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Abstract: This paper proposes a new test of the null hypothesis that a generalized method of moments model is identified. The test can detect local or global underidentification, and underidentification in some or all directions. The idea of the test is to compare the volume of two confidence sets -- one that is robust to lack of identification and one that is not. Under the null hypothesis the relative volume of these two sets is $O_p(1)$, but under the alternative, the robust confidence set has infinite relative volume.

Keywords: identification, robust confidence sets, weak instruments, generalized method of moments.

JEL Classifications: C12, C20, C30.

1. Introduction.

One of the key assumptions of the standard linear instrumental variables (IV) model is that the instruments and endogenous variables are correlated. This is the identification assumption, on which the conventional asymptotic theory for the IV model depends. Indeed even if the correlation between the instruments and the endogenous variables is nonzero, but slight, then the conventional Gaussian asymptotic theory can nevertheless provide a very poor approximation to the actual sampling distribution of estimators and test statistics (see, for example, Bound, Jaeger and Baker (1995)). A large literature has considered the exact sampling distribution of the two stage least squares (TSLS) and limited information maximum likelihood (LIML) estimators in models with nonstochastic instruments and Gaussian innovations. These exact distributions are far from the limits obtained from conventional asymptotic theory when the instruments are weak. TSLS is severely biased in the direction of the probability limit of OLS and the associated t-statistic is highly nonnormal and can even be bimodal. Recently, alternative asymptotic nestings have been proposed, which provide much better approximations to the actual sampling distribution of estimators and test statistics in the IV model. Bekker (1994) models the number of instruments as being an increasing function of the sample size. Staiger and Stock (1997) model the correlation between the instruments and endogenous variables as being local to zero.

The generalized method of moments (GMM) model (Hansen (1982)) nests the

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linear IV model as a special case. It is not surprising that analogous issues arise in this model. Many researchers have found that, in a wide variety of contexts, the conventional Gaussian asymptotic theory provides a poor approximation to the sampling distribution of GMM estimators and test statistics. There are many possible reasons why this could happen, but they include identification problems. The identification condition in the GMM model requires the moment condition to have a unique zero at the true parameter value and to have a full rank gradient and is crucial, just as it is in the linear IV special case.

Fortunately, approaches to inference are available that are robust to failure or near-failure of the identification condition. These robust approaches to inference do not permit precise inference on a structural coefficient that is not well identified - that is of course impossible. Robust approaches to inference consist of hypothesis tests and confidence sets that correctly reflect the lack of identification. If the instruments are completely irrelevant then a robust confidence set should contain the whole parameter space (or at least any point in it with probability equal to the coverage of the confidence set). In the context of the linear IV model with fixed instruments and Gaussian errors, Anderson and Rubin (1949) proposed an exact test of the hypothesis that the entire vector of structural coefficients takes on a specified value. An exact confidence set for the entire vector of structural coefficients can be computed as the set of hypothesized coefficient vectors for which this test does not reject. This confi-

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dence set has coverage close to the nominal level with stochastic instruments and/or nonnormal errors, even if the instruments are weak (Staiger and Stock (1997)). It is thus a robust confidence set. Stock and Wright (2000) construct a nonlinear analog of the Anderson-Rubin confidence set applicable in a general GMM model with possible identification difficulties. Other robust confidence sets were proposed by Wang and Zivot (1998), Kleibergen (2001, 2002) and Moreira (2002), and are discussed by Stock, Yogo and Wright (2002).

All of these robust confidence sets that control coverage whether the model is identified or not are formed by inverting the acceptance region of a test statistic. None of them give us point estimates - consistent point estimation is impossible without identification. There is a strong case to be made for saying that researchers should always report only robust confidence sets. However, practitioners are fond of reporting point estimates and associated standard errors. Part of this attachment may be force of habit and part of it may be that there is real value in a point estimate. In addition, robust confidence sets do not generally give us confidence sets for individual elements of the whole parameter vector, other than conservative confidence sets obtained by projection methods. Point estimates and conventional standard errors of course yield confidence intervals for individual elements of the parameter vector. For these reasons, it seems important to provide a diagnostic so as to indicate whether there is an identification problem or not. If the diagnostic

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indicates identification difficulties, the researcher should be warned to use only robust confidence sets. Otherwise, the researcher may rely on conventional point estimates and confidence sets.

In the linear IV model, the first-stage F-test involves running a regression of the endogenous variables on the instruments and testing the null hypothesis of the joint insignificance of the slope coefficients. The null hypothesis is one of a lack of identification. Although an important and useful diagnostic, a significant first-stage F-statistic by no means implies that issues of weak instruments can be ignored (see, for example, Hall, Rudebusch and Wilcox (1996), Staiger and Stock (1997) and Stock and Yogo (2001)). A computationally intensive and asymptotically conservative analog of this test for the GMM model was developed by Wright (2002): this is the only extant test for identification or lack of identification in the nonlinear-in-parameters context that I am aware of.

The first-stage F-test tests the hypothesis that the model is *not* identified. Recently, Hahn and Hausman (2002) proposed a test of the hypothesis that the linear IV model *is* identified. In the case of a single right hand side endogenous variable, the idea is to compare the forward and reverse TSLS regressions. I am not aware of any existing test of the hypothesis that the model *is* identified that is applicable in the nonlinear-in-parameters context.

In this paper, I propose a new test of the hypothesis that the model *is* identified,

applicable in the general GMM model provided that the model has more moment conditions than parameters. The idea is to compare the volume of a Wald confidence set (not robust to identification difficulties) with the volume of a robust S-set. Under the null that the model is identified, this ratio is $O_p(1)$. Under the alternative, the robust confidence set has infinite relative volume.

I argue that for a test of either the null of identification or of a lack of identification to be useful, it must indicate a lack of identification not only when the model is completely unidentified but also when the identification is so weak that conventional Gaussian asymptotics works very poorly. Comparing the first-stage F-statistic with χ_k^2/k critical values, where k denotes the number of instruments, does not satisfy this requirement. In the context of the linear IV model, Stock and Yogo (2001), using the weak instrument asymptotics of Staiger and Stock (1997), show how to solve for the critical value of a first-stage F-statistic that ensures that the weak instrument asymptotic coverage of the TSLS Wald confidence set is no smaller than some bound, or that the asymptotic bias of TSLS is no greater than some bound. These critical values are much higher than the χ_k^2/k critical values with which the first-stage F-statistic is usually compared. Although the test of a null of identification in GMM that I propose in this paper does not have any weak identification asymptotic motivation along the lines provided by Stock and Yogo in the linear IV model, the proposed test focuses directly on the comparison of robust and non-robust confidence sets and

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rejects the null of proper identification whenever robust and non-robust confidence sets are very different. As such, it might be hoped that the proposed test will indicate circumstances in which identification is so weak that conventional asymptotics are seriously misleading and that a strategy of using the robust confidence set when the proposed test rejects, but using the regular Wald confidence set otherwise will control coverage quite well (while also indicating circumstances in which a researcher may safely use point estimates and conventional standard errors). This will of course be demonstrated in the Monte-Carlo simulations below.

The plan for the remainder of this paper is as follows. The GMM model is introduced in section 2. Section 3 describes the proposed test and derives its asymptotic distribution. Section 4 contains a Monte-Carlo simulation evaluating the properties of the proposed test. Section 5 contains two empirical applications. Section 6 concludes.

2. The GMM Model.

The GMM model specifies that $\{Y_t\}_{t=1}^T$ is an observed time series and θ is an nx1 parameter vector with a true value θ_0, in the interior of a compact space Θ, such that

$$E(\phi(Y_t, \theta_0)) = 0$$

where $\phi(.,.)$ is a k-dimensional function, $k \geq n$. The GMM estimator of θ is

$$\hat{\theta} = \arg\min_\theta S(\theta)$$

where

$$S(\theta) = \phi^*(\theta)' W_T \phi^*(\theta),$$

$\phi^*(\theta) = [T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta)]$ and W_T is a symmetric positive definite kxk weighting matrix which converges almost surely to a symmetric nonstochastic $O(1)$ positive definite matrix W. Here are the standard assumptions for the GMM model:

Assumption A1: $\phi^*(\theta)$ is twice continuously differentiable, for all θ in Θ.

Assumption A2: $T^{-1}\Sigma_{t=1}^T \phi(Y_t, \theta) \to_{as} E(\phi(Y_t, \theta))$ and $T^{-1}\Sigma_{t=1}^T \frac{d\phi(Y_t,\theta)}{d\theta} \to_{as} E[\frac{d\phi(Y_t,\theta)}{d\theta}]$, uniformly in θ.

Assumption A3: $T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta_0) \to_d N(0, A)$, where A is 2π-times the zero-frequency spectral density matrix of $\phi(Y_t, \theta_0)$.

Assumption A4: The kxn matrix $B = E[\frac{d\phi(Y_t,\theta_0)}{d\theta}]$ has rank n.

Assumption A5: $E(\phi(Y_t, \theta))$ has a unique zero at $\theta = \theta_0$.

Assumption A6: $V_T(\theta)$ is an estimator of 2π-times the zero-frequency spectral density matrix of $\phi(Y_t, \theta)$ that is consistent, uniformly in θ.

Assumptions A2 and A3 are high level convergence assumptions. Assumption A4 is the local identification assumption. Assumption A5 is the global identification assumption (Hsiao (1983)). Under these assumptions, $\hat{\theta} \to_p \theta$ and

$$\sqrt{T}(\hat{\theta} - \theta) \to_d N(0, (B'WB)^{-1}B'WAWB(B'WB)^{-1})$$

The asymptotically efficient estimator is obtained by choosing a weighting matrix such that $W = A^{-1}$; the variance of this asymptotic distribution is then $(B'A^{-1}B)^{-1}$.

One possible choice of the weighting matrix is the identity matrix. This yields the objective function

$$S_{OS}(\theta) = [T^{-1/2}\Sigma_{t=1}^{T}\phi(Y_t, \theta)]'[T^{-1/2}\Sigma_{t=1}^{T}\phi(Y_t, \theta)]$$

Denote the resulting estimator by $\hat{\theta}_{OS} = \arg\min_{\theta} S_{OS}(\theta)$. This estimator is not asymptotically efficient. A feasible asymptotically efficient estimator can be obtained by setting the weighting matrix equal to $V_T(\hat{\theta}_{OS})^{-1}$, yielding the objective function

$$S_{TS}(\theta) = [T^{-1/2}\Sigma_{t=1}^{T}\phi(Y_t, \theta)]'V_T(\hat{\theta}_{OS})^{-1}[T^{-1/2}\Sigma_{t=1}^{T}\phi(Y_t, \theta)].$$

Denote the resulting estimator, called the two-step estimator, by $\hat{\theta}_{TS} = \arg\min_{\theta} S_{TS}(\theta)$. Another feasible asymptotically efficient estimator can be obtained by setting the weighting matrix equal to $V_T(\theta)$, yielding the objective function

$$S_{CU}(\theta) = [T^{-1/2}\Sigma_{t=1}^{T}\phi(Y_t, \theta)]'V_T(\theta)^{-1}[T^{-1/2}\Sigma_{t=1}^{T}\phi(Y_t, \theta)].$$

Denote the resulting estimator, called the continuous-updating estimator, by $\hat{\theta}_{CU} = \arg\min_{\theta} S_{CU}(\theta)$. This estimator was proposed by Hansen, Heaton and Yaron (1996). If $k = n$, the two-step and continuous-updating estimators are numerically equivalent.

2.1 *Problems with Standard Gaussian Asymptotics for GMM.*

The above asymptotic theory often works poorly in practice. Often, in empirically relevant sample sizes, $\hat{\theta}_{TS}$ and $\hat{\theta}_{CU}$ are biased and have sampling distributions far from those predicted by this asymptotic theory, and the associated t- and F-statistics have erratic rejection rates. These problems, documented in numerous Monte-Carlo studies, could arise because $T^{-1/2}\Sigma_{t=1}^{T}\phi(Y_t, \theta_0)$ fails to converge to normality, or converges only very slowly. Alternatively, it could be that $E(\phi(Y_t, \theta))$ is zero, or close to zero, even for $\theta \neq \theta_0$ - in violation of assumptions A4 and A5.

The focus of this paper is on problems with the asymptotic theory underlying GMM which arise from this latter source: an identification problem. Stock and Wright (2000) proposed an alternative asymptotic nesting in which $E(\phi(Y_t, \theta)) = O(T^{-1/2})$, uniformly in θ. They derive an alternative asymptotic theory which nests the completely unidentified model $(E(\phi(Y_t, \theta)) = 0$, uniformly in $\theta)$ as a special case. This alternative asymptotic theory works much better than the conventional Gaussian asymptotic theory in providing an approximation to the finite sample distributions of GMM estimators and test statistics when identification is weak. In the linear IV model, it reduces to the nesting proposed by Staiger and Stock (1997).

2.2 *S-sets.*

The weak identification problem in GMM may be effectively circumvented by the use of S-sets, as proposed in Stock and Wright (2000). The approach dispenses with point

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estimation and instead forms a confidence set for θ directly from an objective function, using a nonlinear analog of the Anderson-Rubin confidence set. If assumption A3 holds, and if $V_T(\theta_0) \to_p A$, then the continuous-updating objective function evaluated at the true parameter vector, $S_{CU}(\theta_0)$, converges to a χ^2 distribution on k degrees of freedom. No identification assumption (assumption A4 or A5) is required for this to hold. The confidence set for θ is formed as the inverse of the acceptance region of this test, i.e. the confidence set of coverage 1-α is $S_\theta^*(\alpha) = \{\theta : S_{CU}(\theta) \le F_{\chi^2}(k, \alpha)\}$ where $F_{\chi^2}(a, b)$ is the $100b$ percentile of a χ^2 distribution on a degrees of freedom. In a completely unidentified model $(E(\phi(Y_t, \theta)) = 0$, uniformly in $\theta)$ or a locally asymptotically underidentified model $(E(\phi(Y_t, \theta)) = O(T^{-1/2})$, uniformly in $\theta)$, such a confidence set will have infinite expected volume. But this is the correct statement of our uncertainty about θ in the presence of weak identification. More formally, under these circumstances, any confidence set that is valid (i.e. controls coverage) must have infinite expected volume (Dufour (1997)).

2.3 *The Homoskedastic Linear IV Model*

The linear IV model with iid errors specifies that

$$y = X\beta + u$$

$$X = Z\Pi + v$$

where y and X are Tx1 and Txn matrices of endogenous variables, Z is a Txk matrix

of instruments $(k \geq n)$, and u and v are conformable matrices of errors such that $w_t = (u_t, v_t')'$ has variance-covariance matrix Ω (the t subscript on any matrix denotes the tth observation for that variable). The first of these equations is the structural equation of interest, and the researcher wants to do inference on the coefficient vector β. Without loss of generality, there are no included exogenous variables in this equation. If they were present, they would just be projected out. Because u_t and v_t are correlated, OLS is biased.

This model is a special case of the general GMM model with $\phi(Y_t, \theta) = (y_t - X_t'\beta)Z_t$, $\theta = \beta$ and $V_T(\theta) = T^{-1}\Sigma_{t=1}^T Z_t Z_t' T^{-1} \Sigma_{t=1}^T (y_t - X_t'\beta)^2$. Since $B = E[\frac{d\phi(Y_t, \theta_0)}{d\theta}] = E[Z_t X_t'] = E[Z_t Z_t']\Pi$, the local identification condition reduces to requiring that the matrix Π has rank n. If this condition is satisfied, assumption A5 is satisfied too. The TSLS estimator is

$$\hat{\beta}_{TSLS} = (X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'y$$

which is the two-step GMM estimator. The continuous-updating GMM estimator reduces to LIML. The S-set reduces to

$$\{\beta : \frac{(y-X\beta)'P_Z(y-X\beta)}{(y-X\beta)'(y-X\beta)/T} \leq F_{\chi^2}(k, \alpha)\}$$

where $P_Z = Z(Z'Z)^{-1}Z'$. This is the Anderson-Rubin confidence set (Anderson and Rubin (1949)). If $n = 1$ (a single right hand side endogenous variable), the confidence set can be written as

$$\{\beta : (X'P_Z X - \tfrac{F_{\chi^2(k,\alpha)}}{T} X'X)\beta^2 + 2\{\tfrac{F_{\chi^2(k,\alpha)}}{T} y'X - y'P_Z X\}\beta + \{y'P_Z y - \tfrac{F_{\chi^2(k,\alpha)}}{T} y'y \le 0\}$$

This is a quadratic inequality the solution to which is a confidence set for β that must be an interval, the whole real line, the complement of an interval or an empty confidence set. The Anderson-Rubin confidence set is robust to weak instruments. Staiger and Stock (1997) show that it controls coverage in their weak instrument asymptotics. The test is indeed an exact test, if the instruments are fixed and errors are Gaussian (after replacing χ^2 critical values with F critical values).

The first-stage F-test tests the hypothesis that $\Pi = 0$. A rejection of this hypothesis by no means implies that there are no identification difficulties. The identification may not be strong enough for conventional asymptotic theory to work well, as discussed in the introduction. There is an additional issue if $n > 1$. The identification condition requires Π to have rank n, which is a stronger requirement than $\Pi \neq 0$ if $n > 1$. The case in which Π has nonzero rank, smaller than n, can be called partial identification and also leads to failure of conventional asymptotic theory, as discussed by Choi and Phillips (1992). Cragg and Donald (1993) extend the first-stage F-test to test the hypothesis that Π has a specified rank smaller than n. It is a convenient feature of testing the null hypothesis of identification, as I do in this paper, that the test is consistent against the alternative that Π has any rank smaller than n (or that B has any rank smaller than n, in the general GMM case).

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2.4 *Identification Problems in Other Cases.*

The linear IV model with iid errors is only one special case of the general GMM model. The GMM model can accommodate heteroskedastic and/or dependent errors. Indeed, any maximum likelihood estimator is a special case of GMM in which the moment condition is the score function.

A leading application of the GMM model is the consumption capital asset pricing model (CAPM) of Lucas (1978). With constant relative risk aversion (CRRA) preferences, the consumption CAPM Euler equation implies the GMM moment condition $E(Z_t \otimes [\delta R_{t+1}(\frac{C_{t+1}}{C_t})^{-\gamma} - 1]) = 0$ where C_{t+1} and R_{t+1} denote consumption and a vector of gross asset returns, respectively, δ is the discount factor, γ is the coefficient of relative risk aversion and Z_t is any variable in the information set at time t. This can be interpreted as a nonlinear-in-parameters instrumental variables model with Z_t as the instrument vector. However, consumption growth and asset returns are notoriously hard to predict - meanwhile the identification assumptions (A4 and A5) require nonlinear functions of these to be forecastable. Weak identification is thus a prime issue in this context (see Stock and Wright (2000), Wright (2002) and Stock, Wright and Yogo (2002)). The first-stage F-statistic does not apply in the nonlinear-in-parameters GMM model.

Identification in GMM essentially requires that the objective function be locally quadratic (assumption A4) and that it does not have multiple local minima which

give the same value of the population objective function (assumption A5). These conditions apply even where there are no instrumental variables. For example, a logit or probit model in which most of the dependent variables are zeros (or in which most of the dependent variables are one) will have a likelihood that is flat around the true parameter value, causing an identification problem. Likewise, the pseudo-Gaussian likelihood function for a ratio of two parameters will be flat if the denominator is close to zero. Pagan and Robertson (1998) argue that weak identification problems can arise in the structural VAR literature.

This motivates the construction of a test of the identification conditions in the general GMM model (assumptions A4 and A5).

3. The Proposed Test.

As discussed above, identification is a key requirement for conventional asymptotic theory. I am however only aware of one extant test for the null hypothesis that the model *is* identified. That is the test proposed by Hahn and Hausman (2002) that compares forward and reverse TSLS estimators. It applies only in the linear IV model.

This paper proposes a new test of the null that the model is identified, which applies in the general GMM context so long as $k > n$. Define V_1 and V_2 as the volume of the robust S-set and of the usual two-step Wald confidence set for θ, i.e.

$$V_1 = V(\theta : S_{CU}(\theta) \le F_{\chi^2}(k, \alpha))$$

$$V_2 = V(\theta : T(\hat{\theta}_{TS} - \theta)'B'A^{-1}B(\hat{\theta}_{TS} - \theta) \le F_{\chi^2}(n, \alpha))$$

where $V(\theta : .)$ denotes the volume of θ satisfying the specified condition. Define V_3 as the volume of points that are both in the robust S-set and in the usual Wald confidence set, i.e.

$$V_3 = V\{\theta : S_{CU}(\theta) \le F_{\chi^2}(k, \alpha); \; T(\hat{\theta}_{TS} - \theta)'B'A^{-1}B(\hat{\theta}_{TS} - \theta) \le F_{\chi^2}(n, \alpha)\}$$

Let $L_1 = V_1/V_2$, denoting the relative volume of the two confidence sets. The relative volume may be zero or infinite. Let $L_2 = 1(V_1 > 0)V_3/V_1 + 1 - 1(V_1 > 0)$, denoting the fraction of the S-set that is also included in the Wald set (if $V_1 = 0$, L_2 is defined to be 1). The tests proposed in this paper use L_1 and L_2. Throughout the paper, I use the Wald confidence sets associated with the two-step estimator, but exactly the same asymptotic distribution theory would apply to Wald confidence sets associated with the continuous-updating estimator[1].

The numerical computation of these test statistics simplifies in the linear IV model with $n = 1$. In this case, there exists a closed form expression for the robust

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[1] There are two reasons for focussing on the two-step estimator. Firstly, practitioners using a non-robust estimator use the two-step estimator (TSLS) far more frequently than they use the continuous-updating estimator (LIML). Secondly, the continuous-updating estimator is not robust to lack of identification, but still works somewhat better than the two-step estimator when identification is weak. Accordingly, it might be expected (and it turns out in simulations to be true) that a test for a lack of identification that compares the S-set with the two-step Wald confidence set is more powerful in the sense that it has a higher rejection rate than the test that compares the S-set with the continuous-updating Wald confidence set when identification is weak.

S-set, which is the solution to the quadratic equation, equation (1). If the confidence

set is an interval, its volume (V_1) is the width of this interval. If the confidence set

is empty, its volume is zero. If the confidence set is the whole real line or is the

complement of an interval, its volume is infinite. The volume of the Wald confidence

set (V_2) is just $2\sqrt{\frac{F_{\chi^2}(1,\alpha)}{s_u^2 X'P_Z X}}$ where $s_u^2 = T^{-1}(Y - X\hat{\beta}_{TSLS})'(Y - X\hat{\beta}_{TSLS})$. The test

statistic L_1 is just the ratio of these two volumes. The test statistic L_2 can be

approximated by taking a fine grid of points in the robust S-set, and evaluating what

fraction of these are also in the Wald confidence set. If the robust S-set is unbounded,

set $L_2 = 0$. If it is empty, set $L_2 = 1$.

The limiting distributions of L_1 and L_2 under the null of identification (as-

sumptions A4 and A5) are provided in Theorem 1, the proof of which is given in the

appendix. It involves consideration of the behavior of the two-step and continuous-

updating GMM objective functions in a $T^{-1/2}$-neighborhood of the true parameter

value.

Theorem 1: Under assumptions A1-A6, if $k > n$,

$$L_1 \to_d V(b : (\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k,\alpha))/\{\frac{2F_{\chi^2}(n,\alpha)^{n/2}\pi^{n/2}}{n|G'G|^{1/2}\Gamma(n/2)})$$

$$L_2 \to_d I^*V(b : (b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \leq F_{\chi^2}(n,\alpha);$$

$$(\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k,\alpha))/V(b : (\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k,\alpha)) + 1 - I^*$$

where ϕ is a $N(0, I_k)$ kx1 vector, $G = A^{-1/2}B$, $\Gamma(.)$ denotes Euler's gamma function

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and $I^* = 1[\inf_b (\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k, \alpha)]$.

Under the alternative hypothesis of a lack of identification (violation of assumptions A4 and A5), the test statistic L_1 diverges to infinity, while L_2 converges to zero. The tests are one-tailed tests which reject for large values of L_1 and for small values of L_2. The test has power against the alternative that the model is locally unidentified and against the alternative that it is locally but not globally identified[2], unlike any extant test I am aware of[3].

The numerical computation of the null asymptotic distribution of the test statistics simplifies in the case $n = 1$. After a little algebra, the null limiting distribution of L_1 reduces to

$$\frac{1}{\sqrt{F_{\chi^2}(1,\alpha)}} \sqrt{\frac{(G'\phi)^2}{G'G} - \phi'\phi + F_{\chi^2}(k, \alpha)} \, 1[\frac{(G'\phi)^2}{G'G} - \phi'\phi + F_{\chi^2}(k, \alpha) \geq 0]$$

which can easily be simulated, given a consistent estimate of G. For a given ϕ, $b : (\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k, \alpha)$ is just given by the interval $\frac{-G'\phi \pm \sqrt{(G'\phi)^2 - G'G(\phi'\phi - F_{\chi^2}(k,\alpha))}}{G'G}$.

Setting up a grid across this interval, it can be calculated what fraction of these bs

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[2] A variant on the test that would be more powerful in detecting global but not local failure of identification would involve replacing the robust S-set by the convex hull of the robust S-set. This would make no difference to the statement of Theorem 1 (because the robust S-set is convex with probability one asymptotically under the null of identification).

[3] It would actually be possible to do two-tailed tests as well. If the robust S-set is empty, or excessively small in volume (relative to the Wald confidence set), this indicates a specification problem. A two-tailed test would be a joint test of identification and specification that would have power in both directions. Many important papers including Bound, Jaeger and Baker (1995) have been concerned simultaneously about identification and specification. But in this paper I focus exclusively on testing for identification difficulties, and so adopt a one-tailed test.

also satisfy the condition $(b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \leq F_{\chi^2}(n, \alpha)$. The asymptotic distribution of L_2 can be simulated by taking random draws of ϕ and calculating this fraction for each ϕ^4. These would both be degenerate distributions, equal to 1, if $n = k = 1$. Indeed the distributions of L_1 and L_2 are always degenerate if $n = k$. But the statement of Theorem 1 ruled out this case. If $k = n$, the robust S-set is asymptotically equivalent to the Wald confidence set[5]. This is not so in the case $k > n$, where the robust S-set wastes power relative to the Wald set, if the model really is identified. Alternative confidence sets have been proposed that are robust to weak identification. Some of these have the additional feature that they *are* asymptotically equivalent to the Wald confidence set if the model really is identified and $k > n$ (Kleibergen (2001) in the linear IV model and Kleibergen (2002) in the general GMM context). A test for a null of identification could be conducted by comparing the volume of a Wald and a robust confidence set in cases where these two are first-order asymptotically equivalent. This would however be harder as the test would be based on the second-order asymptotic difference between the two confidence sets. The fact that the robust S-set wastes power if $\dot{k} > n$ greatly

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[4]For any ϕ such that $(G'\phi)^2 - G'G(\phi'\phi - F_{\chi^2}(k, \alpha)) < 0$, the set of b satisfying $(\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k, \alpha)$ is empty, and this fraction is set to 1.

[5]In the case $k = n$ (and in this case alone), the sets $b : (\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k, \alpha)$ and $b : (b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \leq F_{\chi^2}(n, \alpha)$ are both the same as the set $b : (G^{-1}\phi + b)'G'G(G^{-1}\phi + b) \leq F_{\chi^2}(n, \alpha)$ which in turn has the same volume (but a different location) as the set $b : b'G'Gb \leq F_{\chi^2}(n, \alpha)$.

simplifies the derivation of an expression for the relative volume of the robust and non-robust confidence sets.

For general n, the set $b : (\phi+Gb)'(\phi+Gb) \leq F_{\chi^2}(k, \alpha)$ is convex. For any ϕ such that $(\phi - G(G'G)^{-1}G'\phi)'(\phi - G(G'G)^{-1}G'\phi) \leq F_{\chi^2}(k, \alpha)$, this set is empty and so has zero volume. Otherwise, the volume of this set for a given ϕ can be calculated by (i) solving for b_i^U and b_i^L, the maximum and minimum of the ith element of b satisfying the equation[6] $(\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k, \alpha)$, (ii) taking a uniform grid of values of b such that the ith element is between b_i^U and b_i^L and (iii) computing the volume as the fraction of points in the grid that satisfy the equation $(\phi + Gb)'(\phi + Gb) \leq F_{\chi^2}(k, \alpha)$ multiplied by $\Pi_{i=1}^n(b_i^U - b_i^L)$. For a given ϕ, the fraction of bs such that $(\phi+Gb)'(\phi+Gb) \leq F_{\chi^2}(k, \alpha)$ that also satisfy the condition $b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \leq F_{\chi^2}(n, \alpha)$ can be calculated from this grid as well. The asymptotic distributions of L_1 and L_2 can thus be simulated by taking random draws of ϕ and conducting these calculations for each ϕ.

The proposed test works for any coverage rate of the Wald and S-sets, α. For all numerical work in this paper, I set $\alpha = 0.95$.

The proposed test is similar in spirit to a Hausman specification test. Any test that compares two estimators one of which is consistent under the null and alternative

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[6]Concretely, in the case $n = 2$, b_1^U and b_1^L may be computed as the solutions to the quadratic equation $[(G_1'G_2)^2 - G_1'G_1G_2'G_2]\tilde{x}^2 + [2\phi'G_1G_1'G_2 - 2G_1'G_1\phi'G_2]\tilde{x} + (\phi'G_1)^2 + G_1'G_1F_{\chi^2}(k, \alpha) - G_1'G_1\phi'\phi = 0$ where G is partitioned as $[G_1\ G_2]$.

hypotheses and the other of which is consistent under the null hypothesis alone is a Hausman specification test (Hausman (1978)). Consistent estimation is however impossible in a model that is not identified[7]. The test that I am proposing instead compares the volume of two confidence sets, which are of the same order under the null, but not under the alternative.

4. Monte-Carlo Results.

4.1 *The Linear IV Model with a Single Included Endogenous Regressor.*

In my first set of Monte-Carlo results, I focus on the linear IV model with $n = 1$. The experimental design follows Hahn and Hausman (2002). I normalize β to zero. The instruments are k independent standard normal random variables. The errors w_t are Gaussian with $\Omega = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ - the parameter ρ governs the endogeneity of x_i. I set $\Pi = (\phi, ...\phi)$. The population R^2 in the first-stage regression is $\tilde{R}_f^2 = k\phi^2/(k\phi^2 + 1)$, so $\phi = \sqrt{\tilde{R}_f^2/(k(1 - \tilde{R}_f^2))}$. I use the following parameter combinations:

T=100, 250, 1000

ρ=-0.9, -0.5, 0.5, 0.9

k=5, 10, 30

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[7]This does not necessarily prevent one from constructing a test based on the difference between two estimators neither of which is consistent under the underidentified alternative. The test of Hahn and Hausman (2002) is such a test. A test based on the difference between the continuous-updating and two-step GMM estimators would be too.

Results are reported for $\tilde{R}_f^2$=0, 0.01, 0.1, 0.3 and 0.5 in Tables 1-5, respectively. In each experiment, I do 1,000 replications. I calculate (i) the coverage of the usual Wald confidence interval (which is 1 minus the size of the TSLS t-test testing the hypothesis that $\beta = 0$), (ii) the coverage of the Anderson-Rubin confidence set, (iii) the rejection rate of the proposed tests L_1 and L_2, (iv) the coverage of the confidence set that is the Anderson-Rubin confidence set if L_1 rejects and the Wald otherwise, (v) the coverage of the confidence set that is the Anderson-Rubin confidence set if L_2 rejects and the Wald otherwise, (vi) the acceptance rate of the first-stage F-test, (vii) the coverage of the confidence set that is the Wald confidence set if the first-stage F-test rejects and the Anderson-Rubin otherwise, (viii) the acceptance rate of the test comparing the first-stage F-test with the weak identification asymptotic critical values of Stock and Yogo (2001)[8] and (ix) the coverage of the confidence set that is the Wald confidence set if the first-stage F-test rejects using these latter critical values and the Anderson-Rubin otherwise. All confidence intervals have 95% nominal coverage and all tests have 5% nominal size. I report the acceptance rate for the first-stage F-test but the rejection rate for my proposed test. This gives some comparability, since the first-stage F-test tests a null hypothesis of underidentification while the proposed test tests a null hypothesis of identification. However, these tests are conceptually quite

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[8]Specifically these are the critical values to ensure that the effective size of a TSLS Wald test is no greater than 25% under the weak identification asymptotics of Staiger and Stock (1997).

distinct.

The model is formally identified in all the experiments except those for which $\tilde{R}_f^2 = 0$. But we would want the tests L_1 and L_2 to reject if the identification is so weak that the TSLS t-statistic exhibits severe size distortions. The potential practical usefulness of the proposed test is as a pretest as in (iv) and (v). The hope, to be evaluated in these experiments, is that the effective coverage of the confidence set that is robust if the test rejects and non-robust otherwise will generally be reasonably close to the nominal level. The researcher is however better off with this strategy than simply using the robust confidence set always in the sense that the researcher will sometimes be allowed to use point estimates and standard errors, which may be preferable for reasons discussed in the introduction.

The effective coverage rate of the Wald confidence interval can be far below the nominal level when $\rho > 0$, as is well known. In extreme cases, its simulated effective coverage is under 1 percent. The Anderson-Rubin confidence set effectively circumvents this problem. The proposed tests have low rejection rates when conventional asymptotic theory works well, but high rejection rates when it works poorly.

The rejection rate of the proposed tests is consistently above 90% in the case $\tilde{R}_f^2 = 0.01$. The test of Hahn and Hausman (2002) has rejection rates of around 10% in many of these simulations (their Table 3). While it is true that the model is formally identified with $\tilde{R}_f^2 = 0.01$, the model cannot be said to be well identified

with such a low theoretical first-stage R-squared in a sample size of 1,000 or less. Since I am thinking of the test as testing for the adequacy of conventional asymptotic theory, I believe that it is a useful feature of the proposed test that it rejects in such cases.

The coverage rates of the confidence sets that condition on the results of the pretest proposed in this paper are never below 72% (for L_1) and never below 76% (for L_2) and are typically higher than this. The coverage rate of the confidence set that conditions on the result of the first-stage F-test can be as low as 16%. It seems intuitive that the test proposed in this paper works relatively well as a pretest because it is based on the direct comparison of robust and non-robust confidence sets and only allows the researcher to use the non-robust approach when this gives results that are close to the robust approach.

The coverage rates of the confidence set that conditions on the comparison of the first-stage F-statistic with the critical values of Stock and Yogo (2001) is never below 79%[9]. There are however several cases in which the test of a null of identification rejects about 60% of the time (allowing the researcher to use conventional point estimates and standard errors), while the test that compares the first-stage F-statistic

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[9]This is not surprising, since these critical values are designed to ensure that the effective coverage of the TSLS Wald confidence set is bounded below by 75% under weak instrument asymptotics. Stock and Yogo provide critical values for different maximal biases or size distortions. I picked the critical values that bound the effective size at 25% because the effective coverage rates of the confidence set for β obtained with them are comparable with the effective coverage rates of the confidence sets that condition on the results of the test for identification proposed in this paper.

with the critical values of Stock and Yogo does not reject the null of underidentification

in any simulations.

Of course, the researcher who never wants effective coverage to be appreciably

different from the nominal level should just always use the robust confidence sets.

4.2 *The Consumption CAPM with CRRA Preferences.*

An important feature of the proposed test is that it is applicable in all GMM models,

not just in the linear IV model, unlike the first-stage F-test or the test of a null of

identification proposed by Hahn and Hausman (2002). My second set of Monte-Carlo

results evaluate the proposed test in the context of the consumption CAPM Euler

equation with CRRA preferences.

To simulate data from the consumption CAPM, I follow the approach of Tauchen

and Hussey (1991) (also used in Tauchen (1986), Kocherlakota (1990), Hansen, Heaton

and Yaron (1996) and Stock and Wright (2000)). This involves fitting a 16-state

Markov chain to consumption and stock-market dividend growth (the state variables)

calibrated so as to approximate the first-order VAR:

$$
\begin{pmatrix} \log(\frac{C_t}{C_{t-1}}) \\ \log(\frac{D_t}{D_{t-1}}) \end{pmatrix} = \mu + \Phi \begin{pmatrix} \log(\frac{C_{t-1}}{C_{t-2}}) \\ \log(\frac{D_{t-1}}{D_{t-2}}) \end{pmatrix} + \begin{pmatrix} u_{ct} \\ u_{dt} \end{pmatrix}
$$

where D_t is the stock-market dividend at date t and $(u_{ct}, u_{dt})'$ is iid normal with mean

zero and variance Λ. I use the same values of μ, Φ and Λ as Kocherlakota (1990), who

chose these by fitting a bivariate VAR(1) to historical US annual real dividend growth

and real consumption growth data[10]. I set the discount factor, δ, to 0.97 and the coefficient of relative risk aversion, γ, to 1.3. Taking random draws of consumption growth and dividend growth from this Markov chain, numerical quadrature is then used to calculate the prices of a stock and a riskfree asset in each period implied by the consumption CAPM with intertemporally separable CRRA preferences. In this way, time series of consumption growth, stock returns and bond returns may be simulated[11]. I then consider GMM estimation of the parameters δ and γ, using both stock and bond returns as the elements of R_{t+1}, using as instruments either instrument set A: a constant, one lag of stock and bond returns and one lag of consumption growth or instrument set B: a constant and one lag of consumption growth[12]. These instrument sets were used by Hansen, Heaton and Yaron (1996). I use sample sizes of 100 (mirroring the available sample sizes for U.S. annual data) and 250 and 1,000 (to see the effects of hypothetical larger sample sizes).

For these simulations, I report the coverage rate of a Wald confidence set and an S-set for $\theta = (\delta, \gamma)'$ in Table 6. I also report the rejection rate of the proposed tests L_1 and L_2. Lastly, I report the coverage rate of the confidence set for θ that

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[10]These are $\mu = \begin{pmatrix} 0.021 \\ 0.004 \end{pmatrix}, \Phi = \begin{pmatrix} -0.161 & 0.017 \\ 0.414 & 0.117 \end{pmatrix}$ and $\Lambda = \begin{pmatrix} 0.0012 & 0.00177 \\ 0.00177 & 0.014 \end{pmatrix}$.

[11]I am grateful to George Tauchen for his Gauss code for implementing this.

[12]In GMM estimation of the consumption CAPM, I set $V_T(\theta) = T^{-1}\Sigma_{t=1}^{T}\phi(Y_t, \theta)\phi(Y_t, \theta)'$ in this paper (heteroskedasticity-robust).

is the S-set[13] if L_1 or L_2 rejects and the Wald set otherwise. In a sample size of 100, the effective coverage rates of the Wald confidence set are far below the nominal level for both instrument sets (about 40%), mirroring the inadequacy of conventional asymptotic approximations in this case, as documented by Hansen, Heaton and Yaron (1996). The effective coverage rate rises with the sample size. The proposed tests have rejection rates over 90% in the sample size of 100, but have lower rejection rates in the larger sample sizes. The rejection rate for L_2 can fall to under 10% in the sample size of 1000 when the conventional asymptotic approximation is not too bad. The confidence set that is the S-set if L_1 or L_2 rejects and the Wald set otherwise yields coverage of above 80% in all cases.

5. Empirical Applications.

5.1 *Financial Intermediation and Growth using Legal Origin Dummies as Instruments.* One of the applications of instrumental variables methods that has received considerable attention recently has been the regression of growth rates on measures of financial development. There is a clear problem of endogeneity in this regression.

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[13]Numerical computation of the S-set involves evaluating the continuous updating objective function at each point in a grid. The bounds of the grid do not affect the coverage of the S-set provided that the true parameter value is within these bounds. Provided that the S-set includes the entire Wald set, setting bounds that are too small can only lower the rejection rate (and thus the power) of the proposed tests L_1 and L_2. In numerical evaluation of S-sets, I take the wider of two possible sets of bounds: (i) a bound for δ between -0.5 and 1.5 and γ between -5 and 50, and (ii) the two-step estimate +/- 30 standard errors.

Recognizing this, authors such as Levine, Loayza and Beck (2000) have used legal origin dummies as instruments. The legal system in each country can usually be unambiguously traced to one of four origins: English Common Law, French Napoleonic Law, German law, or Scandanavian law. It is hoped that the legal origin dummies are correlated with financial development, but do not affect growth rates other than through financial development.

Following Levine, Loayza and Beck (2000), I ran a cross-country regression of real GDP per capita growth rates over the years 1960-1995 on an index of financial development and on three other sets of control variables, treating financial development as endogenous and the controls as included exogenous variables. Financial development was measured by the log of the total value of credits issued by financial intermediaries to the private sector, divided by GDP (the measure of financial intermediation preferred by Levine, Loayza and Beck). Following Levine, Loayza and Beck, the three sets of control variables are (i) the simple conditioning information set: schooling and 1960 real GDP per capita, in logs, (ii) the policy conditioning information set: the simple information set plus the government share in GDP, the trade share in GDP, inflation and the black market premium, and (iii) the full information set: the policy information set plus indicators of revolutions, coups and ethnic fractionalization. I used TSLS, with English, French and German legal ori-

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gin dummies as instruments[14]. The first-stage F-statistic for this regression is 5.63 with the simple conditioning information set, 5.43 with the policy conditioning information set and 5.80 with the full conditioning information set. Meanwhile, the 1% critical value in the test that compares this to a χ^2_k/k distribution is 3.78, so this test clearly rejects in all three cases. The values of the test statistics L_1 and L_2 and the 5% critical values are reported in Table 7. For all three sets of controls and for both test statistics, the null of identification is rejected. I conclude that even if these instruments are uncorrelated with the error term in the structural equation, their correlation with the endogenous regressor is not high enough to allow a researcher to conduct inference in the conventional way. For the simple conditioning information set, the regular Wald confidence set for the effect of financial innovation on growth is 2.34±1.20. The Anderson-Rubin confidence set for this parameter goes from 0.51 to 5.77, which, though quite different, is still an interval containing only strictly positive values. Results with the other sets of controls are similar.

5.2 *The Consumption CAPM with CRRA Preferences.*

In this application, I use US annual data from Campbell and Shiller (1987), updated to cover the years 1889-1999. The data consist of stock returns, bond returns and consumption growth, all in real terms, as described by Campbell and Shiller. I

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[14]I am grateful to Ross Levine for providing me with these data. A more detailed description of the data can be found in Levine, Loayza and Beck.

then consider GMM estimation of the parameters δ and γ, using both stock and bond returns as the elements of R_{t+1}, using instrument sets A and B (as defined the Monte-Carlo simulation above). The values of the test statistics L_1 and L_2 and the 5% critical values are reported in Table 8. For both instrument sets and for both test statistics, the null of identification is rejected. The two-step estimator indicates precisely identified parameters with small risk aversion but the robust S-sets are large and indicate high risk aversion, a pattern that was found by Stock and Wright (2000) with an earlier dataset. Indeed, with either set of instruments, the robust S-set and Wald set are completely disjoint and so the test statistic $L_2 = 0$. This means that the two-step point estimate is not even included in the robust S-set, which inverts the acceptance region of the continuous-updating objective function.

6. Conclusion.

In this paper, I have proposed a test of the null hypothesis of identification, that allows for the detection of a local or global underidentification, and of underidentification in some or all directions. It applies in any GMM model with more moment conditions than parameters. The test is conceptually simple, working by comparing the volume of confidence sets that are robust to underidentification with the volume of the non-robust Wald confidence set. When the test rejects, inference should be conducted only by methods that are robust to underidentification.

Appendix: Proof of Theorem 1.

Let $b = T^{1/2}(\theta - \theta_0)$ where θ_0 denotes the true parameter value. The continuous-updating GMM objective function can be written as

$$S_{CU}(\theta) = [T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta)]' V_T(\theta)^{-1}[T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta)] =$$

$$[T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta_0) + T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \theta^*)}{d\theta} b]' V_T(\theta)^{-1}[T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta_0) + T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \theta^*)}{d\theta} b]$$

where θ^* is on the line segment between θ and θ_0. So

$$S_{CU}(\theta) = [V_T(\theta)^{-1/2'} T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta_0) +$$

$$V_T(\theta)^{-1/2'} T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \theta^*)}{d\theta} b]' [V_T(\theta)^{-1/2} T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta_0) +$$

$$V_T(\theta)^{-1/2} T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \theta^*)}{d\theta} b]$$

which converges to $(\phi + Gb)'(\phi + Gb)$. But $V_1 = T^{-1/2}V(b : S_{CU}(\theta_0 + T^{-1/2}b) \le F_{\chi^2}(k, \alpha))$, so $T^{1/2}V_1 \to_d V(b : (\phi + Gb)'(\phi + Gb) \le F_{\chi^2}(k, \alpha))$. From the first order condition for the two-step estimator,

$$[T^{-1/2}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \hat\theta_{TS})}{d\theta}]' V_T(\hat\theta_{OS})^{-1}[T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \hat\theta_{TS})] = 0$$

$$[T^{-1/2}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \hat\theta_{TS})}{d\theta}]' V_T(\hat\theta_{OS})^{-1}[T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta_0) + T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \tilde\theta)}{d\theta} T^{1/2}(\hat\theta_{TS} - \theta_0)] = 0$$

where θ^* is on the line segment between θ and θ_0. So

$$T^{1/2}(\hat\theta_{TS} - \theta_0) =$$

$$-[T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \hat\theta_{TS})}{d\theta}' V_T(\hat\theta_{OS})^{-1} T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \tilde\theta)}{d\theta}]^{-1}[T^{-1}\Sigma_{t=1}^T \tfrac{d\phi(Y_t, \hat\theta_{TS})}{d\theta}' V_T(\hat\theta_{OS})^{-1} T^{-1/2}\Sigma_{t=1}^T \phi(Y_t, \theta_0)]$$

which converges to $-(G'G)^{-1}G'\phi$. So

$$T(\hat\theta_{TS} - \theta)' B' A^{-1} B(\hat\theta_{TS} - \theta) = [b - T^{1/2}(\hat\theta_{TS} - \theta_0)]' G'G[b - T^{1/2}(\hat\theta_{TS} - \theta_0)]$$

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which converges to $[b + (G'G)^{-1}G'\phi]'G'G[b + (G'G)^{-1}G'\phi]$ and so

$$T^{1/2}V_2 \rightarrow_d V(b : (b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \le F_{\chi^2}(n, \alpha)) = V(b :$$

$$b'G'Gb \le F_{\chi^2}(n, \alpha))$$

with the last equality because setting $(G'G)^{-1}G'\phi$ to an n-vector of zeros shifts the

location of the set $b : (b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \le F_{\chi^2}(n, \alpha)$ but leaves

its volume unchanged. In turn,

$$V(b : b'G'Gb \le F_{\chi^2}(n, \alpha)) = \frac{1}{|G'G|^{1/2}}V(\tilde{b} : \tilde{b}'\tilde{b} \le F_{\chi^2}(n, \alpha)) = \frac{1}{|G'G|^{1/2}}\frac{2F_{\chi^2}(n, \alpha)^{n/2}\pi^{n/2}}{n\Gamma(n/2)}$$

using the equation for the volume of a hypersphere. So $T^{1/2}V_2 \rightarrow_d \frac{1}{|G'G|^{1/2}}\frac{2F_{\chi^2}(n, \alpha)^{n/2}\pi^{n/2}}{n|G'G|^{1/2}\Gamma(n/2)}$

and

$$L_1 \rightarrow_d V(b : (\phi + Gb)'(\phi + Gb) \le F_{\chi^2}(k, \alpha))/\{\frac{2F_{\chi^2}(n, \alpha)^{n/2}\pi^{n/2}}{n|G'G|^{1/2}\Gamma(n/2)}\}$$

Also,

$$T^{1/2}V_3 \rightarrow_d V(b : (b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \le F_{\chi^2}(n, \alpha);$$

$$(\phi + Gb)'(\phi + Gb) \le F_{\chi^2}(k, \alpha))$$

and $1(V_1 > 0) \rightarrow_d 1[\inf_b(\phi + Gb)'(\phi + Gb) \le F_{\chi^2}(k, \alpha)] = I^*$ so that

$$L_2 \rightarrow_d I^*V(b : (b + (G'G)^{-1}G'\phi)'G'G(b + (G'G)^{-1}G'\phi) \le F_{\chi^2}(n, \alpha);$$

$$(\phi + Gb)'(\phi + Gb) \le F_{\chi^2}(k, \alpha))/V(b : (\phi + Gb)'(\phi + Gb) \le F_{\chi^2}(k, \alpha)) + 1 - I^*$$

as required. ∎

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Table 1: Monte-Carlo Results: Linear IV Model $\tilde{R}_f^2 = 0$

T	ρ	k	Coverage		Reject Rate		Accept Rate		Pretest Coverage			
			Wald	AR	L_1	L_2	F_1	F_2	L_1	L_2	F_1	F_2
100	-0.9	5	100.0	94.6	99.7	99.8	93.9	100.0	94.9	94.8	98.5	94.6
100	-0.9	10	100.0	96.5	99.9	99.9	94.5	100.0	96.6	96.6	98.6	96.5
100	-0.9	30	100.0	98.3	100.0	100.0	90.7	100.0	98.3	98.3	99.9	98.3
100	-0.5	5	100.0	94.6	99.9	99.9	94.1	100.0	94.6	94.6	95.1	94.6
100	-0.5	10	100.0	96.5	100.0	100.0	93.2	100.0	96.5	96.5	97.4	96.5
100	-0.5	30	100.0	98.3	100.0	100.0	91.3	100.0	98.3	98.3	98.5	98.3
100	0.5	5	81.7	94.6	100.0	100.0	94.6	100.0	94.6	94.6	92.3	94.6
100	0.5	10	58.7	96.5	99.9	100.0	93.9	100.0	96.4	96.5	92.8	96.5
100	0.5	30	13.8	98.3	100.0	100.0	90.6	100.0	98.3	98.3	89.4	98.3
100	0.9	5	11.3	94.6	99.7	99.8	93.9	100.0	94.6	94.6	91.8	94.6
100	0.9	10	0.8	96.5	100.0	100.0	93.0	100.0	96.5	96.5	92.4	96.5
100	0.9	30	0.0	98.3	100.0	100.0	91.4	100.0	98.3	98.3	91.0	98.3
250	-0.9	5	100.0	95.5	99.7	99.8	95.4	100.0	95.8	95.7	97.5	95.5
250	-0.9	10	100.0	95.8	99.9	99.9	95.8	100.0	95.9	95.9	97.4	95.8
250	-0.9	30	100.0	96.2	100.0	100.0	94.8	100.0	96.2	96.2	98.4	96.2
250	-0.5	5	100.0	95.5	99.8	99.9	94.3	100.0	95.6	95.6	96.3	95.5
250	-0.5	10	100.0	95.8	99.8	99.8	94.9	100.0	95.9	95.9	96.3	95.8
250	-0.5	30	100.0	96.2	100.0	100.0	93.7	100.0	96.2	96.2	97.0	96.2
250	0.5	5	81.0	95.5	99.6	99.7	93.8	100.0	95.4	95.4	93.3	95.5
250	0.5	10	57.4	95.8	99.6	99.8	94.8	100.0	95.7	95.8	92.2	95.8
250	0.5	30	12.7	96.2	99.9	99.9	92.9	100.0	96.2	96.2	90.2	96.2
250	0.9	5	12.1	95.5	99.7	99.8	94.8	100.0	95.5	95.5	92.9	95.5
250	0.9	10	1.4	95.8	99.8	99.8	93.9	100.0	95.8	95.8	92.2	95.8
250	0.9	30	0.0	96.2	100.0	100.0	93.9	100.0	96.2	96.2	92.6	96.2
1000	-0.9	5	100.0	94.8	99.9	99.9	94.8	100.0	94.9	94.9	97.9	94.8
1000	-0.9	10	100.0	94.3	99.4	99.4	94.4	100.0	94.9	94.9	97.2	94.3
1000	-0.9	30	100.0	95.9	99.9	99.9	94.7	100.0	96.0	96.0	97.9	95.9
1000	-0.5	5	100.0	94.8	99.8	99.8	95.6	100.0	94.8	94.8	95.7	94.8
1000	-0.5	10	100.0	94.3	99.4	99.4	94.7	100.0	94.6	94.6	95.4	94.3
1000	-0.5	30	100.0	95.9	99.8	99.9	93.8	100.0	96.0	96.0	96.5	95.9
1000	0.5	5	79.9	94.8	100.0	100.0	96.0	100.0	94.8	94.8	93.4	94.8
1000	0.5	10	58.4	94.3	99.5	99.5	93.7	100.0	94.3	94.3	90.9	94.3
1000	0.5	30	16.6	95.9	99.9	99.8	93.8	100.0	95.9	95.9	90.6	95.9
1000	0.9	5	10.3	94.8	100.0	100.0	95.3	100.0	94.8	94.8	93.1	94.8
1000	0.9	10	1.7	94.3	99.4	99.6	93.9	100.0	94.3	94.3	91.8	94.3
1000	0.9	30	0.0	95.9	99.7	99.7	94.0	100.0	95.9	95.9	92.6	95.9

Notes: The coverage columns give the coverage rates of Wald and AR confidence sets. The reject rate columns give the rejection rates of the proposed tests. The accept rate columns give the rejections rates of the tests based on comparing the first-stage F-statistic with χ_k^2/k critical values and the critical values of Stock and Yogo (2001), designed to ensure that the TSLS Wald test size is no larger than 25%. For these columns, I report an acceptance rate, rather than a rejection rate because the null hypothesis is that the model is *not* identified, whereas for my test the null hypothesis is that the model *is* identified. The last four columns report the effective coverage rate of the confidence set that is the Wald or AR confidence set depending on the results of each identification/underidentification test.

Table 2: Monte-Carlo Results: Linear IV Model $\tilde{R}_f^2 = 0.01$

T	ρ	k	Coverage Wald	AR	Reject Rate L_1	L_2	Accept Rate F_1	F_2	Pretest Coverage L_1	L_2	F_1	F_2
100	-0.9	5	100.0	94.6	99.7	99.8	93.9	100.0	94.9	94.8	98.5	94.6
100	-0.9	10	100.0	96.5	99.9	99.9	94.5	100.0	96.6	96.6	98.6	96.5
100	-0.9	30	100.0	98.3	100.0	100.0	90.7	100.0	98.3	98.3	99.9	98.3
100	-0.5	5	100.0	94.6	99.9	99.9	94.1	100.0	94.6	94.6	95.1	94.6
100	-0.5	10	100.0	96.5	100.0	100.0	93.2	100.0	96.5	96.5	97.4	96.5
100	-0.5	30	100.0	98.3	100.0	100.0	91.3	100.0	98.3	98.3	98.5	98.3
100	0.5	5	81.7	94.6	100.0	100.0	94.6	100.0	94.6	94.6	92.3	94.6
100	0.5	10	58.7	96.5	99.9	100.0	93.9	100.0	96.4	96.5	92.8	96.5
100	0.5	30	13.8	98.3	100.0	100.0	90.6	100.0	98.3	98.3	89.4	98.3
100	0.9	5	11.3	94.6	99.7	99.8	93.9	100.0	94.6	94.6	91.8	94.6
100	0.9	10	0.8	96.5	100.0	100.0	93.0	100.0	96.5	96.5	92.4	96.5
100	0.9	30	0.0	98.3	100.0	100.0	91.4	100.0	98.3	98.3	91.0	98.3
250	-0.9	5	100.0	95.5	99.7	99.8	95.4	100.0	95.8	95.7	97.5	95.5
250	-0.9	10	100.0	95.8	99.9	99.9	95.8	100.0	95.9	95.9	97.4	95.8
250	-0.9	30	100.0	96.2	100.0	100.0	94.8	100.0	96.2	96.2	98.4	96.2
250	-0.5	5	100.0	95.5	99.8	99.9	94.3	100.0	95.6	95.6	96.3	95.5
250	-0.5	10	100.0	95.8	99.8	99.8	94.9	100.0	95.9	95.9	96.3	95.8
250	-0.5	30	100.0	96.2	100.0	100.0	93.7	100.0	96.2	96.2	97.0	96.2
250	0.5	5	81.0	95.5	99.6	99.7	93.8	100.0	95.4	95.4	93.3	95.5
250	0.5	10	57.4	95.8	99.6	99.8	94.8	100.0	95.7	95.8	92.2	95.8
250	0.5	30	12.7	96.2	99.9	99.9	92.9	100.0	96.2	96.2	90.2	96.2
250	0.9	5	12.1	95.5	99.7	99.8	94.8	100.0	95.5	95.5	92.9	95.5
250	0.9	10	1.4	95.8	99.8	99.8	93.9	100.0	95.8	95.8	92.2	95.8
250	0.9	30	0.0	96.2	100.0	100.0	93.9	100.0	96.2	96.2	92.6	96.2
1000	-0.9	5	100.0	94.8	99.9	99.9	94.8	100.0	94.9	94.9	97.9	94.8
1000	-0.9	10	100.0	94.3	99.4	99.4	94.4	100.0	94.9	94.9	97.2	94.3
1000	-0.9	30	100.0	95.9	99.9	99.9	94.7	100.0	96.0	96.0	97.9	95.9
1000	-0.5	5	100.0	94.8	99.8	99.8	95.6	100.0	94.8	94.8	95.7	94.8
1000	-0.5	10	100.0	94.3	99.4	99.4	94.7	100.0	94.6	94.6	95.4	94.3
1000	-0.5	30	100.0	95.9	99.8	99.9	93.8	100.0	96.0	96.0	96.5	95.9
1000	0.5	5	79.9	94.8	100.0	100.0	96.0	100.0	94.8	94.8	93.4	94.8
1000	0.5	10	58.4	94.3	99.5	99.5	93.7	100.0	94.3	94.3	90.9	94.3
1000	0.5	30	16.6	95.9	99.9	99.8	93.8	100.0	95.9	95.9	90.6	95.9
1000	0.9	5	10.3	94.8	100.0	100.0	95.3	100.0	94.8	94.8	93.1	94.8
1000	0.9	10	1.7	94.3	99.4	99.6	93.9	100.0	94.3	94.3	91.8	94.3
1000	0.9	30	0.0	95.9	99.7	99.7	94.0	100.0	95.9	95.9	92.6	95.9

See notes to Table 1.

Table 3: Monte-Carlo Results: Linear IV Model $\tilde{R}_f^2 = 0.1$

T	ρ	k	Coverage Wald	AR	Reject Rate L_1	L_2	Accept Rate F_1	F_2	Pretest Coverage L_1	L_2	F_1	F_2
100	-0.9	5	100.0	94.6	99.0	99.4	88.4	100.0	95.5	95.2	98.1	94.6
100	-0.9	10	100.0	96.5	99.5	99.5	90.3	100.0	97.0	97.0	98.8	96.5
100	-0.9	30	100.0	98.3	100.0	100.0	89.0	100.0	98.3	98.3	99.8	98.3
100	-0.5	5	100.0	94.6	99.6	99.7	90.4	100.0	94.8	94.8	95.8	94.6
100	-0.5	10	100.0	96.5	99.8	99.9	90.2	100.0	96.7	96.6	97.7	96.5
100	-0.5	30	100.0	98.3	100.0	100.0	89.1	100.0	98.3	98.3	98.5	98.3
100	0.5	5	82.4	94.6	99.7	99.8	89.8	100.0	94.7	94.7	91.1	94.6
100	0.5	10	59.9	96.5	99.8	99.8	90.7	100.0	96.5	96.5	91.3	96.5
100	0.5	30	15.7	98.3	100.0	100.0	89.4	100.0	98.3	98.3	88.8	98.3
100	0.9	5	21.0	94.6	99.2	99.4	88.9	100.0	94.5	94.6	86.7	94.6
100	0.9	10	2.2	96.5	99.6	99.7	90.6	100.0	96.5	96.5	89.3	96.5
100	0.9	30	0.0	98.3	100.0	100.0	89.4	100.0	98.3	98.3	89.1	98.3
250	-0.9	5	100.0	95.5	97.4	98.0	82.0	100.0	97.7	97.5	99.3	95.5
250	-0.9	10	100.0	95.8	98.5	98.8	85.5	100.0	97.1	97.0	98.0	95.8
250	-0.9	30	100.0	96.2	99.1	99.3	90.0	100.0	97.1	96.9	99.3	96.2
250	-0.5	5	100.0	95.5	98.5	98.7	82.2	100.0	96.4	96.2	97.4	95.5
250	-0.5	10	100.0	95.8	99.3	99.3	85.4	100.0	95.9	95.9	96.7	95.8
250	-0.5	30	100.0	96.2	100.0	100.0	90.5	100.0	96.2	96.2	97.0	96.2
250	0.5	5	82.0	95.5	98.5	98.7	80.4	100.0	95.5	95.5	90.0	95.5
250	0.5	10	63.3	95.8	99.5	99.6	85.0	100.0	95.8	95.8	88.2	95.8
250	0.5	30	16.3	96.2	99.8	99.8	87.2	100.0	96.1	96.1	85.2	96.2
250	0.9	5	33.2	95.5	97.6	97.9	80.5	100.0	95.4	95.5	79.3	95.5
250	0.9	10	6.1	95.8	98.9	99.2	85.4	100.0	95.8	95.8	84.2	95.8
250	0.9	30	0.0	96.2	98.8	99.1	88.4	100.0	96.2	96.2	87.1	96.2
1000	-0.9	5	100.0	94.8	93.3	97.1	32.4	99.8	98.9	97.7	99.4	95.0
1000	-0.9	10	100.0	94.3	94.4	96.9	46.8	100.0	98.7	97.4	99.2	94.3
1000	-0.9	30	100.0	95.9	96.7	97.3	68.5	100.0	98.9	98.6	99.6	95.9
1000	-0.5	5	100.0	94.8	92.2	94.1	31.0	99.9	97.7	97.7	99.0	94.8
1000	-0.5	10	100.0	94.3	95.5	96.7	46.3	100.0	96.4	96.0	98.3	94.3
1000	-0.5	30	100.0	95.9	98.0	98.7	66.9	100.0	96.8	96.8	98.4	95.9
1000	0.5	5	86.8	94.8	93.1	94.2	31.4	99.9	94.7	94.6	87.1	94.8
1000	0.5	10	73.2	94.3	95.6	96.5	45.1	100.0	93.5	93.7	78.0	94.3
1000	0.5	30	26.5	95.9	97.7	98.4	65.2	100.0	94.9	95.5	69.3	95.9
1000	0.9	5	65.0	94.8	94.2	97.1	31.9	99.9	92.9	94.8	66.6	94.8
1000	0.9	10	26.8	94.3	95.0	96.8	44.4	100.0	93.2	94.3	47.1	94.3
1000	0.9	30	0.0	95.9	96.2	97.3	66.0	100.0	95.4	95.9	65.5	95.9

See notes to Table 1.

Table 4: Monte-Carlo Results: Linear IV Model $\tilde{R}_f^2 = 0.3$

T	ρ	k	Coverage Wald	AR	Reject Rate L_1	L_2	Accept Rate F_1	F_2	Pretest Coverage L_1	L_2	F_1	F_2
100	-0.9	5	100.0	94.6	94.2	97.9	27.1	99.1	98.2	96.7	99.1	94.9
100	-0.9	10	100.0	96.5	95.9	98.2	40.5	100.0	99.0	98.3	99.6	96.5
100	-0.9	30	100.0	98.3	98.8	99.2	59.6	100.0	99.4	99.1	100.0	98.3
100	-0.5	5	100.0	94.6	94.2	95.1	27.8	99.5	96.5	96.4	97.9	94.7
100	-0.5	10	100.0	96.5	96.3	96.8	40.2	100.0	97.9	97.9	98.7	96.5
100	-0.5	30	100.0	98.3	99.8	99.8	61.6	100.0	98.4	98.4	99.1	98.3
100	0.5	5	88.5	94.6	93.3	94.7	27.8	99.7	94.2	94.2	87.7	94.6
100	0.5	10	77.4	96.5	96.9	97.6	38.4	100.0	95.8	95.9	82.4	96.5
100	0.5	30	27.7	98.3	99.8	99.8	62.2	100.0	98.2	98.2	68.2	98.3
100	0.9	5	68.0	94.6	95.0	98.0	27.5	99.4	92.5	94.6	69.0	94.5
100	0.9	10	31.1	96.5	96.4	98.1	39.0	100.0	95.5	96.5	48.7	96.5
100	0.9	30	0.0	98.3	99.1	99.2	61.2	100.0	98.3	98.3	61.0	98.3
250	-0.9	5	100.0	95.5	86.2	95.8	0.6	82.9	99.3	99.0	100.0	98.1
250	-0.9	10	100.0	95.8	89.4	97.7	2.5	100.0	99.3	98.1	99.9	95.8
250	-0.9	30	100.0	96.2	95.3	97.6	13.6	100.0	99.5	98.6	99.9	96.2
250	-0.5	5	99.9	95.5	77.1	81.7	1.0	83.1	99.5	99.2	99.8	97.3
250	-0.5	10	99.9	95.8	84.3	87.8	2.8	100.0	99.0	98.4	99.7	95.8
250	-0.5	30	100.0	96.2	94.4	95.9	14.2	100.0	98.5	98.3	99.8	96.2
250	0.5	5	91.6	95.5	77.7	81.0	1.0	81.3	95.7	95.1	91.5	92.6
250	0.5	10	84.6	95.8	83.6	86.9	3.8	100.0	92.7	92.5	84.6	95.8
250	0.5	30	42.5	96.2	94.7	95.9	14.2	100.0	93.4	94.5	46.5	96.2
250	0.9	5	82.3	95.5	85.9	96.4	0.8	83.0	91.0	95.6	82.0	86.2
250	0.9	10	59.4	95.8	89.7	97.7	3.0	100.0	89.1	95.8	59.5	95.8
250	0.9	30	0.8	96.2	95.5	97.6	15.8	100.0	94.6	96.2	15.9	96.2
1000	-0.9	5	99.9	94.8	39.1	40.3	0.0	0.0	99.9	99.9	99.9	99.9
1000	-0.9	10	99.8	94.3	46.2	54.9	0.0	41.7	99.9	99.3	99.8	97.9
1000	-0.9	30	100.0	95.9	72.9	96.2	0.0	100.0	100.0	99.6	100.0	95.9
1000	-0.5	5	99.7	94.8	30.0	30.0	0.0	0.0	99.9	99.9	99.7	99.7
1000	-0.5	10	99.5	94.3	33.1	32.4	0.0	42.7	99.7	99.7	99.5	97.6
1000	-0.5	30	100.0	95.9	56.8	60.0	0.0	100.0	100.0	99.5	100.0	95.9
1000	0.5	5	94.9	94.8	30.6	30.4	0.0	0.0	95.6	95.6	94.9	94.9
1000	0.5	10	90.2	94.3	32.1	32.4	0.0	40.9	92.2	92.2	90.2	89.0
1000	0.5	30	73.0	95.9	56.1	59.2	0.0	100.0	82.8	82.6	73.0	95.9
1000	0.9	5	91.5	94.8	39.4	40.9	0.0	0.0	93.0	92.7	91.5	91.5
1000	0.9	10	80.8	94.3	46.4	54.0	0.0	40.8	84.9	84.2	80.8	79.2
1000	0.9	30	29.4	95.9	73.5	96.3	0.0	100.0	74.1	95.9	29.4	95.9

See notes to Table 1.

Table 5: Monte-Carlo Results: Linear IV Model $\tilde{R}_f^2 = 0.5$

			Coverage		Reject Rate		Accept Rate		Pretest Coverage			
T	ρ	k	Wald	AR	L_1	L_2	F_1	F_2	L_1	L_2	F_1	F_2
100	-0.9	5	100.0	94.6	77.9	92.3	0.1	42.8	99.0	98.6	100.0	98.0
100	-0.9	10	100.0	96.5	85.0	96.5	0.5	99.8	99.8	99.1	100.0	96.5
100	-0.9	30	100.0	98.3	96.2	98.8	3.8	100.0	99.9	99.5	100.0	98.3
100	-0.5	5	99.2	94.6	68.8	71.5	0.0	40.3	99.1	98.5	99.2	97.3
100	-0.5	10	100.0	96.5	77.4	81.4	0.7	99.6	99.7	99.3	100.0	96.6
100	-0.5	30	100.0	98.3	96.5	97.3	3.6	100.0	99.3	99.3	100.0	98.3
100	0.5	5	92.3	94.6	69.5	72.4	0.1	41.0	94.7	93.9	92.3	91.3
100	0.5	10	89.6	96.5	77.3	80.7	0.5	99.6	94.4	94.1	89.6	96.5
100	0.5	30	52.6	98.3	95.7	96.9	4.2	100.0	95.7	96.4	53.4	98.3
100	0.9	5	86.8	94.6	79.0	92.2	0.1	41.4	92.3	93.7	86.7	85.4
100	0.9	10	69.0	96.5	85.1	97.0	0.1	99.6	88.6	96.3	69.0	96.3
100	0.9	30	4.0	98.3	97.4	98.8	4.2	100.0	97.1	98.3	7.0	98.3
250	-0.9	5	99.9	95.5	45.5	48.6	0.0	0.1	99.9	99.7	99.9	99.8
250	-0.9	10	99.9	95.8	55.6	63.9	0.0	46.2	99.9	99.1	99.9	98.2
250	-0.9	30	100.0	96.2	79.6	97.3	0.0	100.0	99.9	98.8	100.0	96.2
250	-0.5	5	99.0	95.5	35.2	35.0	0.0	0.0	99.4	99.4	99.0	99.0
250	-0.5	10	99.4	95.8	42.6	42.8	0.0	48.8	99.9	99.7	99.4	98.0
250	-0.5	30	100.0	96.2	65.1	68.5	0.0	100.0	100.0	99.6	100.0	96.2
250	0.5	5	94.3	95.5	35.1	34.9	0.0	0.0	95.3	95.3	94.3	94.3
250	0.5	10	92.2	95.8	42.4	42.3	0.0	50.2	94.1	94.1	92.2	91.4
250	0.5	30	74.1	96.2	65.3	68.7	0.0	100.0	86.0	85.4	74.1	96.2
250	0.9	5	91.5	95.5	45.2	47.0	0.0	0.0	93.3	93.0	91.5	91.5
250	0.9	10	84.5	95.8	56.8	68.1	0.0	50.1	88.5	87.8	84.5	83.8
250	0.9	30	28.1	96.2	79.3	97.2	0.0	100.0	79.7	96.1	28.1	96.2
1000	-0.9	5	99.6	94.8	15.6	15.1	0.0	0.0	99.6	99.6	99.6	99.6
1000	-0.9	10	99.1	94.3	16.4	15.8	0.0	0.0	99.6	99.6	99.1	99.1
1000	-0.9	30	100.0	95.9	28.0	27.7	0.0	100.0	100.0	100.0	100.0	95.9
1000	-0.5	5	99.3	94.8	12.2	11.8	0.0	0.0	99.3	99.3	99.3	99.3
1000	-0.5	10	98.7	94.3	12.8	12.2	0.0	0.0	99.2	99.2	98.7	98.7
1000	-0.5	30	99.7	95.9	18.3	17.9	0.0	100.0	99.8	99.8	99.7	95.9
1000	0.5	5	96.1	94.8	12.1	11.8	0.0	0.0	96.3	96.3	96.1	96.1
1000	0.5	10	93.6	94.3	12.6	13.0	0.0	0.0	94.4	94.5	93.6	93.6
1000	0.5	30	88.6	95.9	18.3	17.8	0.0	100.0	89.3	89.3	88.6	95.9
1000	0.9	5	95.2	94.8	15.3	15.3	0.0	0.0	95.4	95.4	95.2	95.2
1000	0.9	10	90.6	94.3	16.5	15.5	0.0	0.0	92.1	92.0	90.6	90.6
1000	0.9	30	73.2	95.9	27.8	27.5	0.0	100.0	76.3	76.2	73.2	95.9

See notes to Table 1.

Table 6: Monte-Carlo Results: Consumption CAPM

Inst. Set	T	Coverage Wald	S	Rejection Rate L_1	L_2	Pretest Coverage L_1	L_2
A	100	38.1	91.4	93.4	94.3	89.7	90.3
A	250	50.7	92.6	85.9	86.3	85.1	89.5
A	1000	74.9	93.8	60.4	14.0	85.2	80.3
B	100	42.4	94.2	96.0	97.6	91.6	93.0
B	250	55.4	95.0	85.6	81.2	87.2	90.6
B	1000	75.8	94.2	57.2	8.8	86.0	80.4

Notes: The coverage columns give the coverage rates of Wald and S sets. The rejection rate columns give the rejection rates of the proposed tests. The pretest coverage columns report the effective coverage rate of the confidence set that is the Wald or S set depending on the result of the proposed identification test.

Table 7: Growth Regression

Controls	Test Statistic L_1	L_2	5 % Critical Values L_1	L_2
Simple	2.20	0.45	1.42	0.71
Policy	2.29	0.44	1.42	0.71
Full	2.12	0.47	1.42	0.71

Notes: This table reports the identification test statistics and associated critical values in three specifications of the cross-country growth regression using legal origin dummies as instruments, as discussed in the text.

Table 8: Consumption CAPM with Annual US Data

Inst. Set	Test Statistic L_1	L_2	5 % Critical Values L_1	L_2
A	4.65	0	2.32	0.25
B	64.15	0	1.57	0.32

Notes: This table reports the identification test statistics and associated critical values in nonlinear GMM Euler equation estimation of the consumption CAPM for stock returns and bond returns with annual US data, using two alternative sets of instruments, as discussed in the text.