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What Can the Data Tell Us About the Equilibrium Real Interest Rate?

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Abstract

The equilibrium real interest rate ($r*$) is the short-term real interest rate that, in the long run, is consistent with aggregate production at potential and stable inflation. Estimation of $r*$ faces considerable econometric and empirical challenges. On the econometric front, classical inference confronts the "pile-up" problem. Empirically, the co-movement of output, inflation, unemployment, and real interest rates is too weak to yield precise estimates of $r*$. These challenges are addressed by applying Bayesian methods and examining the role of several "demand shifters", including asset prices, fiscal policy, and credit conditions. We find that the data provide relatively little information on the $r*$ data-generating process, as the posterior distribution of this process lies very close to its prior. This result contrasts sharply with those for the trend growth or natural rate of unemployment processes. Second, credit spreads are very important for the estimated links between output and interest rates and hence for estimates of $r*$. Estimates of $r*$ that account for this range of considerations are more stable than other estimates, with $r*$ at the end of 2014 equal to approximately 1-1/4 percent.

JEL Classification Code: E5, E3, E4 Keywords: Equilibrium real interest rate, Potential Output, Bayesian Methods

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1 Introduction

The equilibrium real interest rate ($r*$) is the short-term real interest rate that, in the long run, is consistent with aggregate production at potential and stable inflation. This concept enters a range of policy discussions.[1] For example, monetary policy discussions focus on $r*$ because it provides a gauge of a "neutral" stance of monetary policy.[2] However, estimation of $r*$ faces considerable econometric and empirical challenges.

On the econometric front, classical inference confronts the "pile-up" problem. As emphasized in Stock and Watson [1996], maximum-likelihood estimation of models in which a time series contains a small permanent component and a sizable transitory component tends to drive the estimated role of the permanent component toward zero – that is, the estimated variance of this component "piles-up" near zero. Because real interest rates contain (at most) only a small permanent component, the "pile-up" problem is severe in analyses of the equilibrium real interest rate (Laubach and Williams [2003]). Kim and Kim [2013] suggest that the "pile-up" problem is much less severe for the Bayesian approach. This result is closely related to the well-known result that Bayesian inference regarding unit roots requires no special assumptions, in contrast to the classical approach (e.g., Sims and Uhlig [1991]). However, the Bayesian approach faces challenges of its own: In particular, inference depends upon the prior, and researchers may differ in their priors or eschew the Bayesian perspective altogether (e.g., Stock [1991]). We confront this challenge by considering sensitivity of the key conclusions to changes in priors regarding the $r*$ process.

In the application herein, Bayesian methods aid inference in two ways. First, the focus on the posterior distribution of the variance of the permanent component provides a more comprehensive lens with which to examine the range of reasonable settings for this

[1] For a policy-focused discussion, see the summary of work at the Council of Economic Advisers in Obstfeld and Tesar [2015].

[2] For example, see Yellen [2015]. While some notion of "equilibrium" real interest rates has a long tradition – dating to at least Knut Wicksell – there are also a number of alternative notions of "equilibrium", some of which are more short-term and some of which are more long-term. Ferguson [2004] presents a policymaker's assessment of how these various definitions contribute to (or confuse) policy discussions. Section 2 below references related research.

parameter than the focus on point estimates associated with classical inference. In addition, the Bayesian approach allows the researcher to impose a prior that the variance of the permanent component is likely to be non-zero, but not large, and then to assess the information in the data relative to this prior through examination of the entire posterior distribution. The comparison of the prior and posterior distributions is a central part of our examination of the information in the data regarding the $r*$ process.

Turning to empirical challenges, the co-movement of output, inflation, unemployment, and real interest rates is too weak to yield precise estimates of $r*$. This is intuitive – if the relationship were tight, so that most "shocks" to this relationship were shocks to $r*$ rather than random noise, then it would be relatively straightforward to "observe" shocks to $r*$ by looking at "errors" in the IS curve. The weak empirical links between output and interest rates implies an imprecise estimate of the interest-elasticity of demand and large "errors" in estimated IS curves. Because the models of the equilibrium real interest rate we consider involve filtering the surprise in the output (and other) equations to estimate $r*$, mis-specification in this equation–such as the omission of important drivers of output–may substantially affect inference. To address this concern, we examine the role of several "demand shifters" (including asset prices, fiscal policy, and credit conditions) as influences on output fluctuations in addition to interest rates.

We reach two primary conclusions. First, the data provide relatively little information on the $r*$ data-generating process; indeed, the posterior distribution of this process lies very close to the prior distribution. This results contrasts sharply with those for the trend growth or natural rate of unemployment processes, for which the data are very informative. Second, additional demand shifters–in particular, a credit spread–are very important for the estimated links between output and interest rates and hence for estimates of $r*$–implying that previous research that ignores such factors provide poor inference. Estimates of $r*$ that account for this range of considerations are more stable than other estimates, with $r*$ at the end of 2014 equal to approximately 1-1/4 percent.

Finally, we also compare auxiliary implications of the models herein to those from the frequently-cited study of Laubach and Williams [2003]. The models herein yield estimates of the output gap very similar to those from the Congressional Budget Office, in contrast to that of Laubach and Williams [2003].

2 Previous Literature on the Equilibrium Real Interest Rate

We have defined the equilibrium real interest rate ($r*$) as the level of the real (short-term) interest rate that is consistent, in the long-run, with output at potential, unemployment at its natural rate, and inflation at the monetary policymaker's long-run objective (Laubach and Williams [2003]). This concept is a long-run notion, and may vary over time because of changes in the rate of economic growth, the degree of international capital mobility, or other factors that affect the interest rate that balances savings and investment. In the approach herein, $r*$ is estimated using a "trend/cycle" model.

A variety of approaches have been used to gauge the equilibrium interest rate. One approach is to examine factors that have contributed to the average level of interest rates across decades and countries, with guidance from economic theory. For example, Pescatori and Furceri [2014] and Hamilton, Harris, Hatzius, and West [2015] consider the links between average real interest rates and factors such as the rate of potential growth in output (globally or within individual countries) or the global saving rate. Summers [2014] perceives an imbalance between global saving and investment, stemming in part from reduced growth prospects in advanced economies, as a motivation for a persistently low equilibrium real interest rate (and the risk of prolonged "secular stagnation"–that is, a state in which aggregate demand falls persistently short of aggregate supply).

Laubach and Williams [2003] introduce a relatively simple, trend/cycle approach to estimate the equilibrium real interest rate. In their model, fluctuations in output and inflation move along an "IS-curve" linking the output gap to the real interest rate and a Phillips curve linking inflation and the output gap. This system treats the level of potential output, the output gap, and the equilibrium real-interest rate as unobservable state variables, and derives estimates of these concepts using output, inflation, and the real interest rate as observable variables via the Kalman filter. Estimates from their model for the U.S. equilibrium real interest rate are updated frequently on the website of the Federal Reserve Bank of San Francisco, and their measure was about -0.4 percent in the fourth quarter of 2014 (as of June 2015).

Clark and Kozicki [2005] highlight a number of challenges associated with the estimation of the equilibrium real interest rate using the trend/cycle approach. First, as emphasized in Laubach and Williams [2003], classical inference regarding the equilibrium interest rate is plagued by the "pile-up" problem associated with estimation of a small permanent component of a time series with substantial short-run variation (as discussed in Stock and Watson [1996]). Second, the filtered estimates of $r*$ usually differ substantially between the "one-sided" and "two-sided" estimates – a result that holds even if the true population parameters of the model are known because estimation of long-run trends is more precise after substantial data has been accumulated when time series have important short-run components. Finally, "real-time" challenges associated with measurement and data revisions are substantial. The analysis herein will touch upon the first two issues (but not "real-time" considerations), and our results regarding the uncertainty regarding one- and two-sided filtering will largely confirm those of Clark and Kozicki [2005]. For each of these reasons, Clark and Kozicki [2005] suggest that $r*$ estimates may have limited use in policy discussions.[3]

The model developed herein draws on the approach of Laubach and Williams [2003]. In particular, the model consists of an IS-curve and Phillips curve. As in Kuttner [1994], these equations are augmented with an equation linking cyclical fluctuations in unem-

[3]Orphanides and Williams [2007] analyze this idea in detail, and argue that imprecision in estimates of $r*$ point to a limited role for such estimates in policy discussions. This finding is closely related to the challenges related to using output gap estimates in policy discussions (e.g., Orphanides and van Norden [2002]).

ployment to those in output (i.e., an "Okun's law" equation).[4]. This addition nods in the direction of the literature that emphasizes the role of labor-market indicators in assessments of resource-utilization gaps (e.g., Basistha and Startz [2008] and Fleischman and Roberts [2011]).

Research has also highlighted the importance of a broad array of financial conditions in the business cycle: Gilchrist and Zakrajsek [2012] and Kiley [2014a] document important contributions of credit spreads to output movements (using, respectively, forecasting techniques and simple "IS-curve" analysis). Borio, Disyatat, and Juselius [2013] and Arseneau and Kiley [2014] have found that credit measures may improve estimates of the output gap and natural rate of unemployment. Moreover, the fiscal stance can vary substantially, with effects on aggregate production and income (Follette and Lutz [2010]). Simple model-based approaches to estimating $r*$ (e.g., Laubach and Williams [2003], Clark and Kozicki [2005], and Johanssen and Mertens [2015]) have not included such factors, which may distort their conclusions.[5]

The equilibrium rate of interest is distinct from the short-run Wicksellian natural rate of interest, which is the level of the real (short-term) interest rate that is consistent with price stability in the short-to-medium run (e.g., Woodford [2003] and Edge, Kiley, and Laforte [2008]). In particular, the natural rate of interest should be expected to fluctuate considerably over the business cycle, whereas the equilibrium real interest rate is likely to fluctuate less, if at all, over the business cycle and instead should evolve slowly over time. (In the literature, the distinction between the equilibrium real interest rate and the natural rate is not always drawn as finely as herein; as a result, these definitions provide guidance that should help the reader understand the analysis, but care must be taken when comparing the discussion herein to that in the broader literature.[6])

[4]A large literature has focused on fluctuations in the unemployment gap and natural rate of unemployment, rather than the output gap; perhaps the most well-known reference is Staiger, Stock, and Watson [1997]

[5]An alternative approach looks at long-term interest rates to examine movements in $r*$, e.g., Bomfim [2001].

[6]Kiley [2013] notes the long record, emphasized by Paul Samuelson many decades ago, of economists

3 Implementation and Results

3.1 The Model

The model includes equations for the dynamics of output y, inflation, Δp, and the unemployment rate u. Output and the unemployment have permanent, or trend, and cyclical components. A set of equations governs the cyclical dynamics of inflation π (a Phillips curve), unemployment u (Okun's Law), and the output gap y (the IS curve).The cyclical component of a variable is denoted by a superscript "c" and the trend component by a superscript "T" (e.g., $X = X^c + X^T$).

3.1.1 Cyclical Dynamics

Three equations describe the cyclical dynamics of the model. Because $r*$ captures the real interest rate necessary to maintain output at potential over the long run, the key equation is the link between interest rates and output; indeed, the simple model of Laubach and Williams [2003] relies on this equation for identification of both the business cycle (output gap) and $r*$. We specify the links between (the cyclical component of) output and real interest rates via an "IS-curve" similar to that in related research

$$y_t^c = \gamma_r \sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \rho_1 y_{t-1}^c + \rho_2 y_{t-2}^c + \sum_j \Gamma_j Z_{t-1}^j + \epsilon_t^{yc} \tag{1}$$

The parameter γ_r governs the interest sensitivity of output, while rho_1 and ρ_2 determine the autocorrelation patterns. Crucially, the IS curve allows for "demand shifters" through the vector of variables Z^j. Note that Z^j is lagged one period: This choice was made to focus on the role of persistence in the demand shifters, rather than the contemporaneous correlation between such variables and surprises to either endogenous variables in the system (e.g., output) or exogenous variables to the system (e.g., the real interest rate);

to use the same term for different concepts, or different terms for the same concept, in discussions of unobserved economic states.

for example, lagging credit spreads abstracts from any contemporaneous relationship between the output gap, real interest rates, and spreads. This reduced-form approach does not take a stand on the underlying structural model, and the results below will highlight the importance of understanding credit spreads. Nonetheless, the results were essentially identical whether Z^j entered with a lag or contemporaneously. Finally, our analysis will include a specification without demand shifters, and one in which a (lagged) corporate bond spread, credit growth, and fiscal impetus enter. A more complete description of these variables and the motivation for their inclusion follows in the next subsection on data and empirical analysis.

Unemployment is determined by Okun's law

$$u_t^c = -\beta\left(0.4y_t^c + 0.3y_{t-1}^c + 0.2y_{t-2}^c + 0.1y_{t-3}^c\right) + \epsilon_t^u. \tag{2}$$

Note that the equation assumes that unemployment gap fluctuations are a (fixed) distributed lag of output gap fluctuations, capturing the well-known tendency for unemployment to lag movements in output. This relationship is drawn from Braun [1990].

Inflation dynamics are governed by

$$\pi_t = s_1\pi_{t-1} + s_2\frac{\sum_{j=2}^{4}\pi_{t-j}}{3} + (1-s_1-s_2)E_{t-1}\pi^{LR} + \alpha_u u_t^c + \sum_j B_j X_t^j + \epsilon_t^\pi. \tag{3}$$

In the Phillips curve, inflation is a influenced by the unemployment gap, its own lags, and the level of long-run inflation expectations. The role for long-run inflation expectations in the Phillips curve has considerable empirical support (e.g., Kiley [2014a] and Kiley [2014b]), and survey measures of long-run inflation expectations will be used in the empirical analysis. The model also allows for an influence on inflation from other factors (X^j). Following Arseneau and Kiley [2014] and Gilchrist et al [2013], we will include credit conditions in X^j, potentially capturing the notion that such conditions act as a "cost-push" factor on price inflation. Results are not sensitive to inclusion of this factor.

3.1.2 Trends

Output, unemployment, and the real interest rate include trend components.

The trend for output y^T includes I(1) and I(2) components. The I(1) component is a shift to the trend level

$$y_t^T = y_{t-1}^T + g_t + \eta_t^{yt}. \tag{4}$$

The I(2) component is a shock to the trend growth rate g.

$$g_t = g_{t-1} + \eta_t^{yg} \tag{5}$$

The trend for the unemployment rate is a random walk (as in Staiger, Stock, and Watson [1997] and related literature.

$$u_t^T = u_{t-1}^T + \eta_t^{ut}. \tag{6}$$

Finally, the equilibrium real interest rate is a random walk.

$$r_t^T = r_{t-1}^T + \eta_t^{rt}. \tag{7}$$

The specification follows Laubach and Williams [2003]. Note that the model herein does not link movements in the equilibrium real interest rate to the trend growth rate. Clark and Kozicki [2005] and Hamilton, Harris, Hatzius, and West [2015] question the strength of this relationship, and we take the approach of including as few parameters as possible to assess movements in the equilibrium real interest rate.

3.1.3 Observation Equations

Finally a set of observation equations for output and unemployment link the model back to observable data. The inflation observation equation is the Phillips curve (as its dependent variable is observed inflation).

Output is the sum of its trend and cycle,

$$y_t = y_t^T + y_t^c \tag{8}$$

as is the unemployment rate,

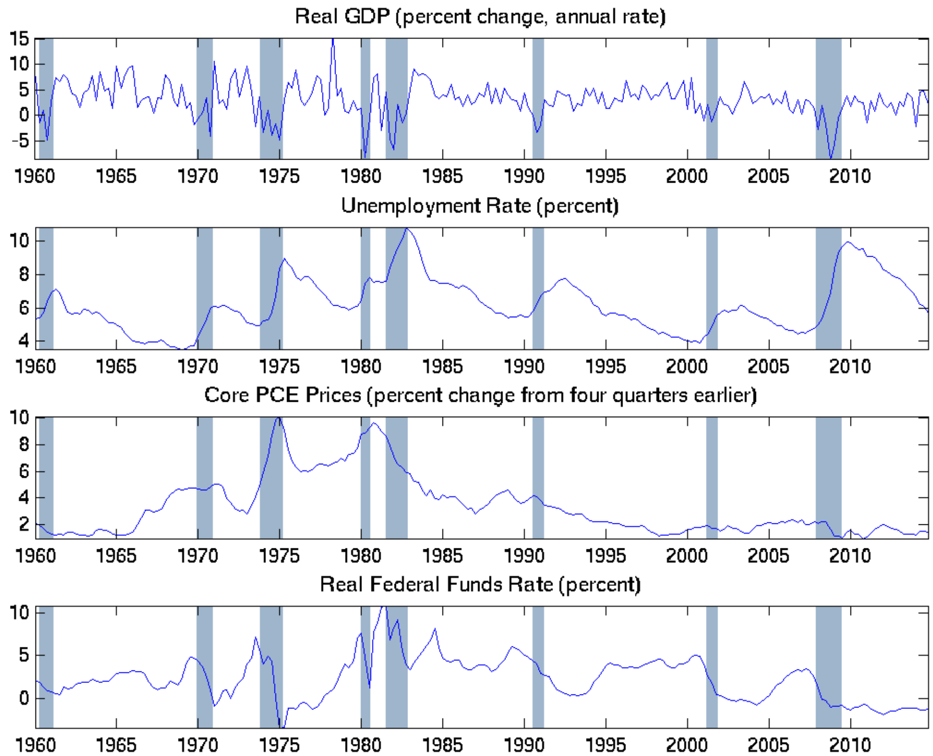
$$u_t = u_t^T + u_t^c \tag{9}$$

Note that the other observable variables that enter the cyclical equations (the real interest rate, long-run inflation expectations, and the vectors of possible inflation and demand shifters (X and Z) are treated as exogenous to the system.

3.2 Data

A look at the data helps understand some of our later results. Output is measured by real GDP, and the empirical analysis considers real GDP divided by the civilian non-institutional population aged 16 and over to remove the effects on trend growth associated with underlying population trends The unemployment rate is for the the civilian non-institutional population aged 16 and over; inflation is measured by the price index for Personal Consumption Expenditures excluding food and energy (core PCE inflation, as the primary interest herein is in the relationship between slack and inflation, and the effects of volatile food and energy prices is largely orthogonal to this interest). Finally, the real interest rate is measured by the nominal federal funds rate minus the change in core PCE prices from four quarters earlier. (This backward-looking measure of the real interest rate is commonly

Figure 1: Key Data Series: Output, Unemployment, Inflation, and the Real Interest Rate



Source: Real GDP and PCE prices, Bureau of Economic Analysis; Unemployment Rate, Bureau of Labor Statistics; Federal Funds Rate, Federal Reserve Board. Inflation and real interest rate computed by author. Data accessed from Federal Reserve FRB/US database available at http://www.federalreserve.gov/econresdata/frbus/us-models-package.htm. Data accessed July 2, 2015. Shaded regions indicate recessions as identiied by the National Bureau of Economic Research.

used in related studies.)[7]

Focusing first on variables common to trend/cycle models and as reported in figure 1, the unemployment rate shows a clear cycle around recessions (as identified by the National Bureau of Economic Research); this cycle, in conjunction with Okun's Law, is highly informative about the overall business cycle. Indeed, a casual look at the change in real GDP illustrates how difficult it would be to gauge the state of overall resource

[7]Real GDP and the core PCE price index are from the Bureau of Economic Analysis; Population and the unemployment rate are from the Bureau of Labor Statistics; and the nominal federal funds rate is from the Federal Reserve Board. All data are taken from the Federal Reserve's FRB/US model database, including any adjustments made by Federal Reserve staff. Brayton, Laubach, and Reifschneider [2014] describes the most recent FRB/US model and an easily-accessible version of the data.

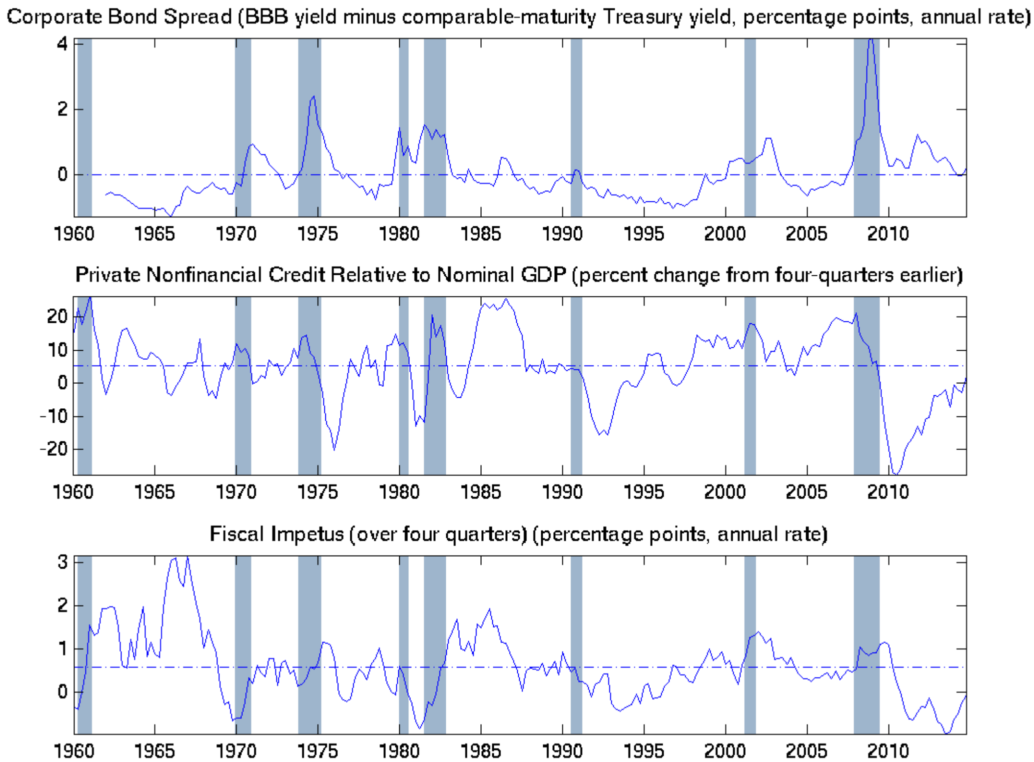
utilization without labor market data. Inflation has a highly persistent component over the period since 1960, but has been fairly stable around 2 percent for the two decades leading up to 2014. Finally, the real interest rate appears to have both a clear cyclical component and a lower frequency component–consistent with the general idea that there may be some trend component to the real interest rate.

As additional control variables, we consider three time series that capture factors previous research or commentary has suggested are important for fluctuations in output and which may be particularly important in assessments or $r*$: A corporate bond spread, aggregate credit growth, and the impetus to demand from fiscal policy. A large literature documents that corporate bond spreads have important information for forecasting output or as a shifter in an IS curve (e.g., Gilchrist and Zakrajsek [2012] and Kiley [2014a]). As shown in figure 2, this spread tends to rise during recessions. Moreover. it was notably below average in the late 1990s and mid-2000s: This observation call to mind the notion from Summers [2014] that the full-employment periods experienced in the late 1990s and mid-2000s were supported by easy financial conditions ("bubbles"), which Summers [2014] suggests may have masked a trend decline in the equilibrium real interest rate. Finally, this spread has bee unusually high from the financial crisis of 2008 through 2014 and only returned to near its average level in 2014.

The middle panel presents aggregate credit growth (relative to GDP): Borio, Disyatat, and Juselius [2013] and Arseneau and Kiley [2014] suggest rapid credit growth was an impetus to the business cycle. Lo and Rogoff [2015] and Rogoff [2015] cast doubt on views of "secular stagnation" and instead view the slow recovery and low level of interest rates in recent years to the deleveraging process. According to this view, demand will recover more notably when credit growth returns to normal–and the middle panel suggests that credit growth had approached a more typical level, relative to GDP, by late 2014.

Finally, the lower panel reports the fiscal impetus measure of Follette and Lutz [2010]: This measure captures the "exogenous" impetus to GDP from fiscal policy. Such a construc-

Figure 2: Key Data Series: A Corporate Bond Spread, Credit Volumes, and Fiscal Impetus



Source: Private Nonfinancial Credit Relative to Nominal GDP, Federal Reserve Board (Financial Accounts of the United States, http://www.federalreserve.gov/releases/z1/Current/) and Bureau of Economic Analysis; Corporate bond spread, Author's computation based on 10-year Treasury yield and Corporate bond yield from Federal Reserve's FRB/US model database (available at http://www.federalreserve.gov/econresdata/frbus/us-models-package.htm. Data accessed July 2, 2015); Fiscal impetus from Follette and Lutz [2010]. Shaded regions indicate recessions as identified by the National Bureau of Economic Research.

tion is fraught with controversy, as it includes both direct, discretionary spending measures and assumptions on how tax changes flow though to spending. Because this measure is controversial, we will explore results with and without this measure. Setting these controversies aside, the fiscal impetus measure has been a notable drag on spending since 2011 and remained far below its typical level in 2014, consistent with the common view that the shift toward a contractionary fiscal stance at that time has been a factor in the relatively tepid pace of economic growth (e.g., Bernstein [2014]).

3.3 Estimation

3.3.1 Estimation Strategy

Output, inflation, unemployment, the real interest rate, and any additional controls are observed variables. Output, inflation, and the unemployment are endogenous in the model, and other controls are treated as exogenous. This follows Laubach and Williams [2003] and Clark and Kozicki [2005]. (An obvious extension is to treat other observables as "endogenous" within the model. Johanssen and Mertens [2015] pursue this idea by including a specification for the endogenous evolution of the short-term interest rate. We do not adopt this approach, and discuss problems that arise by treating control variables as exogenous in subsequent sections.)

The errors in all equations are assumed to follow a Normal distribution, implying that the Kalman filter is the optimal method for parsing the data into trend and cycle. We construct the likelihood of the data via the Kalman filter.

A long literature has emphasized that estimation of "trend/cycle" decompositions via maximum likelihood is problematic because the variance of the trend processes for economic growth "pile-up" near zero (Stock and Watson [1996]). Bayesian methods do not face the same problems (Kim and Kim [2013]). As earlier work on $r*$ has noted the empirical challenges associated with the pile-up problem, a Bayesian approach is attractive. In addition, a Bayesian approach allows imposition of prior views that capture common discussions in the literature, and a clear discussion of the information in the data through comparison of the prior and posterior distributions.

For these reasons, we take a Bayesian approach. The objective is to estimate the parameter vector θ. Under the Bayesian approach, a prior distribution, represented by the density $p(\theta|\mathcal{M})$ is combined with the likelihood function $p(\mathcal{Y}^o|\theta, \mathcal{M})$ for the observed data $\mathcal{Y}^o(= \{y_t\}_{t=1}^T)$, to obtain, via Bayes rule, the posterior:

$$p(\theta|\mathcal{Y}^o, \mathcal{M}) \propto p(\mathcal{Y}^o|\theta, \mathcal{M})p(\theta|\mathcal{M}).$$

To facilitate estimation, we must access the posterior $p(\theta|\mathcal{Y}^o, \mathcal{M})$. Unfortunately, the posteriors are analytically intractable, owing to the complex ways θ enters the likelihood function. To produce draws from the posteriors, we resort to Markov-Chain Monte Carlo.[8]

We employ a Normal(0,2) prior for all "cyclical" parameters in the equations. Given the scaling of variables in the model, this prior is fairly uninformative.

The priors for the standard errors of the cyclical and trend equations assume an Inverted Gamma distribution. For the errors in the cyclical equations, we assume means and standard errors of 2; the exception is the error in Okun's aw, which is assumed to have a mean and standard deviation of 0.25. For the trend errors, we assume means and standard errors of 0.25. These assumptions imply the prior view that fluctuations in trend growth, $r*$, U^T are modest relative to cyclical variation. Our subsequent analysis will compare these prior distributions with the posterior distributions to highlight the information in the data for $r*$.

3.3.2 Results

Our estimation sample spans from 1960Q1 to 2014Q4. We present results for three specifications:

- Case 1 – No controls (as in Laubach and Williams [2003] and Clark and Kozicki [2005]): In this case, both X and Z are empty.
- Case 2 – Only credit spread control: In this case, corporate bond spreads are the only element of Z and X is empty.
- Case 3 – All controls: In this case, Z includes the corporate bond spread, credit growth (with household and business credit separated), and fiscal impetus.[9]

[8]All our estimation procedures use Dynare (Adjemian et al [2011]).

[9]The credit variable is credit growth, measured as the quarterly percent change. It is assumed that it is this growth rate relative to trend growth g that influences the output gap.

The parameters governing dynamics are reported in table 1 and the standard errors of the shocks to each equation are reported in table 2.

	Case 1 - No controls		**Case 2-Bond Spread**		**Case 3- All controls**	
	Mean	s.d.	Mean	s.d.	Mean	s.d.
IS curve						
ρ_1	1.330	0.1008	0.920	0.1071	0.838	0.0961
ρ_2	-0.376	0.0998	0.037	0.0977	-0.016	0.0859
γ_r	0.076	0.0340	0.111	0.0357	0.126	0.0297
Γ_{Spread}	-	-	-0.549	0.0991	-0.494	0.0929
Γ_{Fiscal}	-	-	-	-	-0.047	0.1850
$\Gamma_{Credit,Bus}$	-	-	-	-	0.043	0.0161
$\Gamma_{Credit,HH}$	-	-	-	-	0.022	0.0134
Okun's Law						
β	-0.578	0.0455	-0.581	0.0457	-0.566	0.0440
Phillips Curve						
$s1$	0.632	0.0706	0.637	0.0705	0.644	0.0780
$s2$	0.151	0.0825	0.179	0.0821	0.182	0.0818
α_u	-0.073	0.0528	-0.089	0.0417	-0.087	0.0564
$B_{Credit,Bus}$	-	-	-	-	0.004	0.0202

Table 1: Moments of Posterior Distributions of Parameters for Dynamic Equations

Several results are apparent. As shown in the first two columns, the system without any controls has relatively typical properties: The AR(2) process for output shows a coefficient on the first lag (ρ_1) greater than one and a negative coefficient on the second lag (ρ_2) ; the coefficient on the output gap in the Okun's law equation (β) is near (but slightly greater than) $1/2$ (similar to Fleischman and Roberts [2011], and the estimated mean and standards error of the posterior distribution for the Phillips curve relationship between inflation and unemployment suggests that unemployment is not strongly associated with downward pressure on inflation ($\alpha_u > 0$, but small and estimated somewhat imprecisely).

The importance of the control variables can be seen in columns 3-6, which report cases 2 and 3. Note that the lagged credit spread is strongly associated with cyclical movements in output. In addition, the interest sensitivity of the output gap is larger and more precisely estimated in the presence of the controls. Finally, fiscal impetus is not significant (and enters with the "wrong sign"), and the coefficients on credit are fairly small. Fiscal impetus would enter with a large and precisely estimated coefficient if it entered contemporaneously; this contrast suggests much of the effect of this measure on output is contemporaneous. The significant effect of the credit spread on the output cycle and the estimate of the interest elasticity suggest that the addition of these variables may affect estimates of r* by an economically-meaningful degree.

	Case 1 - No controls		**Case 2-Bond Spread**		**Case 3- All controls**	
	Mean	s.d.	Mean	s.d.	Mean	s.d.
Dynamic Equations						
e^{π}	0.808	0.0399	0.812	0.0421	0.812	0.0402
e^{yc}	0.612	0.0542	0.572	0.0459	0.572	0.0417
e^{u}	0.073	0.0091	0.072	0.0092	0.072	0.0090
Trend Equations						
e^{rt}	0.261	0.1465	0.336	0.2160	0.234	0.1575
e^{Yt}	0.398	0.0674	0.357	0.0675	0.324	0.0733
e^{ut}	0.121	0.0190	0.126	0.0186	0.133	0.0183
e^{yg}	0.086	0.0149	0.088	0.0165	0.089	0.0167

Table 2: Moments of Posterior Distributions of Standard Deviation of Shocks)

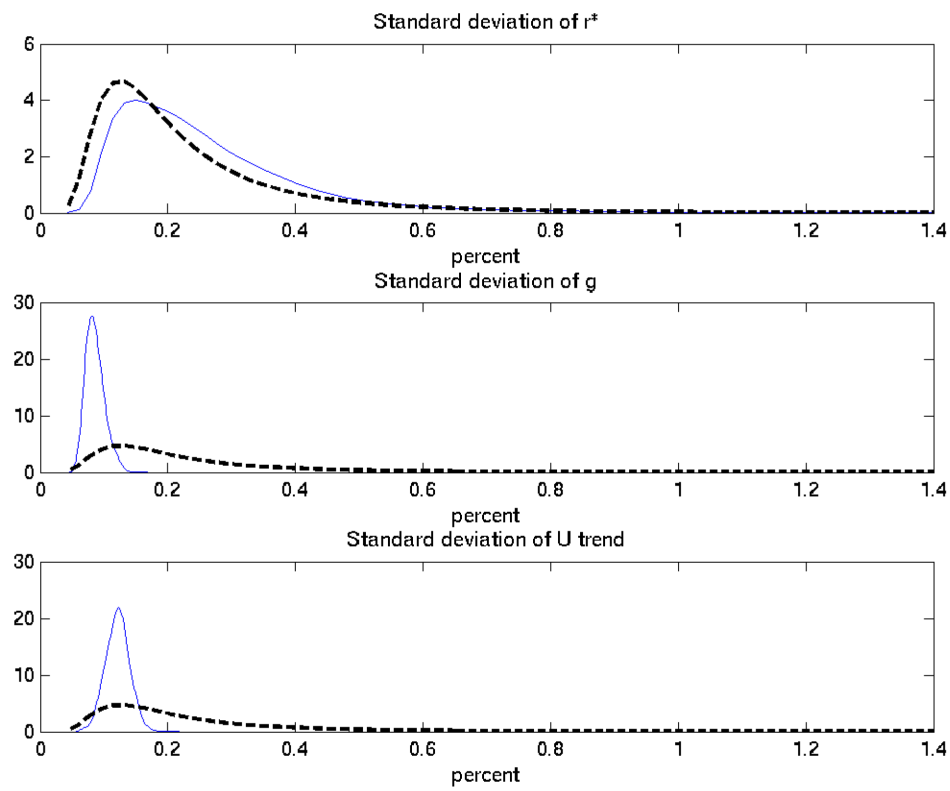
The standard error estimates in table 2 are also informative about the properties of the system. In particular, the posterior distributions suggest that the standard error for the trend growth process and the process for the natural rate of unemployment are fairly tight around their estimated mean, near 0.1 percent per quarter. In contrast, the mean and standard error of the posterior distribution for the $r*$ process lie close to the prior and are imprecise.

4 Implications of the Estimation Results

4.1 The Information in the Data

To illustrate the degree of precision, relative to the prior, in the posterior distribution of the $r*$ process. figures 3 and 4 plot the prior and posterior distributions for the standard error of the low-frequency components of $r*$, g, and U^T for case 1 and case 3.

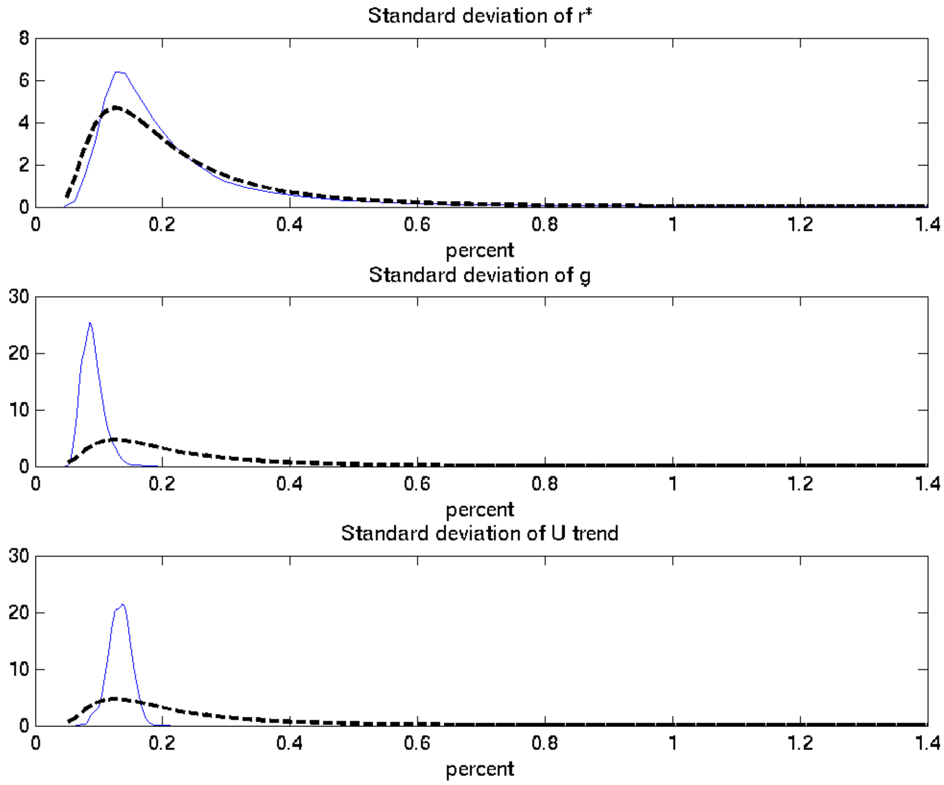
Figure 3: Prior and Posterior Distributions of Standard Errors of Trends: Case 1



Source: Author's computations from model estimates.

The results are similar across cases. The posterior distribution for $r*$ is about as diffuse and centered on similar values as the prior distribution, indicating that the data contain relatively little information about the $r*$ process. This result contrasts sharply with those for the $g*$ and U^T processes, where the posterior distribution is much tighter than the prior distribution.

Figure 4: Prior and Posterior Distributions for Standard Errors of Trends: Case 3



Source: Author's computations from model estimates.

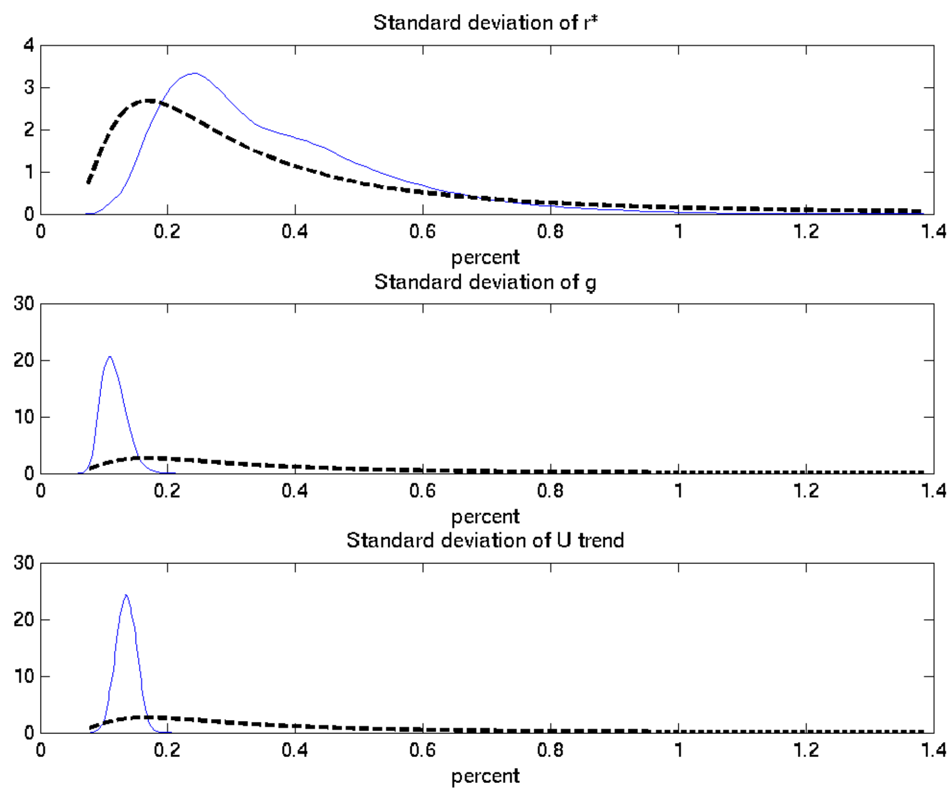
These results suggest that researchers with alternative prior views about the degree to which $r*$ has shifted over time are unlikely to be swayed by the information in the relationship between output and short-term interest rates–there is simply too little information in such co-movement over the past 50 years to provide much guidance. This result is very loosely related to the challenges associated with classical inference and $r*$ estimation, where researchers have noted sharp distinctions between maximum likelihood and median-unbiased estimates of the $r*$ process (e.g., Laubach and Williams [2003] and Clark and Kozicki [2005]).

To illustrate the role of the prior in shaping parameter estimates, three robustness checks were considered. First, following the suggestion in Mueller [2012], we shifted the prior mean of each of the standard deviations for the trends by one (prior) standard deviation and examined the change in the posterior mean. Table 3. reports the ration of the change in posterior mean to the change in prior mean for the standard deviations of $r*$, g, and U^T. This ratio is near one (about 0.8) for the $r*$ process, but much smaller for the g and U^T processes. This indicates that the data are not very informative for the $r*$ process.

	e^{rt}	e^{g}	e^{ut}
$\Delta E\sigma^{j,post}/\Delta E\sigma^{j,prior}$	0.80	0.34	0.08

Table 3: Change in Posterior Mean of Trend Innovations' Standard Deviations with Change in Prior Mean

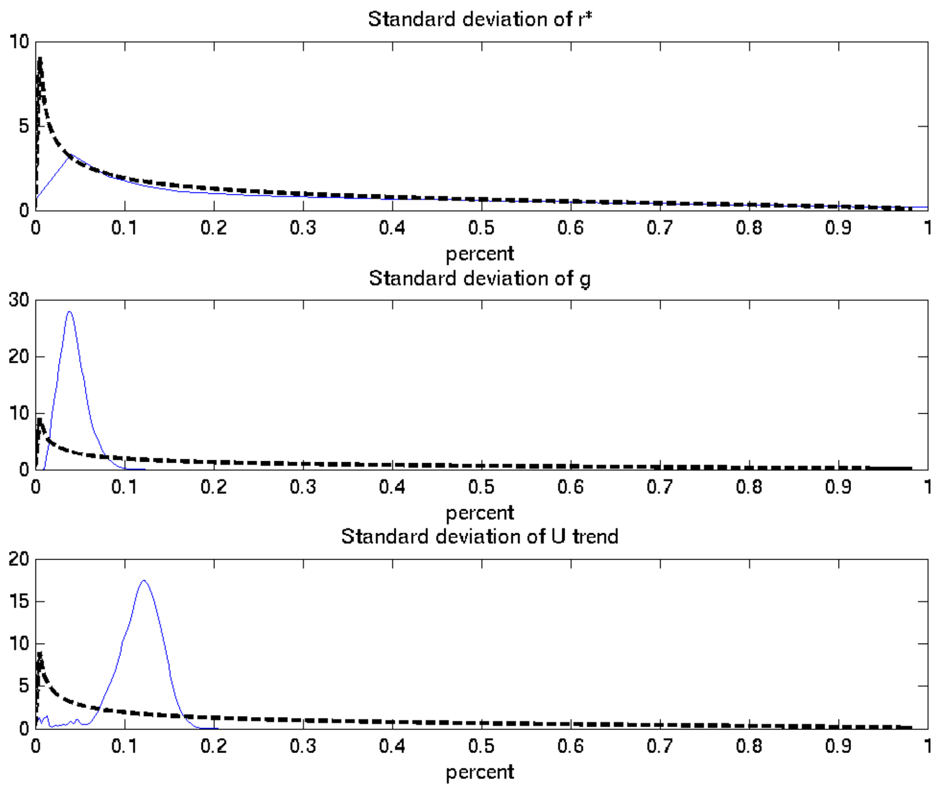
Figure 5: Prior and Posterior Distributions for Standard Errors of Trends: Case 3 with Looser Priors



Source: Author's computations from model estimates.

We also considered estimation of the parameters in which the prior mean for the standard deviations of the trends were held at 0.25, but the standard deviation of the prior was increased to 5 (from 0.25). Figure 5 compares the estimated of the priors and posteriors for the standard deviations of $r*$, g, and U^T. As before, the posterior for $r*$ resembles the prior, while those for the other trend processes are informed by the data.

Figure 6: Prior and Posterior Distributions for Standard Errors of Trends: Case 3 with Alternative Prior Shape



Source: Author's computations from model estimates.

Finally, we examined priors with a different shape. As should be apparent in the figures shown so far, the (inverted Gamma) priors for the standard deviations of the trends have peaks away from zero and place low probability on the possibility that the innovations for the trend have a very low variance. To explore the sensitive of the results for the trend processes to this prior, we considered priors for the standard deviations of $r*$, g, and U^T that peak near zero (and have the same prior mean). In particular, we assumed a Beta distribution for these parameters. Plots of the priors and posteriors under this assumption are shown in figure -6. As before, the posterior for the standard deviation of $r*$ lies very close to the prior, whereas the data appear quite informative about the g and U^T processes.

4.2 Estimates of r*

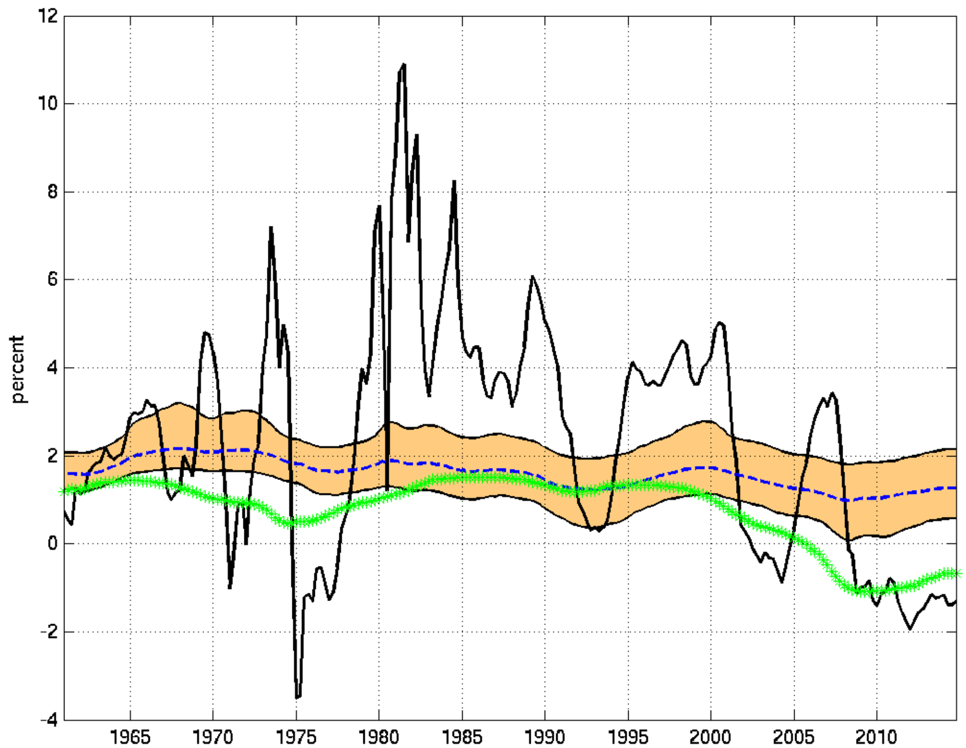
While the process for $r*$ is closely related to the assumed prior distribution, this prior captures the common view (as captured in the literature developing and using $r*$) that there may be some low frequency component to the equilibrium short-term real interest rate. As a result, examination of the model's estimate of $r*$ and the range of estimates consistent with the posterior distribution of the model's parameters carries information about both where the equilibrium rate may currently stand and the degree to which alternative views are "reasonable" in light of the model. Figure -7 presents the smoothed (or two-sided) estimate of r* from case 3 (all controls) along with the 68-percent confidence set based on the posterior distribution of model parameters.[10]

As illustrated by the comparison of the black and blue-dashed line (as well as the associated confidence set), the overwhelming share of movements in the real interest rate are viewed by the model as movements around $r*$, with only modest movements in $r*$. From the early 1960s through the 1980s, $r*$ is estimated to lie near 2 percent, close to the equilibrium rate assumed in the now-classic analysis of monetary policy rules in Taylor [1993]. At the end of 2014, the point estimate of $r*$ equaled 1-1/4 percent

The green-starred line in figure -7 represents the estimate of $r*$ from case 1, without controls in the IS curve. Two results are apparent. First, the estimate from the model without controls usually falls outside the confidence set from case 3– which is another ex-

[10] This confidence set describes the range of point estimates for $r*$ implied by the distribution of coefficients. It is not a complete description of uncertainty. In particular, this assessment abstracts from filtering uncertainty – that is, the inherent uncertainty associated with extracting an unobserved variable via the Kalman filter for known parameters. However, this notion of confidence is what is relevant for comparing results across models, as it indicates whether the coefficients from a model are "close" to those implied by the posterior distribution.

Figure 7: Estimate of $r*$



Source: Author's computations from model estimates.

pression of the finding that the controls are important for describing output gap dynamics and the model without controls lies a far distance from the posterior distribution of the model with controls. (In principle, this need not have been the case, as case 1 is a special case of the general model, and the prior distribution for all of the parameters is centered, loosely, on case 1.)

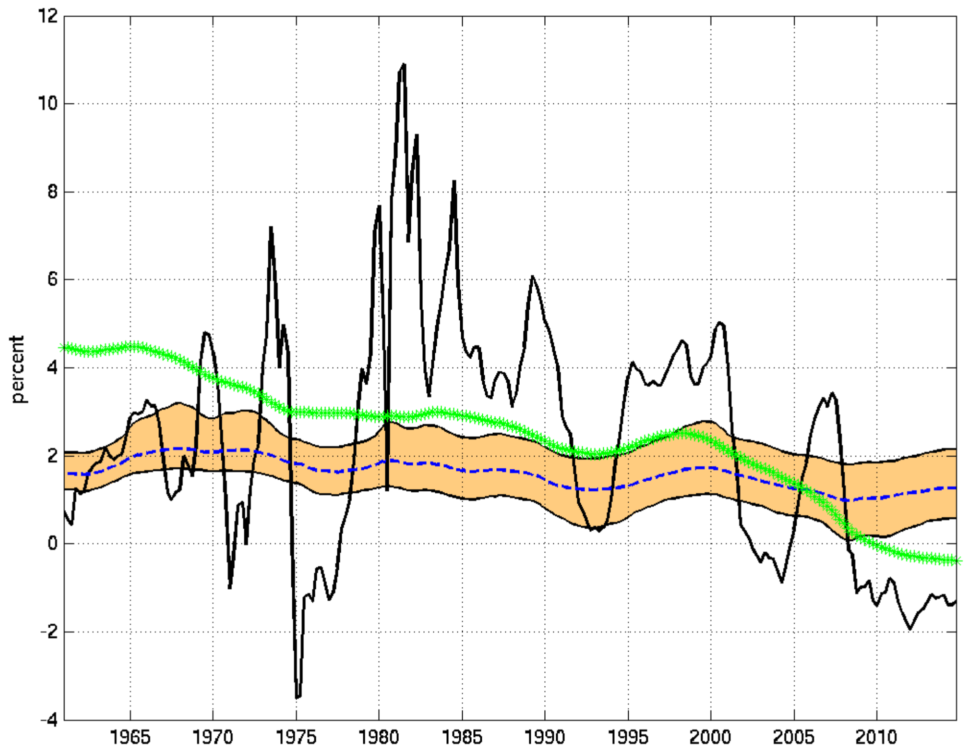
In addition, it is evident that the estimate of $r*$ absent controls is somewhat more variable than that with controls – that is, the controls appear to pick up some of the business-cycle frequency movements affecting the IS curve that models absent controls may tend to attribute to movements in $r*$. In addition, the model without controls estimates that $r*$ has fallen to a very low level since 2008, whereas the model with controls put $r*$ at 1-1/4 percent – a level modestly below that estimated for earlier decades.

This difference in results between models with and without controls raises a central question – to what extent is the model estimating a notion of the "equilibrium rate" that is relevant. The answer to this question depends upon the circumstances. In particular, the control variables are treated as exogenous in the model herein, and any comparison of estimates of the equilibrium real interest rate from the model must consider the current values of the control variables in light of historical norms for such variables. When evaluating this situation, the state of the credit spread is the key factor, as it plays the dominant role among the control factors. In this regard, it is notable from figure -2 that the corporate bond spread essentially equaled its average over the estimation period, in sharp contrast to readings for this variable in the six years prior to 2014. Because this variable is the main control variable influencing $r*$. it may be reasonable to expect the estimated value of $r*$ at end-2014 of 1-1/4 percent to be a plausible candidate for the equilibrium short-term interest rate over the *long run*. That said, the value of short-term interest rates compatible with output at potential *in the short run* will be influenced by many factors. This assessment is made very clear by the explicit consideration of control variables in our model, but is equally true of models that omit relevant factors in their empirical analysis.

4.3 Comparison to Laubach and Williams Results

Indeed, the simpler models analyzed in Laubach and Williams [2003] and Clark and Kozicki [2005] omit any controls. While this approach avoids the challenge of interpreting the state of controls relative to historical values or likely values in the future, it potentially omits important information. As a result, it is useful to examine the estimates from the model herein to that produced by the Laubach and Williams [2003] model (figure 8). As in figure 7, the black and blue-dashed lines present the data and $r*$ estimate from case 3, respectively, while the shaded region is the 68-percent confidence set. The green-starred

Figure 8: Estimate of $r*$ from Laubach and Williams [2003]



Source: Author's computations from model estimates and Federal Reserve Bank of San Francisco (http://www.frbsf.org/economic-research/economists/john-williams/).

line is the estimate from Laubach and Williams [2003].[11]

The comparison illustrates clearly that the models herein behave quite differently from Laubach and Williams [2003]–both over history, and in the recent sample (where the Laubach and Williams [2003] estimate of $r*$ falls to nearly $-1/2$ percent by end-2014, as compared to 1-1/4 percent for the case-3 model. Indeed, the confidence set suggests that there are not likely coefficient combinations for the specification herein that would lead to estimates similar to those in Laubach and Williams [2003].

The dis-similarity of results partially reflects the role of control variables, and the challenges of interpreting $r*$ in the presence of control variables that were discussed in the

[11]This is the two-sided estimate as reported on the website of the Federal Reserve Bank of San Francisco as of June, 2015.

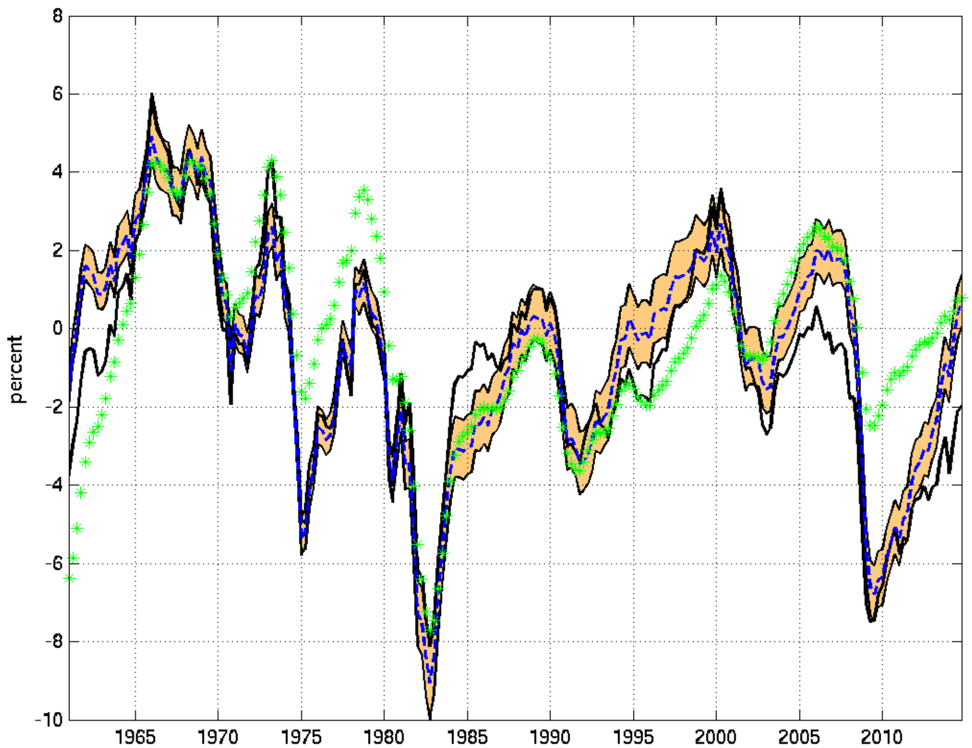
previous subsection. Nonetheless, the models are give quite different results over most of the sample period even abstracting from control variables, as is evident from a comparison of the green-starred lines in figures 7 and 8. This is not especially surprising – the model herein uses more data (output, inflation, unemployment, and real interest rates *vs.* output, inflation, and real interest rates), a different Phillips curve (with a role for long-term inflation expectations), a different estimation sample (including the Great Recession in this analysis, *vs.* a sample period ending in 2000) , and a Bayesian approach. Each of these factors plays some role in accounting for differences.

4.4 Auxiliary Implications of Models

A more direct assessment of the economic implications of these differences can be seen by comparing auxiliary implications of the models. An especially interesting comparison of differences across frameworks can be seen in alternative assessments of the output gap from different frameworks.

Figure -9 presents the estimate of the output gap from the Congressional Budget Office, the case-3 model (which is very similar to that from all specifications analyzed herein), and the Laubach and Williams [2003] model. (The shaded region is the 68-percent confidence set associated with coefficient uncertainty; as above, filter uncertainty is not the focus herein, but is important for all methods). The figure illustrates the similarity in assessments of resource utilization from the early 1960s through about the year 2000 (the end of the estimation sample in Laubach and Williams [2003]). After that period, the CBO and model yield similar assessments of the output gap, but the Laubach and Williams [2003] model is quite different. For example, output is only slightly below potential in 2009 according to the Laubach and Williams [2003] model, but is very far below potential in the alternative assessments. One factor contributing to these different assessments is the role of the unemployment rate, which is central in research on the trend/cycle decomposition of output (e.g., Basistha and Startz [2008] and Fleischman and Roberts [2011]).

Figure 9: Estimate of Output Gap from CBO, Case 3, and Laubach and Williams [2003]



Source: Author's computations from model estimates and Congressional Budget Office (https://www.cbo.gov/sites/default/files/cbofiles/attachments/45066-2015-07-EconomicDataProjections.xlsx).

5 Conclusion

Our analysis points to two central conclusions.

First, the data provide relatively little information on the $r*$ data-generating process, as indicated by a posterior distribution for the $r*$ process that looks like the prior distribution we have assumed. This suggests some caution to researchers adopting either a Bayesian or classical approach to inference regarding $r*$: The data may not be informative, and hence specification choices, including priors (either explicit, as in a Bayesian approach, or implicit, as determined by *a prior* zero restrictions), may dominate results for $r*$. This results contrasts sharply with those for the trend growth or natural rate of unemployment processes, for which the data are very informative.

Second, inclusion of plausible cyclical determinants of output, such as corporate bond spreads, credit growth, or fiscal policy may have important effects on estimates of $r*$. Models that abstract from such considerations may be mis-specified, as they assume that the dynamic influence of such variables is the same across all such "shocks" to the IS curve. (This equivalence is implicit in subsuming such shocks into a single "error-term" in the IS curve.) Models with additional controls, most importantly a corporate bond spread, imply an equilibrium real interest rate of about 1-1/4 percent at the end of 2014. The corporate bond spread had returned to an average value by end-2014, which may imply a neutral contribution from this factor in 2014.

A third implication of our analysis is the potential importance of examining the implications of models used to assess $r*$ for other estimates of the state of the economy. The models herein provide estimates of the output gap that are very similar to those from the Congressional Budget Office, in contrast to the output gap from the model of Laubach and Williams [2003], which provides some indirect support for the approach taken herein.

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