that in equilibrium, aggregate deposits with intermediaries must equal aggregate saving by type s households in excess of b_t^g, the real value of (one-period, riskless nominal) government debt (the evolution of which is also specified as an exogenous disturbance process[7]), which in equilibrium must pay the same rate of interest i_t^d as deposits with intermediaries. It is then possible to derive a law of motion for aggregate private borrowing b_t, of the form

$$\begin{aligned}(1+\pi_b\omega_t)b_t &= \pi_b\pi_s B(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t) - \pi_b b_t^g \\ &\quad + \delta[b_{t-1}(1+\omega_{t-1}) + \pi_b b_{t-1}^g]\frac{1+i_t^d}{\Pi_t}, \end{aligned} \tag{1.10}$$

where the function B (defined in Cúrdia and Woodford, 2009) indicates the amount by which the expenditure of type b households in excess of their current wage income is greater than the expenditure of type s households in excess of their current wage income. This equation, which has no analog in the representative-household model, allows us to solve for the dynamics of private credit in response to various types of disturbances. It becomes important for the general-equilibrium determination of other variables if (as assumed below) the credit spread and/or the resources used by intermediaries depend on the volume of private credit.

We can similarly use the above model of wage determination to solve for the marginal cost of producing each good as a function of the quantity demanded of it, again obtaining a direct generalization of the formula that applies in the representative-household case. This allows us to derive equations describing optimal price-setting by the monopolistically competitive suppliers of the differentiated goods. As in the basic NK model, Calvo-style staggered price adjustment then implies an inflation equation of the form

$$\Pi_t = \Pi(z_t), \tag{1.11}$$

where z_t is a vector of two forward-looking variables, recursively defined by a pair of relations of the form

$$z_t = G(Y_t, \lambda_t^b, \lambda_t^s; \xi_t) + E_t[g(\Pi_{t+1}, z_{t+1})], \tag{1.12}$$

where the vector-valued functions G and g are defined in Cúrdia and Woodford (2009). (Among the arguments of G, the vector of exogenous disturbances ξ_t now includes an exogenous sales tax rate τ_t, in addition to the disturbances already mentioned.)

[7] Our model includes three distinct fiscal disturbances, the processes G_t, τ_t, and b_t^g, each of which can be independently specified. The residual income flow each period required to balance the government's budget is assumed to represent a lump-sum tax or transfer, equally distributed across households regardless of type.

These relations are of exactly the same form as in the basic NK model, except that two distinct marginal utilities of income are here arguments of G; in the case that $\lambda_t^b = \lambda_t^s = \lambda_t$, the relations (1.12) reduce to exactly the ones in Benigno and Woodford (2005). The system (1.11)–(1.12) indicates the nature of the short-run aggregate-supply trade-off between inflation and real activity at a point in time, given expectations regarding the future evolution of inflation and of the variables $\{z_t\}$.

It remains to specify the frictions associated with financial intermediation, that determine the credit spread ω_t and the resources Ξ_t consumed by the intermediary sector. We allow for two sources of credit spreads — one of which follows from an assumption that intermediation requires real resources, and the other of which does not — which provide two distinct sources of "purely financial" disturbances in our model. On the one hand, we assume that real resources $\Xi_t(b_t)$ are consumed in the process of originating loans of real quantity b_t, and that these resources must be produced and consumed in the period in which the loans are originated. The function $\Xi_t(b_t)$ is assumed to be non-decreasing and at least weakly convex. In addition, we suppose that in order to originate a quantity of loans b_t that will be repaid (with interest) in the following period, it is necessary for an intermediary to also make a quantity $\chi_t(b_t)$ of loans that will be defaulted upon, where $\chi_t(b_t)$ is also a non-decreasing, weakly convex function. (We assume that intermediaries are unable to distinguish the borrowers who will default from those who will repay, and so offer loans to both on the same terms, but that they are able to accurately predict the fraction of loans that will not be repaid as a function of a given scale of expansion of their lending activity.) Hence total (real) outlays in the amount $b_t + \chi_t(b_t) + \Xi_t(b_t)$ are required[8] in a given period in order to originate a quantity b_t of loans that will be repaid (yielding $(1 + i_t^b)b_t$ in the following period). Competitive loan supply by intermediaries then implies that

$$\omega_t = \omega_t(b_t) \equiv \chi_t'(b_t) + \Xi_t'(b_t). \tag{1.13}$$

It follows that in each period, the credit spread ω_t will be a non-negative-valued,

[8]It might be thought more natural to make the resource requirement Ξ_t a function of the total quantity $b_t + \chi_t(b_t)$ of loans that are originated, rather than a function of the "sound" loans b_t. But since under our assumptions $b_t + \chi_t(b_t)$ is a (possibly time-varying) function of b_t, it would in any event be possible to express Ξ_t as a (possibly time-varying) function of b_t, with the properties assumed in the text.

non-decreasing function of the real volume of private credit b_t^g. This function may shift over time, as a consequence of exogenous shifts in either the resource cost function Ξ_t or the default rate χ_t.[9] Allowing these functions to be time-varying introduces the possibility of "purely financial" disturbances, of a kind that will be associated with increases in credit spreads and/or reduction in the supply of credit.

Finally, we assume that the central bank is able to control the deposit rate i_t^d (the rate at which intermediaries are able to fund themselves),[10] though this is no longer also equal to the rate i_t^b at which households are able to borrow, as in the basic NK model. Monetary policy can then be represented by an equation such as

$$i_t^d = i_t^d(\Pi_t, Y_t), \tag{1.14}$$

which represents a Taylor rule subject to exogenous random shifts that can be given a variety of interpretations. (This is of course only one simple specification of monetary policy; we consider central-bank reaction functions with additional arguments in section 2.)

If we substitute the functions $\omega_t(b_t)$ and $\Xi_t(b_t)$ for the variables ω_t and Ξ_t in the above equations, then the system consisting of equations (1.1)–(1.3), (1.8)–(1.12), and (1.14) comprise a system of 10 equations per period to determine the 10 endogenous variables $\Pi_t, Y_t, i_t^d, i_t^b, \lambda_t^b, \lambda_t^s, b_t, \Delta_t$, and z_t, given the evolution of the exogenous disturbances. The disturbances that affect these equations include the productivity factor Z_t; the fiscal disturbances G_t, τ_t, and b_t^g; a variety of potential preference shocks (variations in impatience to consume, that may or may not equally affect households of the two types, and variations in attitudes toward work, assumed to be common to the two types) and variations in the wage markup μ_t^w; purely financial shocks (shifts in either of the functions $\Xi_t(b_t)$ and $\chi_t(b_t)$); and monetary policy shocks (shifts in the function $i_t(\Pi_t, Y_t)$). We consider the consequences of systematic monetary policy

[9] Of course, these shifts must not be purely additive shifts, in order for the function $\omega_t(b_t)$ to shift. In our numerical work below, the two kinds of purely financial disturbances that are considered are multiplicative shifts of the two functions.

[10] If we extend the model by introducing central-bank liabilities that supply liquidity services to the private sector, the demand for these liabilities will be a decreasing function of the spread between i_t^d and the interest rate paid on central-bank liabilities (reserves). The central bank will then be able to influence i_t^d by adjusting either the supply of its liabilities (through open-market purchases of government debt, for example) or the interest rate paid on them. Here we abstract from this additional complication by treating i_t^d as directly under the control of the central bank.

for the economy's response to all of these types of disturbances below. Note that this system of equations reduces to the basic NK model (as presented in Benigno and Woodford, 2005) if we identify λ_t^b and λ_t^s and identify i_t^d and i_t^b (so that the pair of Euler equations (1.1)–(1.2) reduces to a single equation, relating the representative household's marginal utility of income to the single interest rate); identify the two expenditure functions $c^s(\lambda;\xi)$ and $c^b(\lambda;\xi)$; set the variables ω_t and Ξ_t equal to zero at all times; and delete equation (1.10), which describes the dynamics of a variable (b_t) that has no significance in the representative-household case.

1.2 Log-Linearized Structural Equations

In our numerical analysis of the consequences of alternative monetary policy rules, we plot impulse responses to a variety of shocks under a candidate policy rule. The responses that we plot are linear approximations to the actual response, accurate in the case of small enough disturbances. These linear approximations to the equilibrium responses are obtained by solving a system of linear (or log-linear) approximations to the model structural equations (including a linear equation for the monetary policy rule). Here we describe some of these log-linearized structural equations, as they provide further insight into the implications of our model, and facilitate comparison with the basic NK model.

We log-linearize the model structural relations around a deterministic steady state with zero inflation each period, and a constant level of aggregate output $\bar{Y}$. (We assume that, in the absence of disturbances, the monetary policy rule (1.14) is consistent with this steady state, though the small disturbances in the structural equations that we consider using the log-linearized equations may include small departures from the inflation target of zero.) These log-linear relations will then be appropriate for analyzing the consequences of alternative monetary policy rules only in the case of rules consistent with an average inflation rate that is not too far from zero. But in Cúrdia and Woodford (2009), we show that under an optimal policy commitment (Ramsey policy), the steady state is indeed the zero-inflation steady state. Hence all policy rules that represent approximations to optimal policy will indeed have this property.

We express our log-linearized structural relations in terms of deviations of the logarithms of quantities from their steady-state values ($\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, etc.), the inflation rate $\pi_t \equiv \log \Pi_t$, and deviations of (continuously compounded) interest rates from their steady-state values ($\hat{\imath}^d_t \equiv \log(1+i^d_t/1+\bar{\imath}^d)$, etc.). We also introduce isoelastic functional forms for the utility of consumption of each of the two types, which imply that

$$c^\tau(\lambda;\xi_t) = \bar{C}^\tau_t \lambda^{-\sigma_\tau}$$

for each of the two types $\tau \in \{b, s\}$, where $\bar{C}^\tau_t$ is a type-specific exogenous disturbance (indicating variations in impatience to consume, or in the quality of spending opportunities) and $\sigma_\tau > 0$ is a type-specific intertemporal elasticity of substitution.

Then as shown in Cúrdia and Woodford (2009), log-linearization of the system consisting of equations (1.1)–(1.3) allows us to derive an "intertemporal IS relation"

$$\begin{aligned}\hat{Y}_t \;=\; & -\bar{\sigma}(\hat{\imath}^{avg}_t - E_t\pi_{t+1}) + E_t\hat{Y}_{t+1} - E_t\Delta g_{t+1} - E_t\Delta\hat{\Xi}_{t+1} \\ & -\bar{\sigma}s_\Omega\hat{\Omega}_t + \bar{\sigma}(s_\Omega + \psi_\Omega)E_t\hat{\Omega}_{t+1}, \end{aligned} \quad (1.15)$$

where

$$\hat{\imath}^{avg}_t \equiv \pi_b\hat{\imath}^b_t + \pi_s\hat{\imath}^d_t \quad (1.16)$$

is the average of the interest rates that are relevant (at the margin) for all of the savers and borrowers in the economy; g_t is a composite "autonomous expenditure" disturbance as in Woodford (2003, pp. 80, 249), taking account of the exogenous fluctuations in $G_t, \bar{C}^b_t$, and $\bar{C}^s_t$ (and again weighting the fluctuations in the spending opportunities of the two types in proportion to their population fractions);

$$\hat{\Omega}_t \equiv \hat{\lambda}^b_t - \hat{\lambda}^s_t,$$

the "marginal-utility gap" between the two types, is a measure of the inefficiency of the intratemporal allocation of resources as a consequence of imperfect financial intermediation; and

$$\hat{\Xi}_t \equiv (\Xi_t - \bar{\Xi})/\bar{Y}$$

measures departures of the quantity of resources consumed by the intermediary sector from its steady-state level.[11] In this equation, the coefficient

$$\bar{\sigma} \equiv \pi_b s_b\sigma_b + \pi_s s_s\sigma_s > 0 \quad (1.17)$$

[11]We adopt this notation so that $\hat{\Xi}_t$ is defined even when the model is parameterized so that $\bar{\Xi} = 0$.

is a measure of the interest-sensitivity of aggregate demand, using the notation s_τ for the steady-state value of c_t^τ / Y_t; the coefficient

$$s_\Omega \equiv \pi_b \pi_s \frac{s_b \sigma_b - s_s \sigma_s}{\bar{\sigma}}$$

is a measure of the asymmetry in the interest-sensitivity of expenditure by the two types; and the coefficient

$$\psi_\Omega \equiv \pi_b(1-\chi_b) - \pi_s(1-\chi_s)$$

is also a measure of the difference in the situations of the two types. Here we use the notation

$$\chi_\tau \equiv \beta(1+\bar{r}^\tau)[\delta + (1-\delta)\pi_\tau]$$

for each of the two types, where $\bar{r}^\tau$ is the steady-state real rate of return that is relevant at the margin for type τ. Note that except for the presence of the last three terms on the right-hand side (all of which are identically zero in a model without financial frictions), equation (1.15) has the same form as the intertemporal IS relation in the basic NK model; the only differences are that the interest rate that appears is a weighted average of two interest rates (rather than simply "the" interest rate), the elasticity $\bar{\sigma}$ is a weighted average of the corresponding elasticities for the two types of households (rather than the elasticity of expenditure by a representative household), and the disturbance term g_t involves a weighted average of the expenditure demand shocks $\bar{C}_t^\tau$ for the two types (rather than the corresponding shock for a representative household).

Equation (1.15) is derived by taking a weighted average of the log-linearized forms of the two Euler equations (1.1)–(1.2), and then using the log-linearized form of (1.3) to relate average marginal utility to aggregate expenditure. If we instead subtract the log-linearized version of (1.2) from the log-linearized (1.1), we obtain

$$\hat{\Omega}_t = \hat{\omega}_t + \hat{\delta} E_t \hat{\Omega}_{t+1}. \tag{1.18}$$

Here we define

$$\hat{\omega}_t \equiv \log(1+\omega_t/1+\bar{\omega}),$$

so that the log-linearized version of (1.9) is

$$\hat{\imath}_t^b = \hat{\imath}_t^d + \hat{\omega}_t. \tag{1.19}$$

and

$$\hat{\delta} \equiv \chi_b + \chi_s - 1 < 1.$$

Equation (1.18) can be "solved forward" for $\hat{\Omega}_t$ as a forward-looking moving average of the expected path of the credit spread $\hat{\omega}_t$. This now gives us a complete theory of the way in which time-varying credit spreads affect aggregate demand, given an expected forward path for the policy rate. One the one hand, higher current and/or future credit spreads raise the expected path of $\hat{\imath}_t^{avg}$ for any given path of the policy rate, owing to (1.19), and this reduces aggregate demand $\hat{Y}_t$ according to (1.15). And on the other hand, higher current and/or future credit spreads increase the marginal-utility gap $\hat{\Omega}_t$, owing to (1.18), and (under the parameterization that we find most realistic) this further reduces aggregate demand for any expected forward path for $\hat{\imath}_t^{avg}$, as a consequence of the $\hat{\Omega}_t$ terms in (1.15). The fact that larger credit spreads reduce aggregate demand for a given path of the policy rate is consistent with the implicit model behind the proposal of McCulley and Toloui (2008) and Taylor (2008). But our model does *not* indicate, in general, that it is *only* the borrowing rate i_t^b that matters for aggregate demand determination. Hence there is no reason to expect that the effect of an increased credit spread on aggregate demand can be fully neutralized through an offsetting reduction of the policy rate, as the simple proposal of a one-for-one offset seems to presume.

Log-linearization of the aggregate-supply block consisting of equations (1.11)–(1.12) similarly yields a log-linear aggregate-supply relation of the form

$$\pi_t = \kappa(\hat{Y}_t - \hat{Y}_t^n) + \beta E_t \pi_{t+1} + u_t + \xi(s_\Omega + \pi_b - \gamma_b)\hat{\Omega}_t - \xi\bar{\sigma}^{-1}\hat{\Xi}_t, \qquad (1.20)$$

where $\hat{Y}_t^n$ (the "natural rate of output") is a composite exogenous disturbance term, a linear combination of the variations in g_t, $\bar{H}_t$ and Z_t (sources of variation in the flexible-price equilibrium level of output that, in the absence of steady-state distortions or financial frictions, correspond to variations in the *efficient* level of output), while the additional exogenous disturbance term u_t (the "cost-push shock") is instead a linear combination of the variations in μ_t^w and τ_t (sources of variation in the flexible-price equilibrium level of output that do *not* correspond to any change in the efficient level of output).[12] The coefficients in this equation are given by

$$\gamma_b \equiv \pi_b \left(\frac{\psi \bar{\lambda}^b}{\psi_b \bar{\bar{\lambda}}} \right)^{1/\nu} > 0;$$

$$\xi \equiv \frac{1-\alpha}{\alpha} \frac{1-\alpha\beta}{1+\omega_y \theta} > 0,$$

where $0 < \alpha < 1$ is the fraction of prices that remain unchanged from one period to the next; and

$$\kappa \equiv \xi(\omega_y + \bar{\sigma}^{-1}) > 0.$$

Note that except for the presence of the final two terms on the right-hand side, (1.20) is exactly the "New Keynesian Phillips curve" relation of the basic NK model (as expounded, for example, in Clarida *et al.*, 1999), and the definitions of both the disturbance terms and the coefficient κ are exactly the same as in that model (except that $\bar{\sigma}$ replaces the elasticity of the representative household). The two new terms, proportional to $\hat{\Omega}_t$ and $\hat{\Xi}_t$, respectively, are present only to the extent that there are credit frictions. These terms indicate that, in addition to their consequences for aggregate demand, variations in the size of credit frictions also have "cost-push" effects on the short-run aggregate-supply tradeoff between aggregate real activity and inflation.

Finally, the central-bank reaction function (1.14) can be log-linearized to yield

$$\hat{\imath}_t^d = \phi_\pi \pi_t + \phi_y \hat{Y}_t + \epsilon_t^m, \qquad (1.21)$$

where ϵ_t^m is an exogenous disturbance term (to which we shall refer as a "monetary policy shock"). Except for the disturbance, this is the form of linear rule recommended by Taylor (1993). The implications of such a rule for the evolution of the composite interest rate $\hat{\imath}_t^{avg}$ that appears in the IS relation (1.15) can be derived by using (1.19) to write

$$\hat{\imath}_t^{avg} = \hat{\imath}_t^d + \pi_b \hat{\omega}_t. \qquad (1.22)$$

[12] See Cúrdia and Woodford (2009) for the precise definition of these composite disturbances. They are in fact exactly the same as in the basic NK model (Woodford, 2003, chap. 3, or Benigno and Woodford, 2005), except that $\bar{\sigma}$ defined in (1.17) replaces the corresponding intertemporal elasticity of expenditure of the representative household.

1.3 Numerical Calibration

The numerical values for parameters that are used in our calculations below are the same as in Cúrdia and Woodford (2009). Many of the model's parameters are also parameters of the basic NK model, and in the case of these parameters we assume similar numerical values as in the numerical analysis of the basic NK model in Woodford (2003, Table 6.1.), which in turn are based on the empirical model of Rotemberg and Woodford (1997). The new parameters that are also needed for the present model are those relating to heterogeneity or to the specification of the credit frictions. The parameters relating to heterogeneity are the fraction π_b of households that are borrowers, the degree of persistence δ of a household's "type", the steady-state expenditure level of borrowers relative to savers, s_b/s_s, and the interest-elasticity of expenditure of borrowers relative to that of savers, σ_b/σ_s.[13]

In the calculations reported here, we assume that $\pi_b = \pi_s = 0.5$, so that there are an equal number of borrowers and savers. We assume that $\delta = 0.975$, so that the expected time until a household has access to the insurance agency (and its type is drawn again) is 10 years. This means that the expected path of the spread between lending and deposit rates for 10 years or so into the future affects current spending decisions, but that expectations regarding the spread several decades in the future are nearly irrelevant.

We calibrate the degree of heterogeneity in the steady-state expenditure shares of the two types so that the implied steady-state debt $\bar{b}$ is equal to 80 percent of annual steady-state output.[14] This value matches the median ratio of private (non-financial, non-government, non-mortgage) debt to GDP over the period 1986-2008.[15] This requires a ratio $s_b/s_s = 1.27$. We calibrate the value of σ_b/σ_s to equal 5. This is an arbitrary choice, though the fact that borrowers are assumed to have a greater willingness to substitute intertemporally is important, as this results in the prediction that an exogenous tightening of monetary policy (a positive value of the residual ϵ_t^m in (1.14)) results in a reduction in the equilibrium volume of credit b_t (see Figures 2

[13]Another new parameter that matters as a consequence of heterogeneity is the steady-state level of government debt relative to GDP, $\bar{b}^g/\bar{Y}$; here we assume that $\bar{b}^g = 0$.

[14]In our quarterly model, this means that $\bar{b}/\bar{Y} = 3.2$.

[15]We exclude mortgage debt when calibrating the degree of heterogeneity of preferences in our model, since mortgage debt is incurred in order to acquire an asset, rather than to consume current produced goods in excess of current income.

and 5 below). This is consistent with VAR evidence on the effects of an identified monetary policy shock on household borrowing.[16]

It is also necessary to specify the steady-state values of the functions $\omega(b)$ and $\Xi(b)$ that describe the financial frictions, in addition to making clear what kinds of random perturbations of these functions we wish to consider when analyzing the effects of "financial shocks." We here present results for two cases. In each case, we assume that the steady-state credit spread is due entirely to the marginal resource cost of intermediation;[17] but we do allow for exogenous shocks to the default rate, and this is what we mean by the "financial shock" in Figures 10 through 12 below.[18] In treating the "financial shock" as involving an increase in markups but no increase in the real resources used in banking, we follow Gerali *et al.* (2008).[19]

The two cases considered differ in the specification of the (time-invariant) intermediation technology $\Xi(b)$. In the case of a *linear intermediation technology,* we suppose that $\Xi(b) = \tilde{\Xi}b$, while in the case of a *convex intermediation technology,* we assume that

$$\Xi(b) = \tilde{\Xi}b^{\eta} \tag{1.23}$$

for some $\eta > 1$.[20] In both cases, in our numerical analyses we assume a steady-state

[16]See, for example, Den Haan *et al.* (2004).

[17]We assume this in the results presented here because we do not wish to appear to have sought to minimize the differences between a model with financial frictions and the basic NK model, and the use of real resources by the financial sector (slightly) increases the differences between the two models.

[18]Note that our conclusions regarding both equilibrium and optimal responses to shocks *other* than the "financial shock" are the same as in an economy in which the banking system is perfectly competitive (and there are no risk premia), up to the linear approximation used in the numerical results reported below.

[19]These authors cite the Eurosystem's quarterly Bank Lending Survey as showing that since October 2007, banks in the euro area had "strongly increased the margins charged on average and riskier loans" (p. 24).

[20]One interpretation of this function is in terms of a monitoring technology of the kind assumed in Goodfriend and McCallum (2007). Suppose that a bank produces monitoring according to a Cobb-Douglas production function, $k^{1-\eta^{-1}}\Xi_t^{\eta^{-1}}$, where k is a fixed factor ("bank capital"), and must produce a unit of monitoring for each unit of loans that it manages. Then the produced goods Ξ_t required as inputs to the monitoring technology in order to manage a quantity b of loans will be given by a function of the form (1.23), where $\tilde{\Xi} = k^{1-\eta}$. A sudden impairment of bank capital, treated as an exogenous disturbance, can then be represented as a random increase in the multiplicative factor $\tilde{\Xi}$. This is another form of "financial shock", with similar, though not identical,

credit spread $\bar{\omega}$ equal to 2.0 percentage points per annum,[21] following Mehra *et al.*, (2008).[22] (Combined with our assumption that "types" persist for 10 years on average, this implies a steady-state "marginal utility gap" $\bar{\Omega} \equiv \bar{\lambda}^b/\bar{\lambda}^s = 1.22$, so that there would be a non-trivial welfare gain from transferring further resources from savers to borrowers.) In the case of the convex technology, we set η so that a one-percent increase in the volume of credit increases the credit spread by one percentage point (per annum).[23] The assumption that $\eta > 1$ allows our model to match the prediction of VAR estimates that an unexpected tightening of monetary policy is associated with a slight reduction in credit spreads (see, *e.g.*, Lown and Morgan, 2002, and Gerali *et al.*, 2008). We have chosen a rather extreme value for this elasticity in our calibration of the convex-technology case, in order to make more visible the difference that a convex technology makes for our results. (In the case of a smaller value of η, the results for the convex technology are closer to those for the linear technology, and in fact are in many respects similar to those for an economy with no financial frictions at all.)

As a first exercise, we consider the implied equilibrium responses of the model's endogenous variables to the various kinds of exogenous disturbances, under the assumption that monetary policy is described by a Taylor rule of the form (1.21). The coefficients of the monetary policy rule are assigned the values $\phi_\pi = 1.5$ and $\phi_y = 0.5$[24] as recommended by Taylor (1993).[25] Among other disturbances, we consider the effects of random disturbances to the error term ϵ_t^m in the monetary policy rule. In section 2, we consider the predicted dynamics under a variety of other monetary policy specifications as well.

In all of the cases that we consider, we assume that each of the exogenous distur-

effects as the default rate shock considered below.

[21] In our quarterly numerical model, this means that we choose a value such that $(1 + \bar{\omega})^4 = 1.02$.

[22] Mehra *et al.* argue for this calibration by dividing the net interest income of financial intermediaries (as reported in the National Income and Product Accounts) by a measure of aggregate private credit (as reported in the Flow of Funds). As it happens, this value also corresponds to the median spread between the FRB index of commercial and industrial loan rates and the federal funds rate, over the period 1986-2007.

[23] This requires that $\eta = 51.6$.

[24] This is the value of ϕ_y if $\hat{\imath}_t^d$ and π_t are quoted as annualized rates, as in Taylor (1993). If, instead, (1.21) is written in terms of quarterly rates, then the coefficient on $\hat{Y}_t$ is only 0.5/4.

[25] See, for example, Taylor (2007) as an example of more recent advocacy of a rule with these same coefficients.

bances ξ_{it} evolves according to an AR(1) process,

$$\xi_{it} = \rho_i \xi_{i,t-1} + \epsilon_{it},$$

where ϵ_{it} is a mean-zero i.i.d. random process, and that these processes are independent for the different disturbances i. We make various assumptions about the size of the coefficient of serial correlation ρ_i and the standard deviation of the innovations ϵ_{it}, that are explained below.

1.4 Credit Frictions and the Propagation of Disturbances

We can explore the consequences of introducing financial frictions into our model by considering the predicted responses to aggregate disturbances of a kind that also exist in the basic NK model, and see how much difference to our results the allowance for credit frictions makes. A special case in which we obtain a simple result is that in which (i) Ξ_t is an exogenous process (i.e., independent of b_t — there are no variable resource costs of intermediation), and (ii) $\chi_t(b)$ is a linear function, $\chi_t(b) = \tilde{\chi}_t b$, at all times (i.e., the default rate is independent of the volume of lending), though the default rate may vary (exogenously) over time. In this case, (1.13) implies that ω_t will also be an exogenous process (equal to $\tilde{\chi}_t$), and (1.18) implies that $\hat{\Omega}_t$ will be an exogenous process (determined purely by the evolution of $\tilde{\chi}_t$).

In this case, the set of equations (1.15), (1.20), (1.21) and (1.22) comprise a complete system for determination of the equilibrium evolution of the variables $\pi_t, \hat{Y}_t, \hat{\imath}_t^d$, and $\hat{\imath}_t^{avg}$, given the evolution of the exogenous disturbances, that now include $\hat{\Xi}_t$ and $\hat{\omega}_t$ disturbances. If we use (1.22) to substitute for $\hat{\imath}_t^d$ in (1.21), we obtain an equation in which the policy rule is written in terms of its implications for the average interest rate $\hat{\imath}_t^{avg}$,

$$\hat{\imath}_t^{avg} = \phi_\pi \pi_t + \phi_y \hat{Y}_t + \pi_b \hat{\omega}_t + \epsilon_t^m. \tag{1.24}$$

Then equations (1.15), (1.20) and (1.24) comprise a complete system for determination of $\pi_t, \hat{Y}_t$, and $\hat{\imath}_t^{avg}$. This system of equations is a direct generalization of the familiar "three-equation system" in expositions of the log-linearized basic NK model, as in Clarida *et al.* (1999).

In fact, this system of equations is *identical* to the structural equations of the basic NK model, if the latter model is parameterized by assigning the representative household an intertemporal elasticity of substitution that is an appropriately weighted average of the interemporal elasticities of the two types in this model.[26] The only differences from the equations of the basic model are that the interest rate $\hat{\imath}_t^{avg}$ in this system need not correspond to the policy rate, and that each of the three equations contains additional additive disturbance terms ($\hat{\omega}_t, \hat{\Omega}_t$, and $\hat{\Xi}_t$ terms) owing to the possibility of time variation in the credit frictions.

This means that the predictions of the model about the equilibrium responses of inflation, output and nominal interest rates to any of the non-financial disturbances — disturbances to tastes, technology, monetary policy, or fiscal policy, all of which are also allowed for in the basic NK model — under a given monetary policy rule are identical to those of the basic NK model. (The linearity of the approximate model equations implies that the impulse responses to any of the non-financial disturbances are independent of what we assume about the size of the financial disturbances.) Hence our conclusions about the desirability of a given form of Taylor rule — at least from the standpoint of success in stabilizing inflation, output or interest rates — will be exactly the same as in the basic NK model, *except* to the extent that we may be concerned about the ability of policy to stabilize the economy in response to purely financial disturbances (which are omitted in the basic NK model).

This conclusion requires somewhat special assumptions; but numerical analysis of our calibrated model suggests that the conclusion is not too different even when one allows ω_t and Ξ_t to vary endogenously with variations in the volume of private credit. In the numerical results shown in Figures 2-9, we plot the equilibrium responses to various types of disturbances (a different disturbance in each figure, with the responses of different variables in the separate panels of each figure). In each figure, we compare the equilibrium responses to the same disturbance in three different models. The "FF" model is the model with heterogeneity and financial frictions described in the previous sections. The "NoFF" model is a model with preference heterogeneity of the same type (and correspondingly parameterized), but in which there are no financial frictions (ω_t and Ξ_t are set equal to zero at all times). Finally, the "RepHH" model is a representative household model, with parameters that are present in the FF model

[26]A key parameter of the basic NK model is $\sigma \equiv s_c\tilde{\sigma}$, where $\tilde{\sigma}$ is the intertemporal elasticity of substitution of private expenditure, and s_c is the steady-state share of private expenditure in total aggregate demand. (See Woodford, 2003, p. 243.) For the equivalence asserted in the text to obtain, it is necessary to parameterize the representative-household model so that σ has the value of the coefficient $\bar{\sigma}$ (defined in (1.17)) in our model.

calibrated in the same way as in the other two models.[27]

We first consider the case of a linear intermediation technology ($\eta = 1$). In this case, the credit spread ω_t still evolves exogenously, as assumed in the discussion above, but Ξ_t is no longer independent of b_t. Nonetheless, in this case we continue to find that for an empirically plausible specification of the quantity of resources used in intermediation, the existence of credit frictions makes virtually no difference for the predicted equilibrium responses to shocks.

This is illustrated in Figures 2-4 for three particular types of exogenous disturbances. In Figure 2 we consider the equilibrium responses to a contractionary monetary policy shock, represented by a unit (one percentage point, annualized) increase in ϵ_t^m. We furthermore assume that the policy disturbance is persistent; specifically, ϵ_t^m is assumed to follow an AR(1) process with coefficient of autocorrelation $\rho = 0.6$.[28] The separate panels of the figure indicate the impulse responses of output, inflation,[29] the deposit rate (policy rate), the credit spread,[30] and aggregate private credit respectively. Figure 3 illustrates the equilibrium responses of the same variables to a unit positive innovation in the productivity factor, where the disturbance is now assumed to have an autocorrelation of 0.9, and monetary policy is conducted in accordance with (1.21) with no random term. Figure 4 shows the corresponding equilibrium responses in response to an increase in government purchases by an amount equal to one percent of total output, again assuming an autocorrelation coefficient of 0.9.

In each of Figures 2 through 4, we observe that the impulse responses of output,

[27] The intertemporal elasticity of substitution of the representative household is a weighted average of the elasticities of the two types in the models with preference heterogeneity, as discussed in the previous footnote.

[28] We assume a lower degree of persistence for this disturbance than for the others considered below, in order to make the shock considered in this figure similar in its implications to the identified monetary policy shocks in VAR studies. This value also makes the results shown in Figure 2 for the "RepHH" model directly comparable to those reported for the case $\rho = 0.6$ in the discussion of the basic NK model in Woodford (2003, chap. 4).

[29] In the plots, both the inflation rate and the interest rates are reported as annualized rates, so that 0.10 means an increase in the inflation rate of 10 basis points per annum. In terms of our quarterly model, what is plotted is not the response of π_t, but rather the response of $4\pi_t$.

[30] In the present model, the spread is exogenously fixed, and so there is necessarily a zero response of this variable, except in the case of a shock to the exogenous credit spread itself. However, we include this panel as we use the same format for the figures to follow, when the spread is an endogenous variable.

inflation, and the two interest rates are virtually identical under all three parameterizations of the model. (The same is true for the other non-financial aggregate disturbances — a common disturbance to the impatience to consume of all households, a disturbance to the disutility of work, a shock to government purchases, a shock to the tax rate, or a shock to the wage markup — though we do not include these figures here.) We have already explained above why this would be true in the case that the resources used in intermediation are independent of the volume of lending. Our numerical results indicate that even when we assume that intermediation uses resources (and indeed that credit spreads are entirely due to the marginal resource cost of making additional loans), and that the required resources are proportional to the volume of lending, heterogeneity and the existence of a steady-state credit spread (of a realistic magnitude) still make only a negligible difference. This is because the contribution of the banking sector to the overall variation in the aggregate demand for produced goods and services is still quite small.[31]

The inclusion of heterogeneity and an intermediary sector in the model does have one important consequence, even in this case, and that is that the model now makes predictions about the evolution of the volume of credit. As noted earlier, under our proposed calibration, a tightening of monetary policy causes credit to contract (as VARs show, especially in the case of consumer credit), despite the fact that there is no mechanical connection between monetary policy and credit supply in our model.[32] The size of the credit contraction in response to a monetary policy shock is smaller in the "FF" model than in the "NoFF" model, but it remains of the same sign. Figure 3 shows that private credit expands during an expansion caused by a productivity improvement. To the extent that this kind of disturbance is considered an important source of business cycles, our model can therefore explain the observed procyclicality of aggregate credit. (The model implies procyclical credit movements in response to a number of other types of disturbances as well — labor supply shocks, wage markup shocks, tax rate shocks, or shocks to the spending opportunities of type b

[31]Note that in each of Figures 2-4, the existence of the credit frictions in the 'FF' model makes a substantial difference for the equilibrium evolution of credit b_t relative to the prediction of the 'NoFF' model. However, this change in the size of the banking sector does not have substantial consequences for aggregate output, employment, or inflationary pressure.

[32]We do not, for example, assume that credit can only be supplied by commercial banks, that in turn can only finance their lending by attracting deposits subject to a reserve requirement — so that variations in the supply of reserves by the central bank have a direct effect on loan supply.

households — though these figures are not shown.) Figure 4 shows, instead, that not all shocks result in procyclical variations in credit. An increase in government purchases increases output, but crowds out expenditure by type b households (the more interest-sensitive ones) in particular, and so is associated with a contraction of credit. Credit similarly moves counter-cyclically in the case of variations in the spending opportunities of type s households; an increase in $\bar{C}_t^s$ increases output, but credit contracts due to the reduced supply of deposits to intermediaries (figure not shown). The observed procyclicality of aggregate credit suggests that variations in G_t and $\bar{C}_t$ are not the main sources of business cycles.

Financial frictions matter somewhat more for equilibrium dynamics if we also assume that credit spreads vary endogenously with the volume of lending. Figures 5-9 show equilibrium responses of the same aggregate variables to a variety of types of exogenous disturbances, in the case of the "convex intermediation technology" calibration ($\eta >> 1$) discussed in the previous section. Figures 5-7 show responses to the same three kinds of shocks as in Figures 2-4 respectively, but for the alternative intermediation technology. Figure 8 shows the corresponding responses to an exogenous increase in the spending opportunities of savers; and Figure 9 shows responses to an exogenous increase in real government debt by an amount equal to 1 percent of GDP. In each case (except the monetary policy shock in Figure 5[33]), the disturbance is modeled as an AR(1) process with autoregressive coefficient 0.9.

In the case of the monetary policy shock (Figure 5), we again find that the equilibrium responses of output and inflation are nearly the same in all three models, though the "FF model" is no longer quite so indistinguishable from the "NoFF" model. The most important effect of allowing for endogeneity of the credit spread is on the implied responses of interest rates to the shock. Because credit contracts in response to this shock (as noted earlier, though now by less than in Figure 2), the spread between the lending rate and the deposit rate decreases, in accordance with the empirical finding of Lown and Morgan (2002). This means that the deposit rate need no longer decline as much as does the lending rate. Moreover, because the reduced spread has an expansionary effect on aggregate demand, output declines slightly less in response to the shock than in the "NoFF" model; this is also a reason for the deposit rate to decline less. Thus the most visible effect is on the predicted response of the deposit rate, which is visibly smaller in the "FF model." The effects

[33] In this case, as in Figure 2, the monetary policy disturbance is an AR(1) process with $\rho = 0.6$.

of financial frictions are similarly mainly on the path of the deposit rate in the case of a shock to the sales tax rate τ_t (not shown).

The effects of financial frictions on variables besides the interest rates are more visible in the case of the technology shock (Figure 6). Though again the largest effect is on the path of the deposit rate, in this case the endogeneity of the markup also has non-negligible effects on the equilibrium response of output. (The primary reason for the difference is that this shock has a larger immediate effect on the path of credit, and hence a larger immediate effect on the equilibrium spread in the case of the convex technology.) Because an increase in productivity leads to an expansion of credit, credit spreads now increase in the 'FF model'; this is has a contractionary effect on aggregate demand, so that output increases less than in the "NoFF model." Similar effects of financial frictions are observed in the case of a disturbance to the disutility of working (an exogenous increase in the multiplicative factor $\bar{H}_t$ in (1.5)). The effects of an increase in the wage markup μ_t^w or the tax rate τ_t are likewise similar, but with opposite signs to the effects shown in Figure 6.

The effects of financial frictions are even more significant in the case of a shock to government purchases (Figure 7) or to the consumption demand of savers (Figure 8).[34] These are both disturbances that crowd out the expenditure of private borrowers (as the most interest-sensitive category of expenditure) to a significant extent, and so substantially reduce equilibrium borrowing and credit spreads. In each case, the reduction in spreads has a further expansionary effect on aggregate demand, so that output increases by more than in the "NoFF" model. Moreover, despite the fact that output expands more, inflation also falls more; this is because of the favorable "cost-push" effect of a reduction of credit spreads.

Note that the effect would be quite different in the case of a shock to the consumption demand of *borrowers* rather than savers (not shown). In this case, private credit would increase rather than decreasing, and by less than in Figure 8, because of the greater interest-elasticity of the demand of borrowers; this would imply a small increase in spreads, making the disturbance slightly less expansionary, but with a less dramatic effect than in Figure 8.[35] The aggregate effects of financial frictions are

[34]The shock considered here increases the value of $c^s(\lambda)$ by one percent for each possible value of λ.

[35]The effect of financial frictions in this case is somewhat similar to the case of the technology shock shown in Figure 5.

even smaller in the case of a uniform increase in the consumption demand of *both* types of households, since in this case the effects of the two types of expenditure on equilibrium credit spreads partially offset one another.

Finally, the consequences of financial frictions are of particular qualitative significance in the case of a disturbance to the path of government debt (Figure 9). Here we consider a disturbance to fiscal policy that temporarily increases the level of government debt, through a lump-sum transfer to households, which is then gradually taken back over a period of time, so that the path of real government debt is eventually the same as it would have been in the absence of the shock. In the case of the "NoFF model", *Ricardian equivalence* holds, as in the representative household model; and so in these cases, the fiscal shock has no effect on output, inflation, or interest rates. However, an increase in government borrowing crowds out private borrowing, and in the case of the convex intermediation technology, the reduced private borrowing implies a reduction in spreads. This has an expansionary effect on aggregate demand, with the consequence that both output and inflation increase, as shown in the figure.[36]

To sum up, we find that under an empirically realistic calibration of the average size of credit spreads, the mere existence of a positive credit spread does not imply any substantial quantitative difference for our model's predictions, either about the effects of a monetary policy shock or about the effects of other kinds of exogenous disturbances under a given systematic monetary policy. What matters somewhat more is the degree to which there is *variation* in credit spreads. If spreads vary endogenously (as in our model with a convex intermediation technology), then the effects of disturbances are somewhat different, especially in the case of types of disturbances — such as variations in government borrowing, or changes in the relative spending opportunities available to savers as opposed to borrowers — that particularly affect the evolution of the equilibrium volume of private credit.

Another important difference of the model with credit frictions is the possibility of exogenous disturbances to the financial sector itself, represented by exogenous

[36] Ricardian equivalence does not hold precisely in the "FF model" even in the case of the linear intermediation technology. However, in this case (not shown) there is no reduction in credit spreads in response to the shock, and the only consequence for aggregate demand comes from the reduction in the resources used by the banking sector, so that shock is actually (very slightly) *contractionary* in this case. But there is very little difference in the predictions of the "NoFF" and "FF" models in the case of that technology, so that we omit the figure here.

variation in either the intermediation technology $\Xi_t(b)$ or the default rate schedule $\chi_t(b)$. Again, these disturbances matter to the determination of aggregate output, inflation and interest rates primarily to the extent that they imply variation in credit spreads. Figure 10 shows the responses (for the "FF" model only, and in the case of the convex intermediation technology) to an exogenous shift up in the schedule $\chi_t(b)$, of a size that would increase the credit spread by 2 percentage points (as an annualized rate) for a given volume of private credit. (Because of the contraction of credit that results, in equilibrium the shock only increases the credit spread by a little over one percentage point.) Under the baseline Taylor rule, this kind of "purely financial" disturbance increases the credit spread and contracts aggregate credit; it also contracts real activity and lowers inflation. (An increase in the credit spread owing to an increase in the marginal resource cost of intermediation has similar effects, not shown.) We show below that these responses to the shock are not desirable on welfare grounds. One of key issues taken up in the next section is whether modification of the baseline Taylor rule to directly respond to financial variables can improve upon these responses.

2 Adjustments to the Baseline Taylor Rule

We turn now to the consequences of modifying the baseline Taylor rule by including a direct response to some measure of financial conditions. We first discuss the welfare criterion that we use to evaluate candidate policy rules, and then turn to our results for some simple examples of modified Taylor rules.

2.1 Welfare criterion

We shall suppose that the objective of policy is to maximize the average ex ante expected utility of the households. As shown in Cúrdia and Woodford (2009), this implies an objective of the form

$$E_0 \sum_{t=0}^{\infty} \beta U(Y_t, \lambda_t^b, \lambda_t^s, \Delta_t; \tilde{\xi}_t) \tag{2.1}$$

where

$$U(Y_t, \lambda_t^b, \lambda_t^s, \Delta_t; \tilde{\xi}_t) \equiv \pi_b u^b(c^b(\lambda_t^b; \xi_t); \xi_t) + \pi_s u^s(c^s(\lambda_t^s; \xi_t); \xi_t) - \frac{\psi}{1+\nu} \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega} \Delta_t, \qquad (2.2)$$

and

$$\tilde{\Lambda}_t \equiv \psi^{\frac{1}{1+\nu}} \left[\pi_b \psi_b^{-\frac{1}{\nu}} \left(\lambda_t^b\right)^{\frac{1+\nu}{\nu}} + \pi_s \psi_s^{-\frac{1}{\nu}} \left(\lambda_t^s\right)^{\frac{1+\nu}{\nu}}\right]^{\frac{\nu}{1+\nu}}.$$

Note that the final term in (2.2) represents the average disutility of working, averaging both over the entire continuum of types of labor j and over the two types of households, using the model of equilibrium labor supply discussed in section 1.1.

Using this welfare criterion, we can compute the equilibrium responses to the various types of shocks in our model under an optimal policy commitment (the Ramsey policy problem). This problem is treated in more detail in Cúrdia and Woodford (2009). Here we are interested not in characterizing fully optimal policy, but in the extent to which various simple modifications of the Taylor would result in a closer approximation to Ramsey policy. One way in which we judge the closeness of the approximation is by comparing the responses to shocks under candidate policy rules to those that would occur under the Ramsey policy.

We also evaluate the level of welfare associated with alternative simple rules (modified Taylor rules of various types), using a method proposed by Benigno and Woodford (2008). Under this approach, one computes (for the equilibrium associated with each candidate policy rule) the value of a quadratic approximation to the Lagrangian for an optimization problem that corresponds to the continuation of a previously chosen Ramsey policy; this approximate Lagrangian is minimized by a time-invariant linear rule under which the responses to shocks are the same (to a linear approximation) as under the Ramsey policy. By computing the value of this Lagrangian under a given time-invariant policy rule, we have a criterion that would rank as best (among all possible linear rules) a rule that achieves exactly the responses to shocks associated with the Ramsey policy. We use this method to rank the benefits from alternative spread-adjusted or credit-adjusted Taylor rules; this is a more formal way of assessing the degree to which a given modification of the Taylor rule leads to responses to shocks that are closer to those implied by Ramsey policy.[37]

[37] See Altissimo *et al.* (2005) for discussion of a numerical method that can be used to compute this welfare measure.

2.2 Spread-Adjusted Taylor Rules

Let us first consider generalizations of (1.21) of the form

$$\hat{\imath}_t^d = \phi_\pi \pi_t + \phi_y \hat{Y}_t - \phi_\omega \hat{\omega}_t, \tag{2.3}$$

for some coefficient $0 \leq \phi_\omega \leq 1$. These rules reflect the idea that the funds rate should be lowered when credit spreads increase, so as to prevent the increase in spreads from "effectively tightening monetary conditions" in the absence of any justification from inflation or high output relative to potential. They essentially correspond to the proposal of authors such as McCulley and Toloui (2008) and Taylor (2008), except that we consider the possible advantages of a spread adjustment that is less than the size of the increase in credit spreads. (The proposal of these authors corresponds to the case $\phi_\omega = 1$; the classic Taylor rule corresponds to the opposite limiting case, $\phi_\omega = 0$.) We now omit the random term ϵ_t^m, as there is nothing desirable about unnecessary randomization of policy.[38]

In the previous section, we have discussed the economy's response to a variety of types of disturbances under this kind of policy rule, in the case that $\phi_\omega = 0$. We now consider the consequences of alternative values for $\phi_\omega > 0$, and compare the equilibrium responses to shocks under this kind of policy to those under Ramsey policy (i.e., an optimal policy commitment). Figures 11-17 present numerical responses in the case of several different types of exogenous disturbances, when the model is calibrated in the same way as in the previous section, for the case of a convex intermediation technology.

Figure 11 shows the responses of endogenous variables to a "financial shock" of the same kind as in Figure 10, but for variant monetary policy rules of the form (2.3). The figure shows the responses in the case of five different possible values of ϕ_ω, ranging between 0 and 1. The response of each variable under the Ramsey policy is also shown (as a solid blue line). We observe that adjusting the intercept of the Taylor rule in response to changes in the credit spread can indeed largely remedy the defects of the simple Taylor rule, in the case of a shock to the economy of this

[38]In fact, in some models arbitrary randomization of monetary policy would raise welfare, as in the example of Dupor (2003). But we verify that under our numerical parameterization, the Lagrangian for our policy problem is locally convex, so that randomization is necessarily welfare-reducing (at least in the case of a small enough random term). See Benigno and Woodford (2008) for discussion of this issue, and the algebraic conditions that must be verified.

Table 1: Optimal value of the spread-adjustment coefficient ϕ_ω in policy rule (2.3), in the case of a convex intermediation technology. Each column indicates a different type of disturbance, for which the policy rule is optimized; each row indicates a different possible degree of persistence for the disturbance.

	Z_t	μ_t^w	τ_t	G_t	b_t^g	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	5.71*	5.71*	5.12	2.87	0.63	5.71*	-0.04	2.08	0.86	0.65
$\rho_\xi = 0.50$	5.71*	5.71*	5.71*	3.90	0.71	5.71*	0.17	1.03	0.84	0.72
$\rho_\xi = 0.90$	5.71*	5.71*	5.71*	-1.31	0.75	5.71*	1.57	-0.36	0.82	0.75
$\rho_\xi = 0.99$	5.71*	5.71*	5.71*	-8.51	0.65	5.71*	5.71*	-3.89	0.70	0.67

* higher number leads to indeterminacy

kind. And the optimal degree of adjustment is close to 100 percent, as proposed by McCulley and Toloui and by Taylor. To be more precise, both inflation and output increase a little more under the 100 percent spread adjustment than they would under Ramsey policy; but the optimal responses of both variables are between the paths that would result from a 75 percent spread adjustment and the one that results from a 100 percent spread adjustment. If we optimize our welfare criterion over policy rules with alternative values of ϕ_ω, assuming that this type of disturbance is the only kind that ever occurs, the welfare maximum is reached when $\phi_\omega = 0.82$, as shown in Table 1.

It is interesting to observe in Figure 11 that, while a superior policy involves a reduction in the policy rate relative to what the unadjusted Taylor rule would prescribe, this does not mean that under such a policy the central bank actually cuts its interest rate target more sharply in equilibrium. The size of the fall in the policy rate (shown in the middle left panel) is about the same regardless of the value of ϕ_ω; but when ϕ_ω is near 1, output and inflation no longer have to decline in order to induce the central bank to accept an interest-rate cut of this size, and in equilibrium they do not decline. (In fact, the nominal policy rate does fall a little more, and since expected inflation does not fall, the *real* interest rates faced by both savers and borrowers fall more substantially when ϕ_ω is near 1.) The contraction of private credit in equilibrium is also virtually the same regardless of the value of ϕ_ω. Nonetheless, aggregate expenditure falls much less when ϕ_ω is positive; the expenditure of borrowers no longer has to be cut back so much in order to reduce their borrowing, because their labor income no longer falls in response to the shock, and there is an offsetting increase in the expenditure of savers.

The figure is very similar in the case of an exogenous shock to the marginal resource cost of intermediation (an exogenous increase in the multiplicative factor $\tilde{\Xi}_t$, not shown). As indicated in Table 1, in this case the optimal response coefficient is only slightly smaller, 0.75. The other comments about the shock to $\tilde{\chi}_t$ apply equally to this case. The responses to a purely financial shock, both under the modified Taylor rules and under optimal policy, do depend greatly on the assumed persistence of the disturbance. Figure 12 shows the responses to a shock to the default rate (an exogenous increase in the factor $\tilde{\chi}_t$) of the same magnitude as in Figure 11, but under the assumption that the disturbance lasts for only one quarter. Under the unadjusted Taylor rule, such a shock again contracts output and reduces inflation in the quarter of the disturbance; but both output and inflation then overshoot their long-run levels in the quarter following the shock, as a consequence of the reduced level of private indebtedness. A spread-adjusted Taylor rule leads both to smaller immediate declines in output and inflation (or even to increases, in the case of a sufficiently large spread adjustment), and to smaller subsequent increases in output and inflation (or even to decreases in both variables in the quarter following the disturbance, if the spread adjustment is large enough). One observes that the responses of output and inflation under optimal policy again lie between those resulting from the simple rules with $\phi_\omega =$ 0.75 and with $\phi_\omega = 1.0$; as Table 1 shows, the optimal value of the response coefficient is actually 0.86. In fact, as Table 1 shows, the optimal value of ϕ_ω in the case that shocks of this kind are the only disturbance to the economy is fairly similar, regardless of the persistence of the shocks; a nearly complete (though not quite complete) offset for the spread variation is optimal in each of the cases considered. Again, we obtain broadly similar conclusions in the case of disturbances to $\tilde{\Xi}_t$; the optimal values of ϕ_ω (as shown in Table 1) are slightly smaller, but in all cases greater than 0.5.

However, our conclusions about the optimal spread adjustment are considerably more varied when we consider other kinds of disturbances. (In the model with an endogenous credit spread, $\eta > 0$, a spread adjustment in the Taylor affects the economy's equilibrium response to disturbances of all types, and not just disturbances originating in the financial sector.) Even if we restrict our attention to disturbances that with a serial correlation $\rho = 0.9$ (the third row of Table 1), we see from the Table that the optimal spread adjustment is quite different for different disturbances. In the case of an exogenous disturbance to the level of government debt b_t^g, the optimal spread adjustment is again a large fraction of 1, regardless of the degree of persistence; this is because in our model, the effects of a government debt shock are essentially equivalent to a disturbance to the financial intermediation technology (as government borrowing crowds out private borrowing). But the results for other disturbances are much less similar.

Figure 13 shows the equilibrium responses to an exogenous increase in the productivity factor Z_t, again for the case $\rho = 0.9$. We observe that equilibrium responses under the unadjusted Taylor rule are quite different than those under optimal policy: output does not increase nearly as much as would be optimal, and inflation sharply declines, while under optimal policy it would not decline (and indeed would very slightly increase). (Essentially, this is because the Taylor rule reacts to a productivity-driven boom by raising interest rates, unless inflation falls sharply enough for this no longer to be required, as it does in equilibrium owing to the monetary tightening.) Because such a boom is associated with a credit expansion, the credit spread rises in the case of an endogenous spread; hence a spread adjustment $\phi_\omega > 0$ adjusts policy in the right direction, accommodating the boom to a somewhat greater extent. But as shown in the figure, even a 100 percent spread adjustment is not nearly enough of a modification of the baseline Taylor rule to correct this problem; monetary policy remains much too contractionary in response to such a shock. The optimal response, in the case that shocks of this kind were the only disturbances in the model, would be a value of ϕ_ω much greater than 1. If we optimize over the value of ϕ_ω, imposing only the constraint that the policy rule (2.3) must lead to a determinate rational-expectations equilibrium,[39] we find that welfare is maximized by making ϕ_ω as large as is possible given the determinacy constraint. (In the case of our calibrated parameter values, determinacy requires that we restrict attention to values $\phi_\omega \leq 5.71$.) The same conclusion is reached in the case of a shock to attitudes toward labor supply $\bar{H}_t$, a shock to the wage markup μ_t^w, or a shock to the tax rate τ_t.

In other cases, the optimal spread adjustment is less extreme, but still greater than 100 percent of the increase in the credit spread. Figure 14 shows the equilibrium responses to an exogenous increase in the factor $\bar{C}_t^b$, representing an increase

[39] For further discussion of this requirement, and its relevance to the choice of a monetary policy rule, see, for example, Woodford (2003, chap. 4).

in the spending opportunities of type b households, again under the assumption that $\rho = 0.9$. As in the case of a productivity shock, the unadjusted Taylor rule tightens policy in response to an output increase that is actually efficient, and so is too contractionary. Because this kind of boom is associated with a credit boom, the credit spread increases, and a spread adjustment $\phi_\omega > 0$ modifies the policy rule in a desirable direction. But again, even a 100 percent spread adjustment is insufficient. As shown in Table 1, the optimal spread adjustment coefficient would be greater than 1.5. (It is much less than in the case of the productivity shock, however, because this kind of disturbance leads to more procyclical credit.)

Our conclusions are quite different, however, if one considers instead a disturbance to the spending opportunities of type s households (increase in $\bar{C}_t^s$). As shown in Figure 15, the unadjusted Taylor rule is too contractionary in this case as well. But because the shock results in counter-cyclical variation in private credit, and hence in the credit spread, a spread adjustment $\phi_\omega > 0$ changes the Taylor rule in the wrong direction: as shown in the figure, this would make monetary policy even more excessively contractionary in response to this kind of disturbance. In fact, as shown in Table 1, the optimal spread adjustment would have the opposite sign ($\phi_\omega = -.36$). We obtain a similar conclusion (for essentially the same reason) in the case of a shock to government purchases G_t (again assuming $\rho = 0.9$), as shown in Figure 16. This is another example of an expansionary shock that reduces private credit (because government purchases crowd out mainly the spending of type b households, which is the more interest-sensitive kind of private expenditure) and so reduces the equilibrium credit spread; a spread adjustment then modifies the baseline Taylor rule in the wrong direction. (As shown in Table 1, the optimal adjustment would actually be $\phi_\omega = -1.31$.)

Many of these results are also quite sensitive to the assumed degree of persistence of the disturbance. For example, in the case of shocks to government purchases, if the disturbance has a coefficient of serial correlation $\rho = 0.5$, as assumed in Figure 17, the optimal spread adjustment is positive. The reason is that in this case, unlike the one shown in Figure 16, monetary policy is *too expansionary* under the baseline Taylor rule; hence welfare is improved by a positive spread adjustment, which in this case would *raise* the policy rate owing to the decline in the credit spread. In fact, the optimal spread adjustment is much larger than 100 percent ($\phi_\omega = 3.90$). Instead, in the case of a higher degree of persistence (for example, the case $\rho = 0.9$ shown in Figure 16), policy is too tight under the baseline Taylor rule, and the optimal spread adjustment is negative. In fact, if the serial correlation is instead $\rho = 0.99$, the optimal spread adjustment is not only negative, but also very large ($\phi_\omega = -8.51$).

Thus in the endogenous-spread case, we certainly cannot say in general that a positive spread adjustment is necessarily an improvement upon the baseline Taylor rule, let alone that the appropriate adjustment will generally be of about the size of the increase in the credit spread. The optimal spread adjustment is quite different in the case of different types of disturbances (including disturbances of different degrees of persistence). It is not possible to offer a general statement about the optimal spread adjustment without reaching a view about the quantitative importance of the different types of theoretically possible disturbances in practice.

When doing this, it is important to consider not only the optimal spread adjustment in the case of a given type of disturbance, but also the size of the change in welfare achieved by a spread adjustment in each case. Table 2 reports the welfare change (relative to the baseline Taylor rule) for each of the types of shocks, for each of several different possible sizes of spread adjustment (the same four values of ϕ_ω considered in the figures). The first part of the table shows results for the case of disturbances with zero persistence, the second part for the case of disturbances with $\rho = 0.9$. In the case of each type of disturbance, the amplitude of the shock is normalized so that the standard deviation of fluctuations in output around trend will be one percentage point, in the case that that disturbance is the only kind that exists.

When considering the overall advantage of a given increase in the spread adjustment, it is necessary to consider the implications for the way in which the economy will respond to *all* of the different types of disturbances to which it is subject at different times. It is possible to determine this, however, by looking across a given row of the table. For example, suppose that in a given economy, 50 percent of output fluctuations (relative to trend) are due to productivity shocks, 25 percent are due to variations in the level of government purchases, and 25 percent are due to credit spread variations resulting from shocks to the default rate. Suppose furthermore that each of the three types of disturbances that occur have serial correlation coefficient $\rho = 0.9$ (so that the second part of Table 2 applies), and that the three disturbances are independent of one another (so that we can simply sum the contributions of the three disturbances to our quadratic loss function).[40] Then a change in the value of ϕ_ω

[40]Of course, there is no reason why these disturbances are necessarily distributed independently

Table 2: Welfare consequences of increasing ϕ_ω, in the case of different disturbances. Each column indicates a different type of disturbance, while each row corresponds to a given degree of spread adjustment.

	Z_t	μ_t^w	τ_t	G_t	b_t^g	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
No persistence ($\rho_\xi = 0$)										
$\phi_\omega = 0.25$	89.5	50.8	35.7	1.5	23.7	89.5	-1.0	6.7	33.4	24.4
$\phi_\omega = 0.50$	179.6	101.6	71.1	2.9	36.2	179.6	-3.7	12.6	56.5	37.7
$\phi_\omega = 0.75$	270.4	152.2	106.1	4.3	36.4	270.4	-8.2	17.7	68.3	38.7
$\phi_\omega = 1.00$	361.9	202.7	140.6	5.7	22.8	361.9	-14.9	22.0	67.3	25.9
Persistence ($\rho_\xi = 0.9$)										
$\phi_\omega = 0.25$	66.2	60.8	58.9	-29.4	47.3	66.2	119.0	-41.2	52.6	47.6
$\phi_\omega = 0.50$	132.4	121.7	117.8	-64.9	76.8	132.4	220.2	-105.4	87.7	77.4
$\phi_\omega = 0.75$	198.6	182.5	176.6	-106.8	86.8	198.6	302.6	-193.9	103.8	87.8
$\phi_\omega = 1.00$	264.8	243.3	235.4	-155.4	75.5	264.8	365.0	-307.9	98.9	77.0

will raise welfare if and only if raises $W^{tot} = 0.5W_Z + 0.25W_G + 0.25W_{\tilde{\chi}}$, where W_Z is the welfare measure reported in the Z_t column of Table 2, W_G is the welfare measure reported in the G_t column, and so on. For example, in the case of an increase in ϕ_ω from 0.75 to 1.00, the table indicates that W_Z increases, while W_G and $W_{\tilde{\chi}}$ both fall. However, the increase in W_Z is larger than the declines in either of the other two quantities. If we use the weights just proposed, W^{tot} increases by a net amount of 19.7, so that the increase to a 100 percent adjustment would be beneficial in welfare terms, despite the fact that it leads to a less optimal response to two of the types of disturbances.[41] Among the cases considered in the table, $\phi_\omega = 1.00$ achieves the highest value of W^{tot}. In fact, W^{tot} would be maximized by setting $\phi_\omega = 1.49$. (This represents a compromise among the values that would be optimal for each of the

of one another. For example, the preferences of type b households and of type s housholds need not fluctuate independently of one another. But to deal with this possibility, we would need additional information beyond that reported in Table 2. In effect, we would have to consider additional types of disturbances besides those reported in the table: a disturbance that raises $\bar{C}_t^b$ and $\bar{C}_t^s$ in the same proportion, a disturbance that raises τ_t by half the amount of the increase in G_t, and so on.

[41]This assumes, of course, that only policy rules within the restricted family (2.3) are considered.

Table 3: Optimal value of the spread-adjustment coefficient ϕ_ω in policy rule (2.3), as in Table 1, but for the case of a linear intermediation technology.

	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	1.72	1.19
$\rho_\xi = 0.50$	1.51	1.30
$\rho_\xi = 0.90$	0.24	0.27
$\rho_\xi = 0.99$	-2.39	-2.32

three types of disturbance individually, as reported in Table 1.)

This result, however, is quite dependent on which types of disturbances are thought to account for greater shares of the variance decomposition of output fluctuations. If, for example, one supposes that 50 percent of output fluctuations are due to productivity shocks and 50 percent to fluctuations in $\bar{C}_t^s$ (again, assuming $\rho = 0.9$ for both disturbances and that they are independent of one another), then the appropriate welfare measure would be $W^{tot} = 0.5W_Z + 0.5W_{C^s}$. One observes that this measure increases when one moves from $\phi_\omega = 0.50$ to 0.75 (W_Z increases by more than W_{C^s} falls), but that it decreases if one increases the spread adjustment further from 0.50 to 0.75 (W_{C^s} falls by more than W_Z increases). In fact, in this case, W^{tot} would be maximized by $\phi_\omega = 0.40$. If we allow for the fact that disturbances with many different degrees of persistence are also possible (the cases in Table 1 representing only a few of the simplest possibilities), then the range of possible conclusions about the optimal spread adjustment are even larger.

The considerations involved in judging the optimal spread adjustment are simpler in the case that we assume a linear intermediation technology (along with our maintained assumption in the above calculations that $\chi_t(b)$ is linear). In this case, the credit spread is an exogenous process, so that a spread adjustment to the Taylor rule has no consequences (in our log-linear approximation) for the economy's response to disturbances other than purely financial disturbances (shocks to $\tilde{\chi}_t$ or to $\tilde{\Xi}_t$, the two determinants of the credit spread). Moreover, the consequences of a spread adjustment are quite similar in the case of these two types of financial disturbances; so it might seem that we should be able to choose the spread adjustment so as to optimize the response to a single type of shock. However, as shown in Table 3, the optimal spread adjustment is quite different depending on the degree of persistence of the financial disturbances. It is positive and even greater than 1, in the case of either type of disturbance, if the degree of persistence is $\rho = 0.5$ or less. But the optimal degree of spread adjustment is much smaller (on the order of 0.25, for either type of disturbance) if instead we assume $\rho = 0.9$. In the case of even more persistent financial disturbances, the optimal spread adjustment changes sign. If, for example, we assume $\rho = 0.99$, the optimal spread adjustment is more negative than -2, for either type of disturbance.

To summarize, while under many assumptions welfare can be improved by a spread adjustment $\phi_\omega > 0$, the optimal size of spread adjustment need not be even approximately one-for-one (as suggested by discussions such as those of Taylor and of McCulley and Toloui), and depends on which kinds of disturbances are most important as sources of aggregate instability. In the case of a convex intermediation technology parameterized to imply a sharply rising marginal cost of intermediation (our baseline case), a spread-adjustment coefficient that is a large fraction of 1 can be justified in the case that the most important disturbances are ones that affect the economy primarily by affecting the efficiency of intermediation (the $\tilde{\chi}_t, \tilde{\Xi}_t$, and b_t^g shocks). A spread adjustment that is positive but much larger than one-for-one is instead preferred if "supply shocks" (the $Z_t, \bar{H}_t, \mu_t^w$, or τ_t shocks) are the main source of instability. To the extent that "demand shocks" are instead important, our results are more complex; the optimal size and even the optimal sign of the spread adjustment depends both on which type of demand disturbance is more important and on the degree of persistence of these disturbances. Nonetheless, unless one thinks that aggregate fluctuations are driven mainly by (certain types of) real "demand shocks" (highly persistent disturbances to G_t or $\bar{C}_t^s$), a modestly positive value of ϕ_ω is almost certainly an improvement over the unadjusted Taylor rule, though the optimal degree of adjustment can easily be less than one-for-one.

2.3 A Spread Adjustment Combined with Adjustment for Natural Rate Variations

One of the main reasons that our conclusions about the optimal spread adjustment in the previous section depend to such an extent on the types of disturbances affecting the economy is that the baseline Taylor rule fails to take appropriate account of the different types of real disturbances that may be causing inflation and output to vary, even in the absence of credit frictions. (For example, when productivity increases, it is desirable to allow output to increase, but the baseline Taylor rule tightens policy in response to the output increase regardless of whether productivity has increased.) In the case of many of the disturbances discussed in the previous section, the optimal spread adjustment is one that moves the policy rate in a way that helps to correct for this defect (because of the way that spreads happen to be affected by the disturbance in question), even though the kind of interest-rate adjustment that is needed has little to do with credit market imperfections.

This suggests that we might expect to obtain a more robust recommendation about the appropriate spread adjustment if the baseline rule involved a more nearly optimal response to each of the various types of real disturbances that should affect interest-rate policy in the absence of credit frictions. A relative simple way of improving upon the simple rule (1.21) is by taking account of variations in both the "natural rate of output" and the "natural rate of interest," so that the rule becomes

$$\hat{\imath}_t^d = r_t^n + \phi_\pi \pi_t + \phi_y (\hat{Y}_t - \hat{Y}_t^n). \tag{2.4}$$

Here we define $\hat{Y}_t^n$ as the equilibrium level of output (given the values of the various exogenous disturbances, except that ω_t and Ξ_t are set equal to their steady-state values) under flexible prices, and r_t^n as the equilibrium real rate of interest under the same hypothetical circumstances. Note that both $\hat{Y}_t^n$ and r_t^n are functions of the exogenous disturbances, and independent of monetary policy.

These adjustments make the simple Taylor rule a closer approximation to optimal policy in the absence of credit frictions.[42] However, they do nothing to change the model's response to purely financial disturbances, so that there is still reason to expect that a spread adjustment might improve upon the basic rule, at least if financial disturbances are sufficiently important. Hence we consider the more general family of modified Taylor rules

$$\hat{\imath}_t^d = r_t^n + \phi_\pi \pi_t + \phi_y (\hat{Y}_t - \hat{Y}_t^n) - \phi_\omega \hat{\omega}_t, \tag{2.5}$$

for alternative values of ϕ_ω.

[42] See Woodford (2003, chaps. 4, 6) for demonstration that at least under certain special assumptions — an undistorted steady state, and no "cost-push" shocks — this rule achieves fully optimal responses to a number of types of real disturbances, including leading examples of both "supply shocks" and "demand shocks", in the representative-household model (basic NK model).

Table 4: Optimal value of the spread-adjustment coefficient ϕ_ω in policy rule (2.5), for the same set of possible disturbances as in Table 1, and a convex intermediation technology.

	Z_t	μ_t^w	τ_t	G_t	b_t^g	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	1.05	-10.28	-16.56	1.48	0.63	1.05	0.64	1.07	0.86	0.65
$\rho_\xi = 0.50$	1.32	-3.00	-5.06	2.11	0.71	1.32	0.59	0.72	0.84	0.72
$\rho_\xi = 0.90$	0.20	-0.21	-0.37	0.29	0.75	0.20	0.15	0.16	0.82	0.75
$\rho_\xi = 0.99$	-1.39	-1.46	-1.48	-1.37	0.65	-1.39	-1.39	-1.39	0.70	0.67

Table 4 reports the optimal spread adjustment coefficient in the rule (2.5), for each of a variety of individual disturbances, using the same format as in Table 1 earlier. (As in Table 1, we now again assume the convex intermediation technology, so that credit spreads are endogenous.) One observes that in the case of the disturbance to Z_t, for example, it is no longer optimal to increase the spread adjustment coefficient to the greatest extent consistent with determinacy of equilibrium.[43] As Figure 18 shows for the case of a productivity disturbance with $\rho = 0.9$ (the same disturbance as in Figure 13), the Taylor rule with no spread adjustment now results in responses of output, inflation and interest rates that are much closer to those required for Ramsey policy than was true before. But since it remains true that the credit spread increases in response to such a disturbance, a spread adjustment ($\phi_\omega > 0$) will again imply greater accommodation of the productivity increase; and in the case shown in the figure, this means that a 100 percent spread adjustment would be much too large. (In fact, as shown in Table 4, the optimal adjustment in this case is only $\phi_\omega = 0.20$.)

Nor is the optimal spread adjustment any longer as *negative* as before, in the case of a highly persistent disturbance to the level of government purchases. Figure 19 considers again the same disturbance as in Figure 16, but for policy rules in the family (2.5). Here too, the responses without any spread adjustment are now closer to those associated with Ramsey policy. And if anything, a small positive spread adjustment now makes the responses closer to the optimal ones (rather than moving

[43]The determinacy threshold itself is unchanged in this case, since the additional terms in (2.5) relative to (2.3) are functions only of the exogenous disturbances, and have no effect on the determinacy of the equilibrium values of the endogenous variables.

them farther away, as in Figure 16); for example, it can bring about the small initial decline in inflation at the time of the shock that one observes under Ramsey policy. As shown in Table 4, the optimal spread adjustment is now a small positive value, 0.29.

Indeed, it is now quite generally true of those real disturbances that do not affect the economy primarily through their effects on the credit spread (the $\tilde{\chi}_t, \tilde{\Xi}_t$, and b_t^g shocks) and that are not sources of inefficiency even in the representative-household, flexible-price version of our model (i.e., the $\hat{\mu}_t^w$ or τ_t shocks) that, when the degree of persistence is on the order of 0.9, the optimal spread adjustment is only slightly positive (varying between 0.15 and 0.29). Our results remain different, however, in the case of the other two types of disturbances. For disturbances that primarily affect credit spreads, the optimal spread adjustment remains a much larger fraction of 1 (though still less than 1); for the natural-rate adjustments in (2.5) do not change the effects of these disturbances (which have no effects on the natural rates). And in the case of the disturbances that affect other microeconomic distortions (the μ_t^w and τ_t shocks), the optimal spread adjustment has the opposite sign. Figure 20 shows why, in the case of a shock to the tax rate. In the case of this kind of disturbance, the natural-rate-adjusted Taylor rule with no spread adjustment is too contractionary, and a spread adjustment $\phi_\omega > 0$ would only make this worse, since the disturbance contracts credit and reduces the credit spread (so that the spread adjustment would further tighten policy). The optimal spread adjustment in this case is actually $\phi_\omega = -0.37$, as reported in Table 4.

Despite these somewhat differing results, we can actually reach a fairly robust conclusion about the best form of spread adjustment, if we are willing to assume that all significant disturbances have a persistence on the order of $\rho = 0.9$. Under almost all plausible assumptions about the variance decomposition of aggregate fluctuations, the optimal spread adjustment will be positive, but well below 1. For example, if we assume that output fluctuations are 50 percent due to productivity disturbances, 25 percent due to variations in government purchases, and 25 percent due to variations in the default rate (as in the example discussed earlier), then in the case of the family of policy rules (2.5), the optimal value of ϕ_ω is only 0.62 (rather than much larger than 1, as concluded earlier).

However, these results turn out to be sensitive to the degree of persistence of the disturbances (in all cases except that of the "financial" disturbances). If one allows for disturbances that are either much less persistent or much more persistent than assumed in our baseline case, one finds cases in which the optimal spread adjustment may either be much greater than 1, or a large negative quantity. Thus it is hard to reach definite conclusions without taking a stand on the quantitative importance of different types of disturbances as sources of aggregate fluctuations.

One conclusion can, however, be offered without undertaking a detailed empirical analysis of that kind. Because the response coefficients that are most appropriate in a rule of the form (2.3) or (2.5) depends on the type of disturbance to which an economy is subject, a targeting approach to the conduct of policy — in which the central bank must decide at each decision point which path for interest rates is most consistent with its *target criterion*, taking into account everything that it knows about the particular disturbances to which the economy has recently been subjected — can implement a better overall policy, given the variety of types of disturbances that actually occur, than any single rule of one of these simple forms.[44] In Cúrdia and Woodford (2009), we give an example of a simple target criterion, involving only the projected paths for inflation and the output gap, the pursuit of which would lead (in our model, and under the assumption of accurate real-time information about disturbances) to nearly optimal responses to *all* of the different types of disturbances considered in our tables.

2.4 Responding to Variations in Aggregate Credit

Some have suggested that because of imperfections in financial intermediation, it is more important for central banks to monitor and respond to variations in the volume of bank lending than would be the case if the "frictionless" financial markets of Arrow-Debreu theory were more nearly descriptive of reality. A common recommendation in this vein is that monetary policy should be used to help to stabilize aggregate private credit, by tightening policy when credit is observed to grow unusually strongly and loosening policy when credit is observed to contract. For example, Christiano *et al.* (2007) propose that a Taylor rule that is adjusted in response to variations in aggregate credit may represent an improvement upon an unadjusted Taylor rule.

In order to consider the possible advantages of such an adjustment, we now pro-

[44] See Svensson (1999, 2003), Svensson and Woodford (2005), or Woodford (2007) for the general advantages of targeting approaches.

pose to replace (1.21) by a rule of the form

$$\hat{\imath}_t^d = \phi_\pi \pi_t + \phi_y \hat{Y}_t + \phi_b \hat{b}_t, \tag{2.6}$$

for some coefficient ϕ_b, the sign of which we shall not prejudge. (Christiano *et al.*, like most proponents of credit-based policies, argue for the desirability of a positive coefficient.) Figure 21 illustrates the consequences of alternative degrees of response (of either sign) to credit variations, in the case of the same kind of increase in government purchases as in Figures 16 and 19, again in an economy with a convex intermediation technology, and with ϕ_π and ϕ_y set at the Taylor values.

Because in the case of a convex intermediation technology (and in the absence of "purely financial" disturbances) the credit spread ω_t is a monotonic function of the aggregate volume of private credit b_t (and in our log-linear approximation, $\hat{\omega}_t$ is a *linear* function of $\hat{b}_t$), any rule of the form (2.6) is actually *equivalent* to a particular rule of the form (2.3), as far as our model's predictions about the responses to all non-financial shocks are concerned. Under our calibration, a rule of the form (2.6) with a coefficient ϕ_b is equivalent to a rule of the form (2.3) with coefficient $\phi_\omega = -\phi_b$. Hence the results shown in Figure 21 (at least for the two cases with $\phi_b < 0$) are actually the same as those in Figure 16 (for the corresponding values of $\phi_\omega > 0$). As noted before, the optimal spread adjustment in this case would actually be *negative*; this means that a positive coefficient ϕ_b would similarly increase welfare, as the equilibrium responses are moved somewhat closer to those associated with Ramsey policy, as shown in the figure. In fact, the optimal adjustment in the case of this one type of disturbance would be $\phi_b = 1.31$.

Table 5 reports the optimal value of ϕ_b in the rule (2.6), in the case of each of the types of individual disturbances considered in Table 1, using the same format as the earlier table.[45] The results for disturbances other than $\tilde{\chi}_t$ and $\tilde{\Xi}_t$ all follow directly from the results in Table 1. As before, our most important finding is that both the sign and magnitude of the optimal response coefficient depends on which types of disturbances one is concerned with. However, to the extent that our previous

[45]The coefficients in the table indicate the desired increase in the policy rate target, expressed in percentage points per year, per percentage point increase in real aggregate credit. Thus $\phi_b = 1.31$ means that a one percent greater volume of aggregate credit raises the operating target for the policy rate by 1.31 percentage points per year, in the absence of any change in inflation or output. If, in equation (2.6), $\hat{\imath}_t^d$ and π_t are understood to be quarterly rates, then the coefficient on $\hat{b}_t$ in that equation should be written as $\phi_b/4$.

Table 5: Optimal value of the response coefficient ϕ_b in policy rule (2.6), for the same set of possible disturbances as in Table 1, and a convex intermediation technology.

	Z_t	μ^w_t	τ_t	G_t	b^g_t	$\bar{H}_t$	$\bar{C}^b_t$	$\bar{C}^s_t$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	-5.71*	-5.71*	-5.12	-2.87	-0.63	-5.71*	0.04	-2.08	1.12	0.95
$\rho_\xi = 0.50$	-5.71*	-5.71*	-5.71*	-3.90	-0.71	-5.71*	-0.17	-1.03	0.41	0.40
$\rho_\xi = 0.90$	-5.71*	-5.71*	-5.71*	1.31	-0.75	-5.71*	-1.57	0.36	0.06	0.06
$\rho_\xi = 0.99$	-5.71*	-5.71*	-5.71*	8.51	-0.65	-5.71*	-5.71*	3.89	0.00	0.00

* lower number leads to indeterminacy

results provided some support for the view that a positive value of ϕ_ω is likely to be beneficial (even in the case of non-financial disturbances), this would correspond to a preference for a *negative* value of ϕ_b, rather than a positive value as assumed in most discussion of this proposal.

However, the results in Table 1 according to which it is desirable for ϕ_ω to be positive in the case of "purely financial" disturbances do *not* imply that it is optimal for ϕ_b to be negative, since these disturbances shift the equilibrium relation between aggregate credit and the credit spread. In fact, Table 5 shows that the optimal ϕ_b in the case of either of the two types of purely financial disturbances is at least slightly positive. As in our discussion of the spread adjustment, we find that it is desirable to loosen policy in response to a shock that increases $\omega_t(\bar{b})$, to a greater extent than would occur under the unadjusted Taylor rule; but because credit *contracts* in response to such a disturbance (at the same time that the credit spread increases), this is achieved by setting $\phi_b > 0$. Nonetheless, the table shows that except when the disruption of financial intermediation is quite transitory, the optimal response coefficient is quite small. Figure 22 shows how alternative sizes of responses to aggregate credit change the equilibrium responses to an increase in the default rate with persistence $\rho = 0.9$, the same kind of disturbance considered in Figure 11. One sees that responses to credit of *either* sign make the economy's equilibrium response farther from what would occur under optimal policy, when the responses are of moderate size (the sizes of response that would be optimal in the case of other types of disturbance).

We can make two general observations about these results. First, there is little support for the idea that responding to variations in aggregate credit in a way that "leans against the wind" (i.e., with a coefficient $\phi_b > 0$) would increase welfare, in a model of the kind that we consider here. (Of course, one might argue that the benefits of such a policy depend on mechanisms that are simply not present in our model.) When $\rho = 0.5$ or less, Table 5 shows that the optimal response coefficient is not positive in the case of any type of non-financial disturbance,[46] and only moderately positive in the case of financial disturbances; when $\rho = 0.9$ or more, it is positive only for two types of non-financial disturbances (the G_t and $\bar{C}_t^s$ shocks, that are not obviously major sources of aggregate fluctuations in practice), and is only very slightly positive even in the case of the financial disturbances. And second, it is even harder to find a policy within the class (2.6) that is reasonably good regardless of the type of disturbance affecting the economy than it is to find a robust rule within the class (2.3). The robustness properties of the two types of rules is the same, if we are concerned only with non-financial disturbances; the question is whether the type of response that is desirable in the case of non-financial disturbances is also desirable in the case of financial disturbances. In the case of the spread-adjusted rules, the optimal sign of ϕ_ω is positive for most non-financial disturbances, and *also* for financial disturbances; in the case of the rules that respond to credit, the optimal sign of ϕ_b is negative for most non-financial disturbances, but at least slightly) *positive* in the case of financial disturbances. If one must choose a policy rule from one of these two classes, one will in many cases do better by choosing the best rule from the family (2.3), because there is less tension between the goals of achieving desirable responses to the different types of disturbances.

In the case of a linear intermediation technology, rules in the family (2.6) are no longer equivalent to any rules in the family (2.3), in the case of non-financial disturbances. It is then a less trivial question to ask what might be achieved by allowing a non-zero value of ϕ_b. However, the answer is that this lowers welfare, regardless of the sign of the response, in almost all cases. Table 6 reports the optimal value of ϕ_b for each of the types of disturbance considered in Table 5, but for the case of the linear technology ($\eta = 1$). The optimal coefficient is close to zero in *all* cases. The reason that the dynamic response of credit to the various shocks makes it not a useful indicator of the way in which monetary policy needs to be adjusted, regardless of the sign with which one responds to it (assuming that one responds only to the

[46]It is very slightly positive only for the $\bar{C}_t^b$ shock in the case that the persistence is reduced to zero.

Table 6: Optimal value of the response coefficient ϕ_b in policy rule (2.6), for the same set of possible disturbances as in Table 5, but a linear intermediation technology.

	Z_t	μ_t^w	τ_t	G_t	b_t^g	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	-0.09	-0.08	-0.08	-0.03	0.01	-0.09	0.00	-0.01	0.01	0.01
$\rho_\xi = 0.50$	-0.08	-0.08	-0.08	-0.02	0.00	-0.08	0.00	0.00	0.01	0.01
$\rho_\xi = 0.90$	-0.08	-0.08	-0.08	0.01	0.00	-0.08	-0.01	0.00	0.00	0.00
$\rho_\xi = 0.99$	-0.07	-0.07	-0.07	0.02	0.00	-0.07	-0.03	0.01	-0.01	-0.01

contemporaneous volume of credit).

This is illustrated in Figure 23, which shows the responses to a productivity disturbance of the kind considered in Figure 13 under alternative rules in the family (2.6). The reason that the optimal ϕ_b is near zero is not that the unadjusted Taylor rule is already optimal; as discussed earlier, the unadjusted Taylor rule is quite suboptimal in the case of this kind of disturbance, as the central bank tightens policy too much in response to an output increase that should instead be accommodated. One might think that since credit expands in response to the disturbance, a coefficient $\phi_b < 0$ would move policy in the right direction. But the figure shows that while a response to credit of that kind can mitigate the excessively disinflationary effect of the disturbance on impact, it is at the price of making policy be too inflationary later, as credit continues to surge.

The problem is that the time path of the response of credit (only slightly positive in the quarter of the shock, and then growing larger over the next several years) is very different from the time path of the distortions that need to be corrected (greatest in the quarter of the shock, and decaying substantially over the next few years). This is a fairly general problem with the usefulness of aggregate credit as an indicator for monetary policy in the model with a linear intermediation technology: the aggregate volume of credit is a stock that reflects the cumulative effects of disturbances over many previous years, rather than recent (and still relevant) disturbances alone. Once again, we find that responding to credit spreads provides a more useful rule of thumb than responding to the volume of credit. (Under this parameterization of our model, a spread adjustment cannot improve the economy's response to non-financial disturbances, but at least it does not cause *worse* responses to those shocks, either; and a positive spread adjustment does improve the economy's response to financial disturbances, regardless of the type and of the degree of persistence of the disturbance.)

3 Conclusion

We have considered two possible ways in which the standard Taylor rule might be modified to include a response to financial conditions: by adding a response to variations in a credit spread, as proposed by McCulley and Toloui (2008) and Taylor (2008); or by adding a response to a measure of aggregate private credit, as proposed by Christiano *et al.* (2007) among others. According to the model that we have used to analyze this issue, either type of adjustment, if of an appropriate magnitude, can improve equilibrium responses to disturbances originating in the financial sector. (In the case of the spread adjustment, this would require that the policy rate be reduced, relative to what the standard Taylor rule would prescribe, when credit spreads are larger than normal; in the case of a credit response, it would require that the policy rate be reduced when the volume of credit is smaller than normal.) However, even if this is the only kind of disturbance with which we must be concerned, the optimal degree of spread adjustment is likely to be less than a 100 percent offset for the increase in credit spreads; and the optimal degree of response to a reduction in credit will be much less strong than would be required to fully stabilize aggregate credit at some target level.

Neither type of simple proportional adjustment is ideal, however, since the time path of the distortions that one would like to offset is in general not the same as the dynamic response of either of these two indicators to the disturbance. In the model of Cúrdia and Woodford (2009), the most important perturbations of the model structural relations due to credit frictions are direct functions of the path of the credit spread ω_t; but many of the additional terms involve the marginal-utility gap Ω_t (which, to a linear approximation, is a forward-looking moving average of the credit spread) rather than the contemporaneous credit spread alone. The dynamic response of aggregate credit is even less similar to that of the distortions. At the time of a financial disturbance, credit contracts while the distortions (measured either by ω_t or Ω_t) increase; but subsequently, as the underlying disturbance (the shift in the functions $\Xi_t(b)$ or $\chi_t(b)$) dissipates but its effects persist, a lower volume of credit will be associated with *lower* values of both ω_t and Ω_t.

Simple proposals of this kind are even less adequate once one considers their consequences for the economy's responses to other kinds of disturbances besides purely financial ones. Disturbances of all sorts should cause endogenous variation in the volume of private credit, but the response coefficient ϕ_b that would represent the best modification of a standard Taylor rule is quite different in the case of different types of disturbances; in particular, in many cases, it would be better for monetary policy to be loosened when credit *expands* (rather than when it contracts) as a consequence of a non-financial disturbance, though the optimal sign of ϕ_b is positive in the case of most financial disturbances. In the case that the credit spread is endogenous (as in the "convex intermediation technology" case treated above), disturbances of all sorts cause credit spreads to vary as well, and a spread adjustment would also have implications for the economy's response to each of these disturbances. The tension between what is desirable in the case of different types of disturbances is somewhat less acute in this case, as a positive value for ϕ_ω is preferred in the case of many non-financial disturbances as in the case of the purely financial disturbances; but there are considerable differences in the size of adjustment that is best in the case of different types of disturbances.

A superior approach to either kind of simple rule, at least in principle, is to adjust the policy instrument so as to imply economic projections for inflation and real activity that are consistent with a *target criterion*, as discussed in Cúrdia and Woodford (2009). Assuming that the model used to produce these projections takes correct account of the implications of financial conditions for aggregate demand and supply, this will imply a response to changing financial conditions in the way that the central bank sets its target for the policy rate. But the response that is called for is not a simple proportional response to any single measure of financial conditions. A forecast-targeting central bank will properly take account of many credit spreads rather than just one; it will take account of whether changes in credit spreads indicate disruptions of the financial sector as opposed to endogenous responses to developments elsewhere in the economy; and it will calibrate its response depending on its best guess about the likely persistence of disturbances on a particular occasion. Of course, the degree to which such an approach should be expected to improve upon a simple rule depends on the quantity and quality of information available for use in the construction of projections; and the use of a more complex (and inevitably more judgmental) approach creates greater challenges with regard to transparency and accountability. Nonetheless, the advantages of such an approach seem to us even more salient under the more complex circumstances associated with financial market disruptions.

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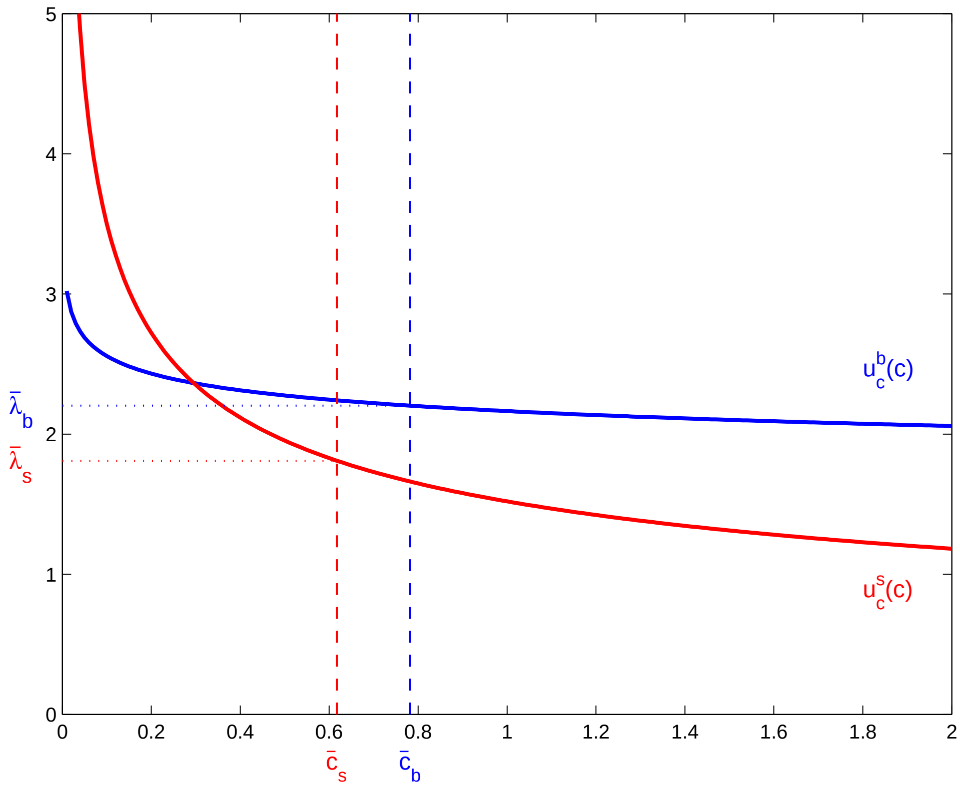

Figure 1: Marginal utilities of consumption for households of the two types. The values $\bar{c}^s$ and $\bar{c}^b$ indicate steady-state consumption levels of the two types, and $\bar{\lambda}^s$ and $\bar{\lambda}^b$ their corresponding steady-state marginal utilities.

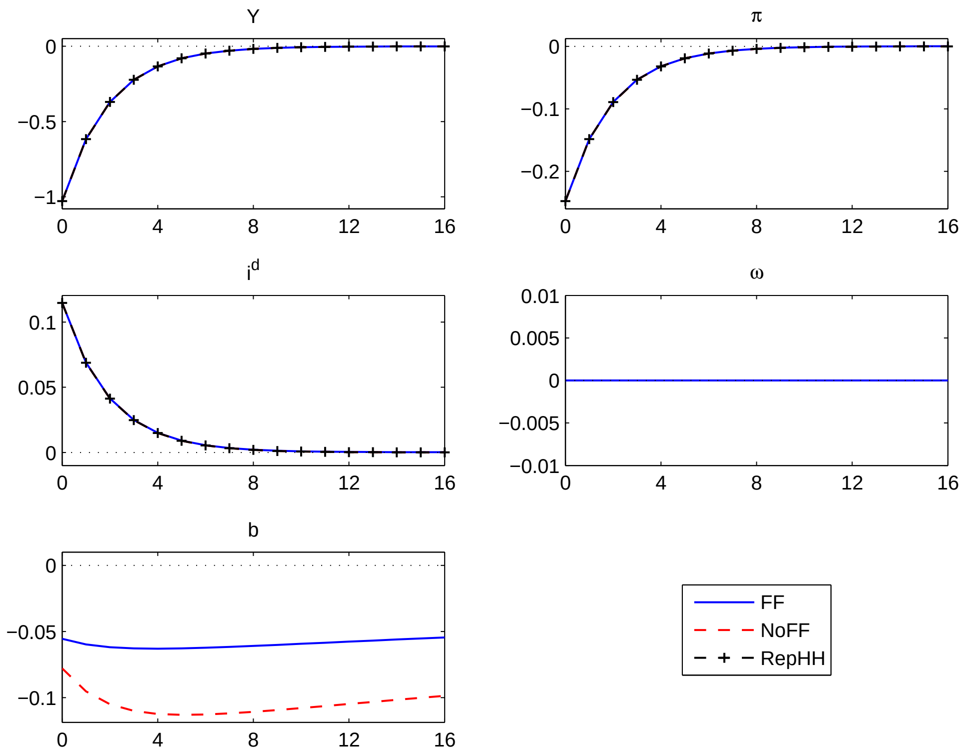

Figure 2: Impulse responses to a 1 percent (annualized) shock to ϵ_t^m, in three different models with a linear intermediation technology.

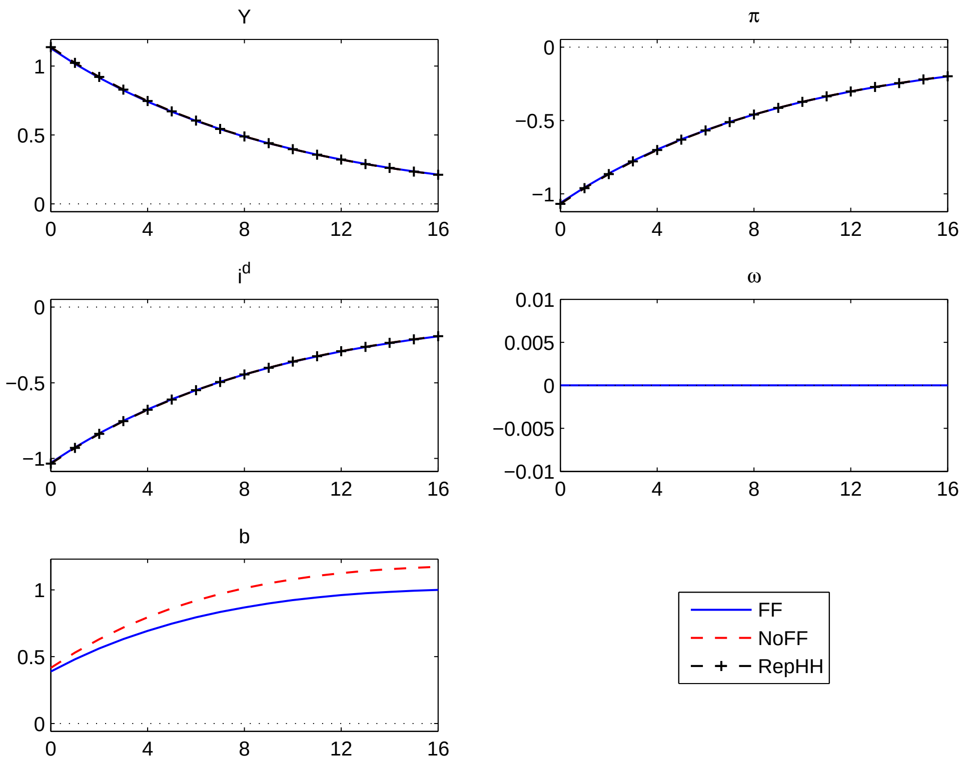

Figure 3: Impulse responses to a 1 percent shock to Z_t, in three different models with a linear intermediation technology.

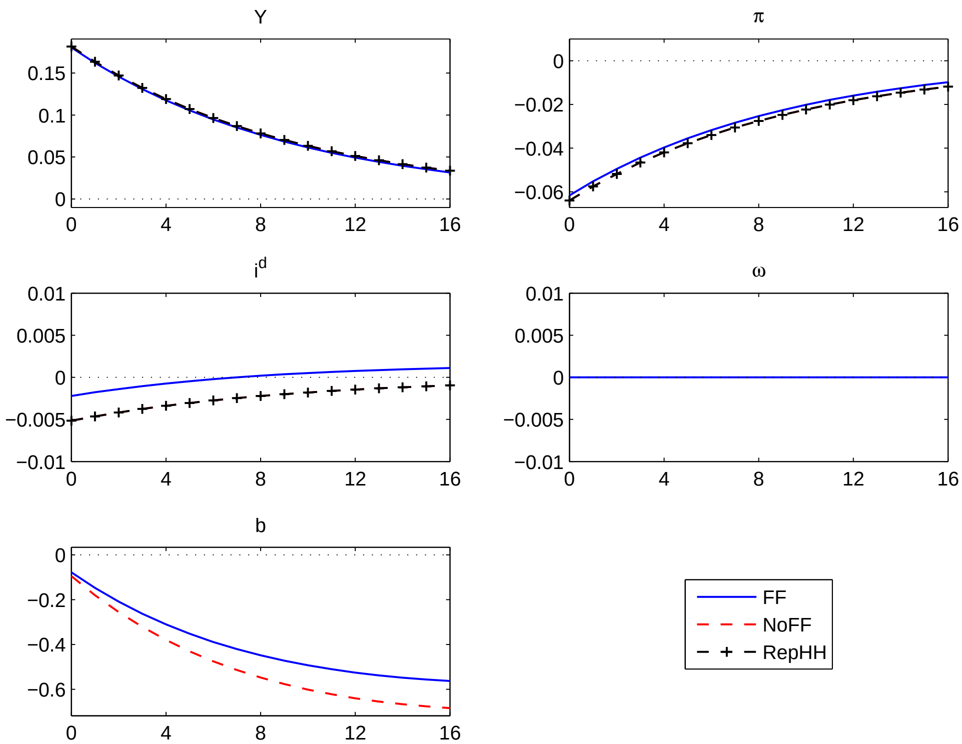

Figure 4: Impulse responses to a shock to G_t equal to 1 percent of steady-state output, in three different models with a linear intermediation technology.

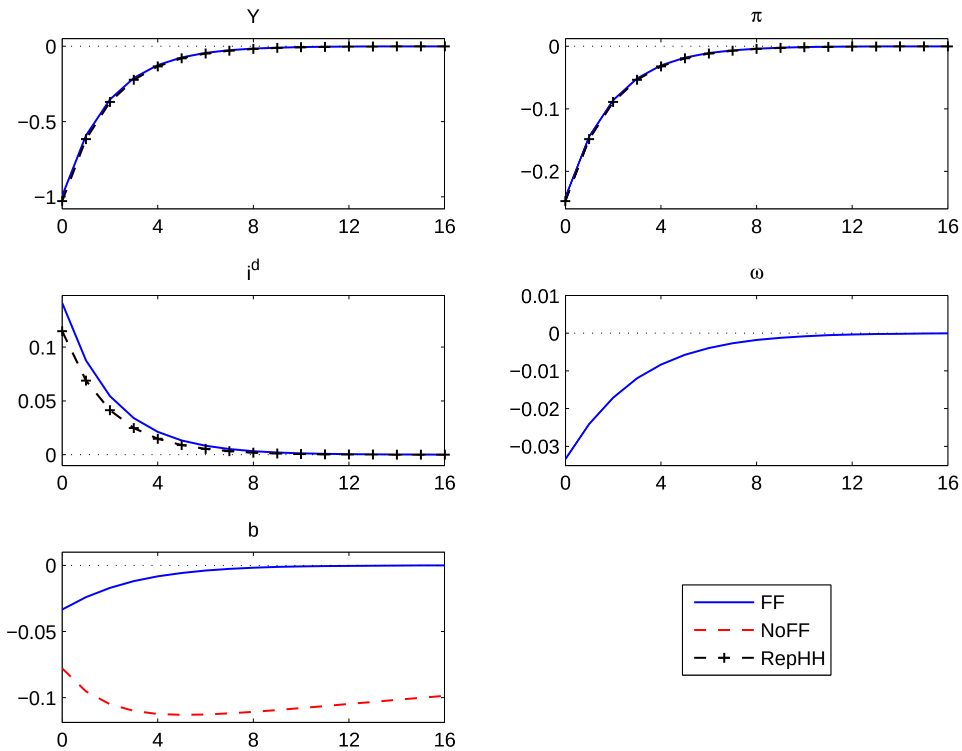

Figure 5: Impulse responses to a 1 percent (annualized) shock to ϵ_t^m, in three different models with a convex intermediation technology.

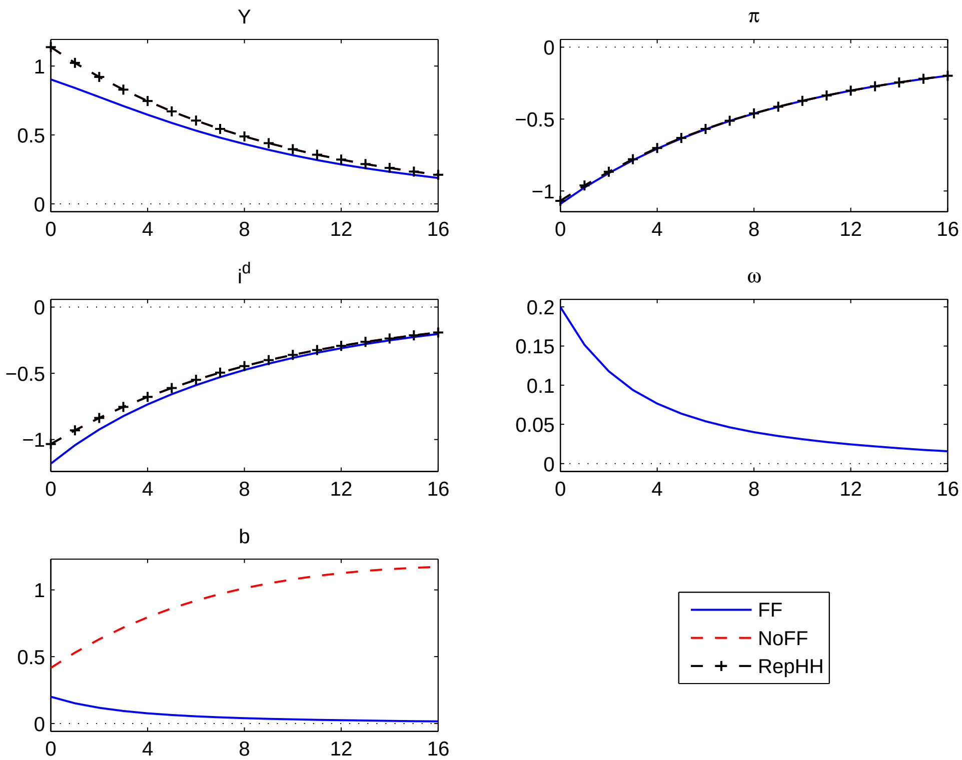

Figure 6: Impulse responses to a 1 percent shock to Z_t, in three different models with a convex intermediation technology.

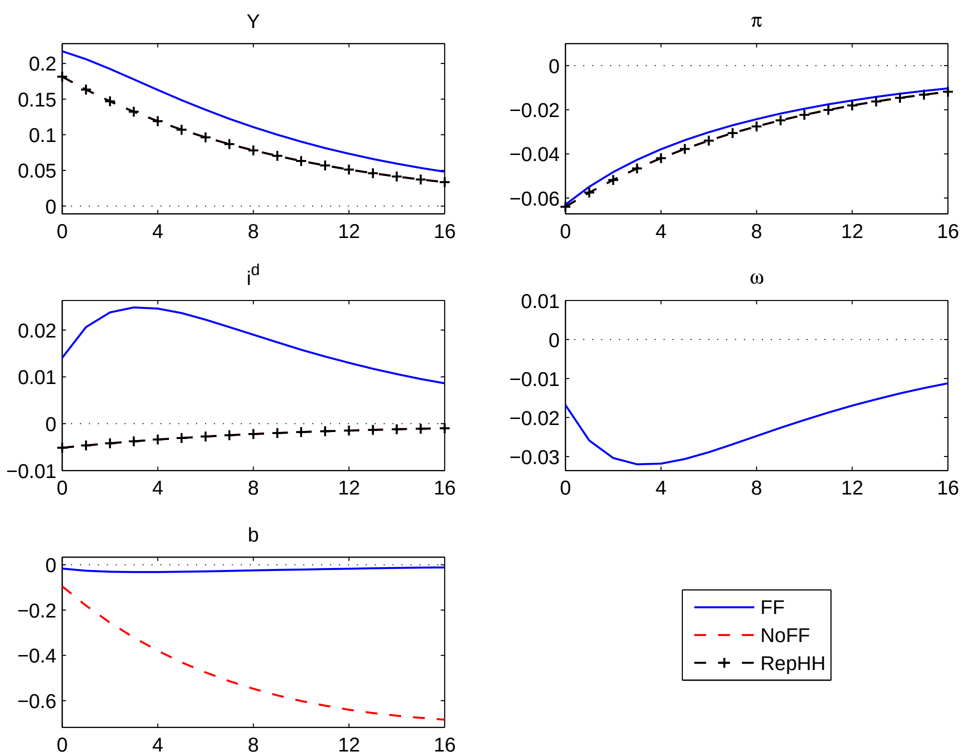

Figure 7: Impulse responses to a shock to G_t equal to 1 percent of steady-state output, in three different models with a convex intermediation technology.

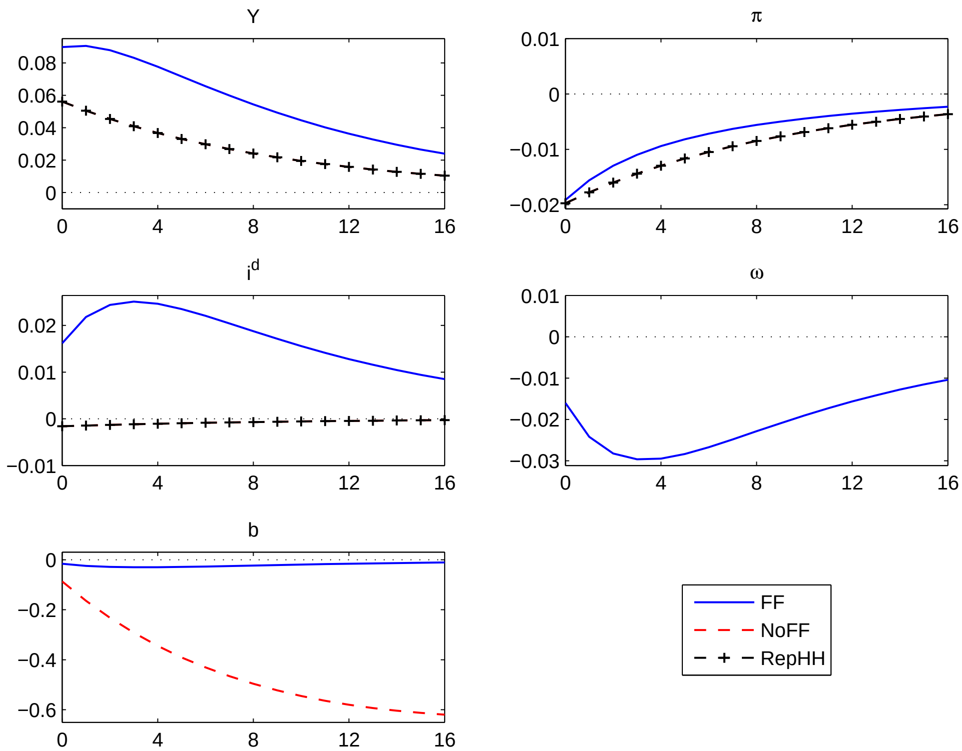

Figure 8: Impulse responses to a 1 percent shock to type s expenditure, in three different models with a convex intermediation technology.

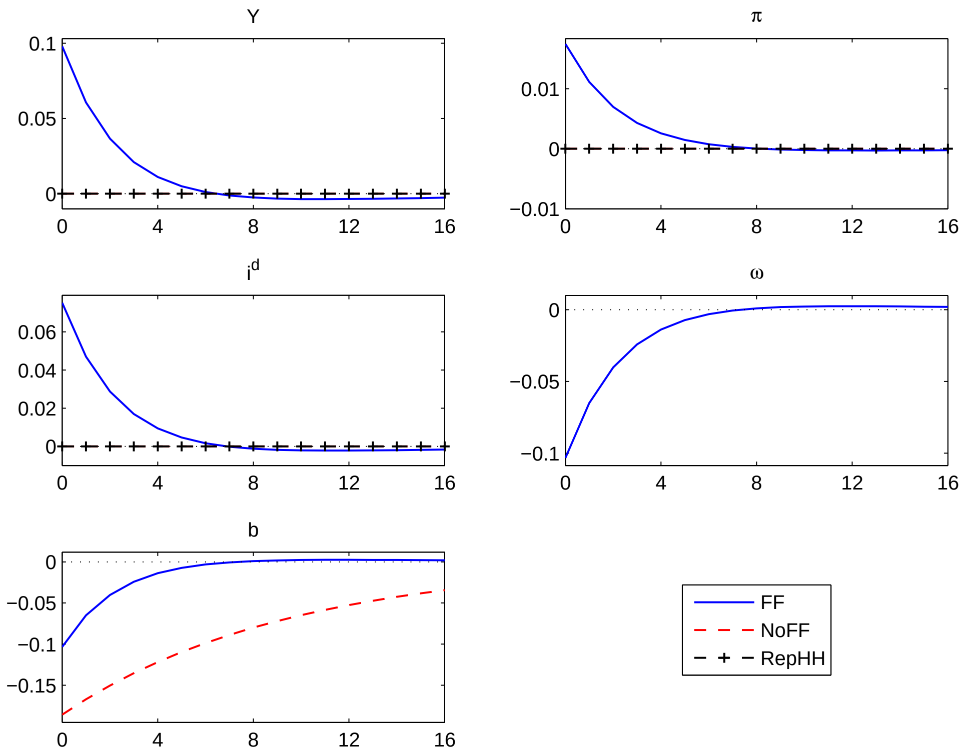

Figure 9: Impulse responses to a shock to b_t^g equal to 1 percent of steady-state output, in three different models with a convex intermediation technology.

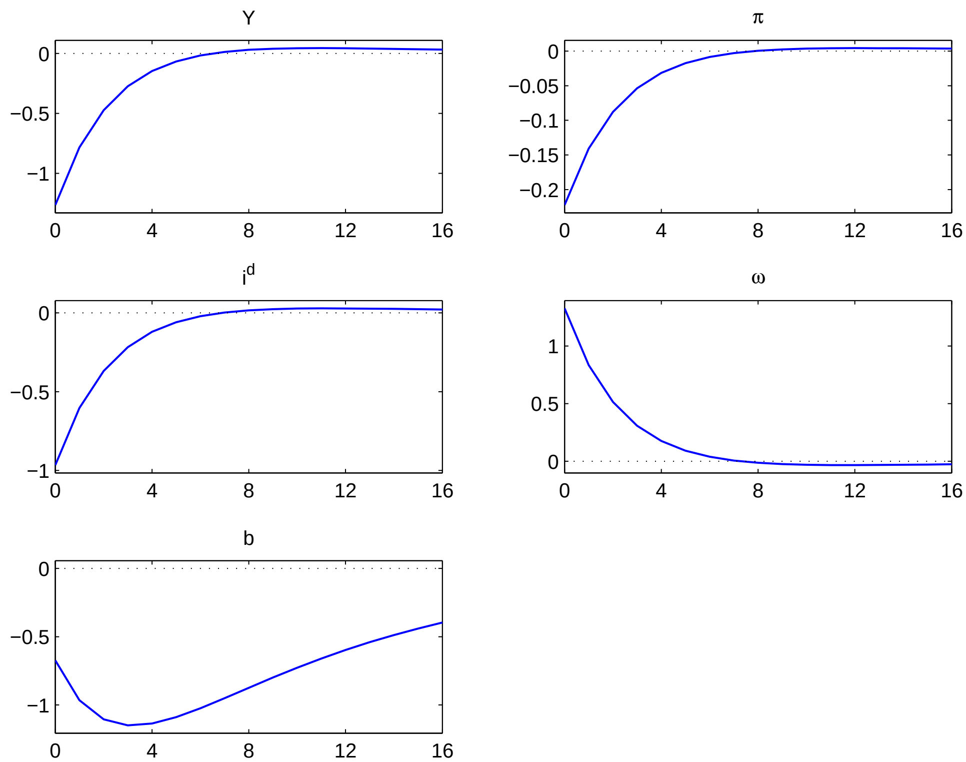

Figure 10: Impulse responses to a shock to $\tilde{\chi}_t$ that increases $\omega_t(\bar{b})$ by 2 percentage points (annualized), in the case of the financial frictions model with convex intermediation technology.

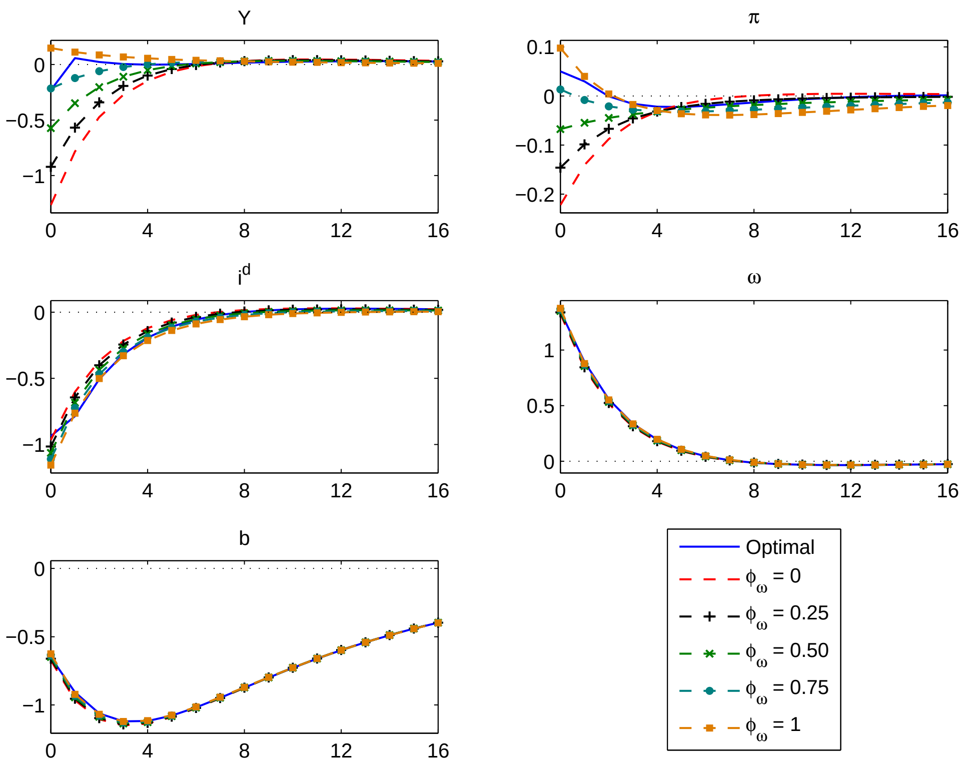

Figure 11: Impulse responses to a shock to $\tilde{\chi}_t$ that increases $\omega_t(\bar{b})$ initially by 2 percentage points (annualized), with persistence $\rho = 0.9$, under alternative degrees of spread adjustment.

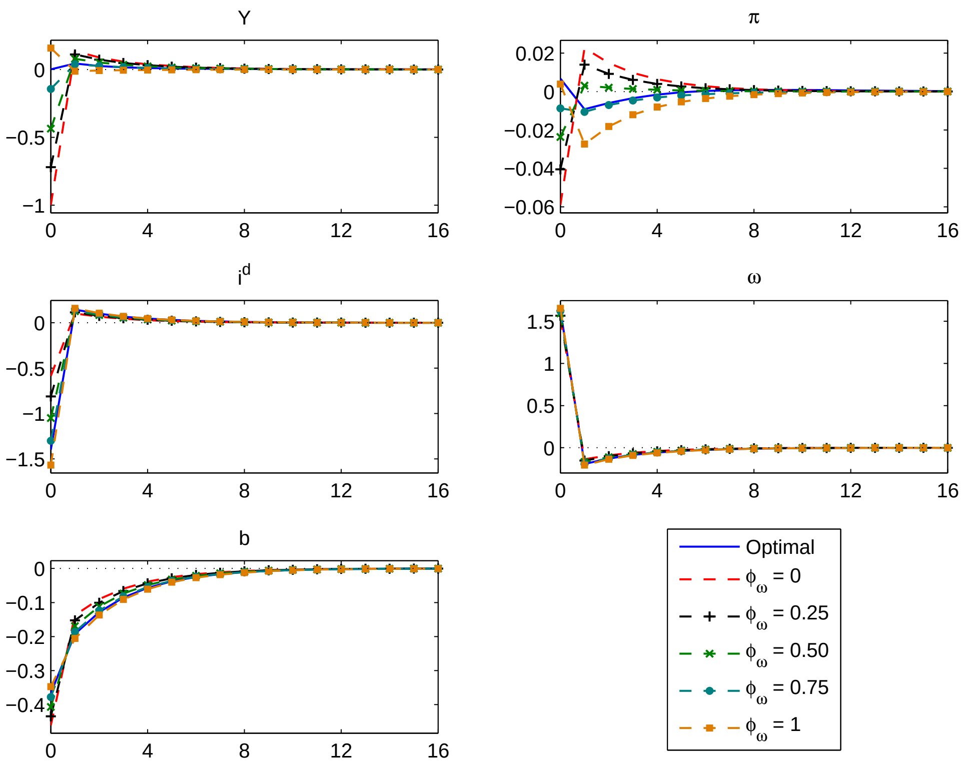

Figure 12: Impulse responses to a shock to $\tilde{\chi}_t$ that increases $\omega_t(\bar{b})$ temporarily by 2 percentage points (annualized), assuming no persistence ($\rho = 0$), under alternative degrees of spread adjustment.

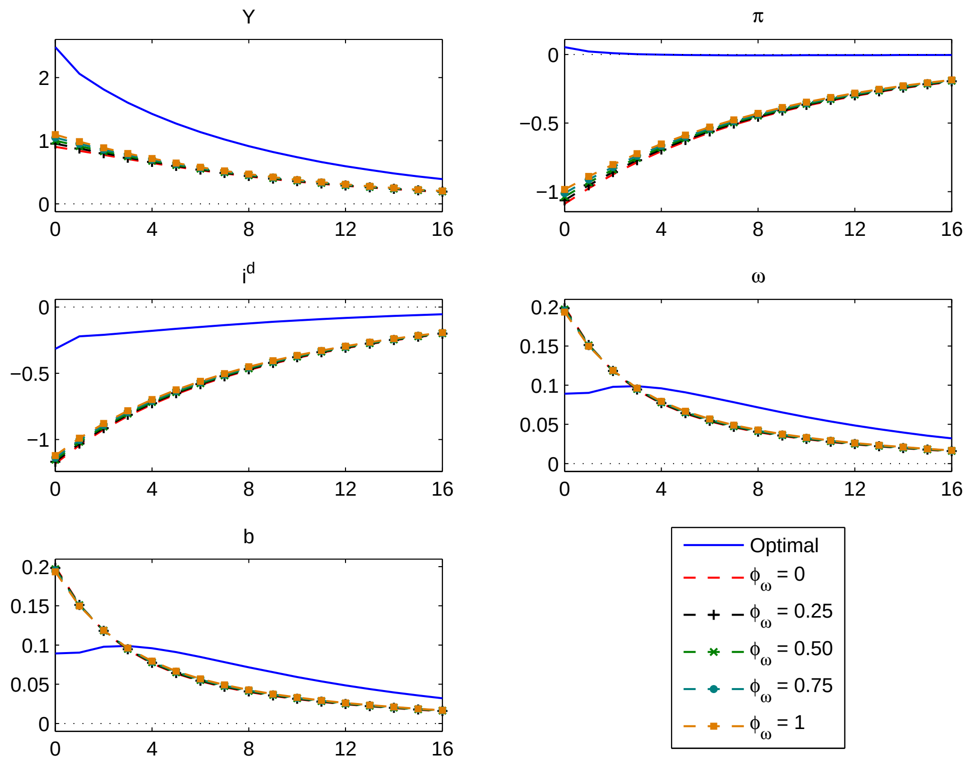

Figure 13: Impulse responses to a 1 percent shock to Z_t, with persistence $\rho = 0.9$, under alternative degrees of spread adjustment.

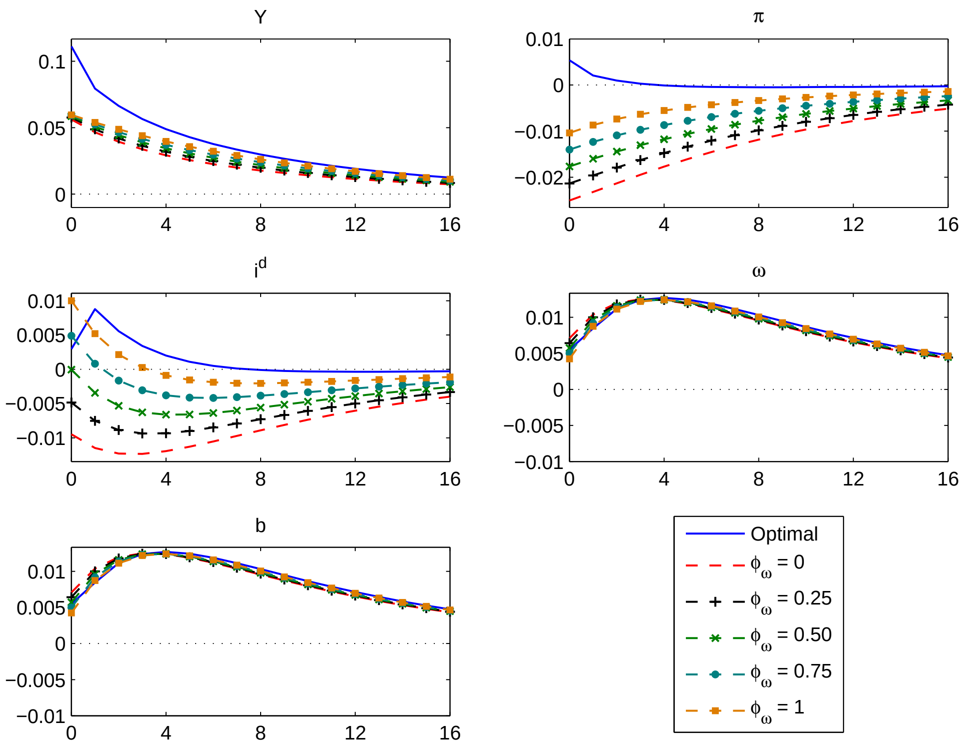

Figure 14: Impulse responses to a 1 percent shock to type b expenditure, with persistence $\rho = 0.9$, under alternative degrees of spread adjustment.

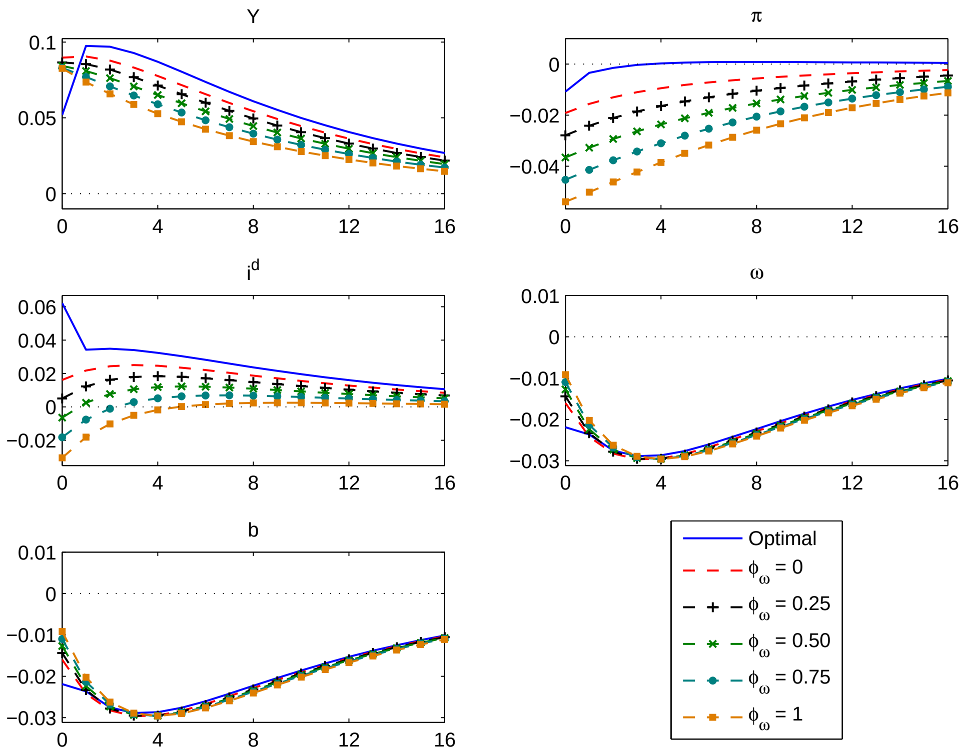

Figure 15: Impulse responses to a 1 percent shock to type s expenditure, with persistence $\rho = 0.9$, under alternative degrees of spread adjustment.

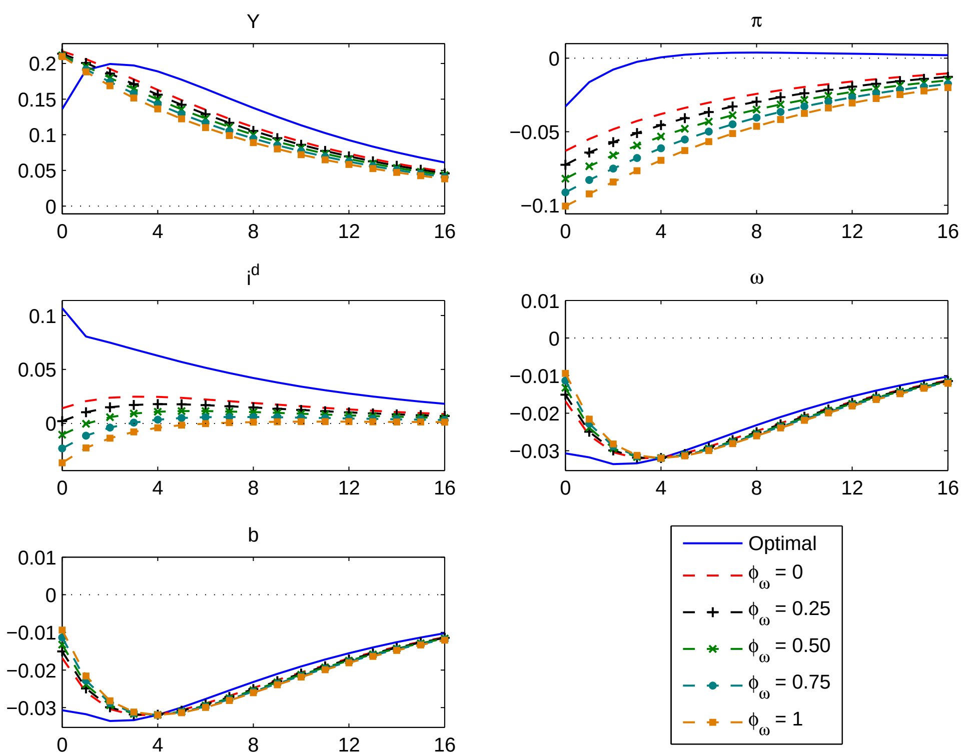

Figure 16: Impulse responses to a shock to G_t equal to 1 percent of steady-state output, with persistence $\rho = 0.9$, under alternative degrees of spread adjustment.

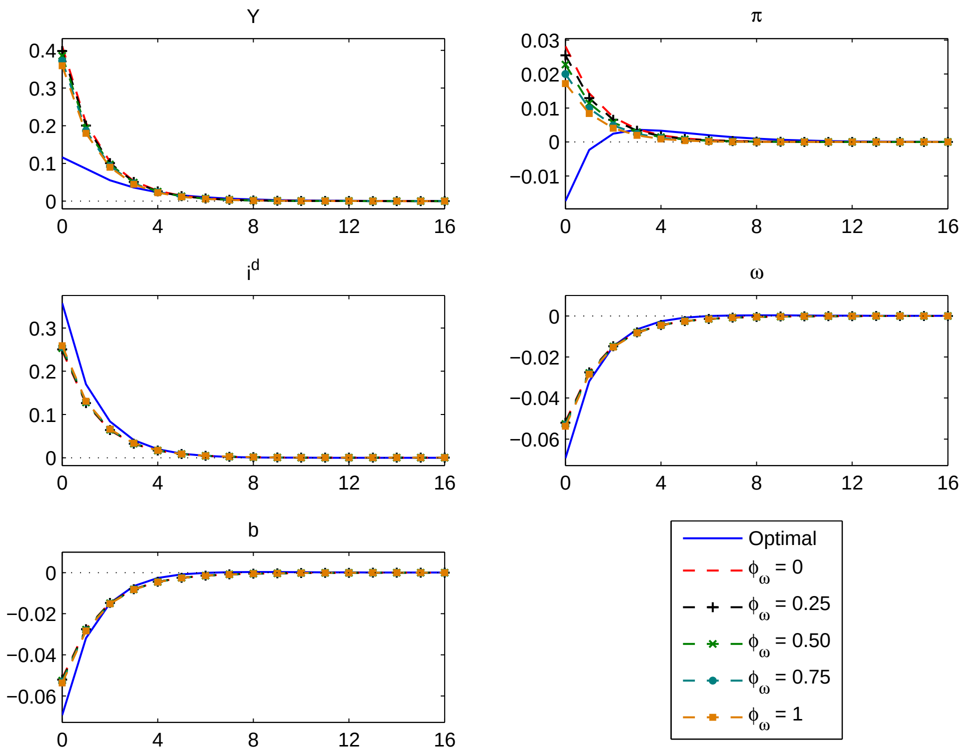

Figure 17: Impulse responses to a shock to G_t equal to 1 percent of steady-state output, but with persistence $\rho = 0.5$, under alternative degrees of spread adjustment.

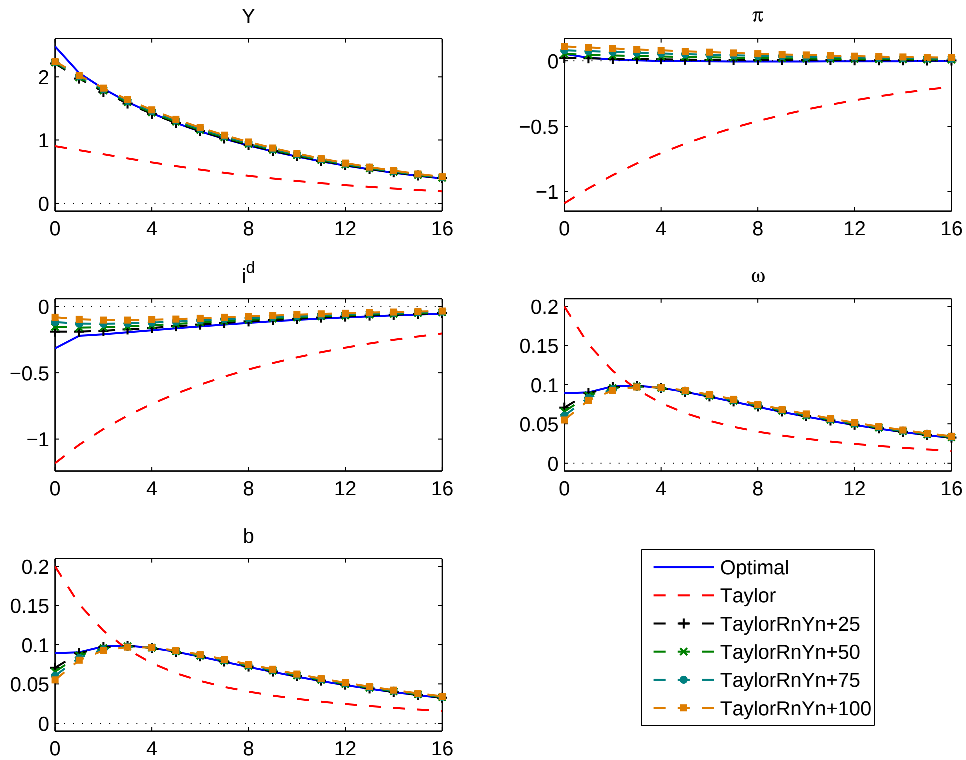

Figure 18: Impulse responses to a 1 percent shock to Z_t, under alternative degrees of spread adjustment, when the Taylor rule takes account of changes in the natural rates.