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Real Interest Rates over the Long-Run

Kei-Mu Yi and Jing Zhang[1]

1. Executive Summary

Long-run averages of short-term real interest rates may provide a useful reference point to help calibrate the future path of the policy interest rate so that it provides the appropriate level of accommodation. This memo presents evidence on the long-run behavior real interest rates for 20 countries and extending back up to 60 years. This evidence is useful in discerning trends over time and across countries. The memo also presents the evolution over time of several key long-run determinants of real interest rates and assesses their influence on the observed trends.

To ascertain what these determinants could be, we review a simple conceptual model: the saving-investment framework. This framework highlights two key forces that help determine long-run and steady-state real interest rates. They are the marginal product of capital – the additional output obtained from an extra unit of capital – and the risk premium on capital. In addition, the marginal product of capital itself can be unpacked into several forces, including total factor productivity and the capital/labor ratio.

Our main findings and conclusions are as follows:

- Real interest rates across countries have not followed a single trend over the long-run, but there are three sub-periods with different trends. There is a decline from the early 1960s through the mid-1970s, then a rising trend until the late 1980s, and, a downward trend since then. Moreover, long-run averages of real interest rates across countries have converged in the past quarter-century, consistent with an increasingly financially integrated world.
- Trends in the long-run marginal product of capital in the 1960s and early 1970s are consistent with the trend in long-run average real interest rates. However, over the past three-to-four decades, the relationship between the two variables has weakened. This implies that movements in the long-run risk premium have risen in importance in understanding long-run real interest rate trends.
  - o Trends in the marginal product of capital are consistent with trends in total factor productivity growth and growth in the capital/labor ratio.
- Going forward, it is difficult to predict what will happen to long-run averages of real interest rates in the United States and abroad. But, two recent studies calculate steady-state measures of the U.S. real-interest rate; both measures point to historically low values now. In addition, growth of U.S. productivity, and of global working-age population, are projected to be lower. These forces will continue to put downward pressure on real interest rates.
- These findings are consistent with the hypothesis that monetary policy rates in the United States and other countries are more likely than prior to the Great Recession to hit the effective lower bound (ELB) in the years ahead.

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2. Real Interest Rates Over the Long-Run: Definitions and Motivation

This section defines what we mean by real interest rates over the long-run and motivates why the FOMC should care about measurements of these concepts. As in the memo "r*: Concepts, Measures and Uses", we employ two definitions of the long-run.

The first concept is the **long-run real rate**, which we define as the average short-term real interest rate measured over a long period of time. To be more precise, it is the long run average of the real interest rate on a short-term (risk-free) asset. The main rationale underlying this concept is that movements in real interest rates owing to frictions such as sticky prices and wages, as well as short-run movements in productivity, oil prices, monetary and fiscal policy, and other forces, "wash out" over long periods of time, leaving only trends in fundamentals driving the real rate over the long run. To be clear, this concept is distinct from the "real long-term rate", which is the real return on long-term bonds.

The second concept is the **steady-state real rate**, which we define as the short-term real interest rate that would prevail in the long-run once all shocks have died down. This feature – the absence of any temporary shocks – is the key rationale for this measure.

The two concepts are distinct, but they share two features – despite their long-run nature, they are time varying; moreover, they attempt to capture something that is inherently unobservable, and must be inferred through the use of statistical and/or economic methods. In the remainder of this memo, we will refer to the "long-run real rate" or the "steady-state real rate". We will refer to "r*" only in the context of one of the short-run r* measures discussed in the conceptual r* memo.

The gap between the current real federal funds rate and measures of short-run r* is informative about the level of monetary policy accommodation.[2] Hence, accurate real-time measures of short-run r* can better inform policy decisions. The memo "Estimates of Short-Run r* from DSGE models" and the memo "Monetary Policy at the Lower Bound with Imperfect Information about r*" deal with important issues on the measurement and policy implications of short-run r*.

In this context, why should policymakers care about the long-run real rate or steady-state real rate? The most important answer to this question is that optimal monetary policy involves an entire time path of real federal funds rates. Setting the optimal amount of accommodation requires estimates of the future path of short-run r*. The long-run real rate and the steady-state real rate characterize the future path of short-run r* once short and medium-run shocks die down. Hence, estimates of long-run and steady-state real rates serve as a reference point or (time-varying) anchor. In this sense, the long-run and steady-state estimates complement the estimates of short-run r*.

On a related point, policy rules such as the Taylor rule typically have an intercept term that is usually interpreted as the long-run or steady-state real federal funds rate. To understand the

[2] The "r*: Concepts, Measures and Uses" memo by Gust *et al* highlights the point that the gap between the current real federal funds rate and measures of short-run or long-run r* is informative about the monetary policy stance.

implications of these types of rules for monetary policy, estimates of the long-run or steady-state real rate are needed.

In addition, methods for computing short-run r* sometimes embed long-run assumptions (such as constant trend productivity growth). Estimates of long-run or steady-state real interest rates can suggest when it is appropriate to change such long-run assumptions.

It is important to reiterate that the long-run and steady-state real interest rates are time varying. Estimates of trends in the long-run or steady-state real rate can shed light on the probability of hitting the ELB in the long-run, as well as on the amount of accommodation available during times of low employment and/or inflation. In turn, these findings can help inform discussions about the long-run goals and framework of monetary policy.

3. **Conceptual Frameworks for Long-Run and Steady-State Real Interest Rates.**

Many of the main forces affecting the long-run or steady-state real rate can be highlighted via a simple saving-investment (or supply and demand for funds) framework from macroeconomic theory. We also briefly discuss a framework that informs two sets of evidence from recent research papers on steady-state real interest rates that we will present later.

Figure 1 shows a graph of a textbook saving and investment diagram. Desired saving is positively related to the real interest rate, and desired investment is negatively related to the real interest rate. The equilibrium occurs at the intersection of the saving and investment curves. The real interest rate that leads desired investment to equal desired saving is known as the equilibrium real interest rate.

Figure 1: Basic Saving-Investment Framework (Global economy):

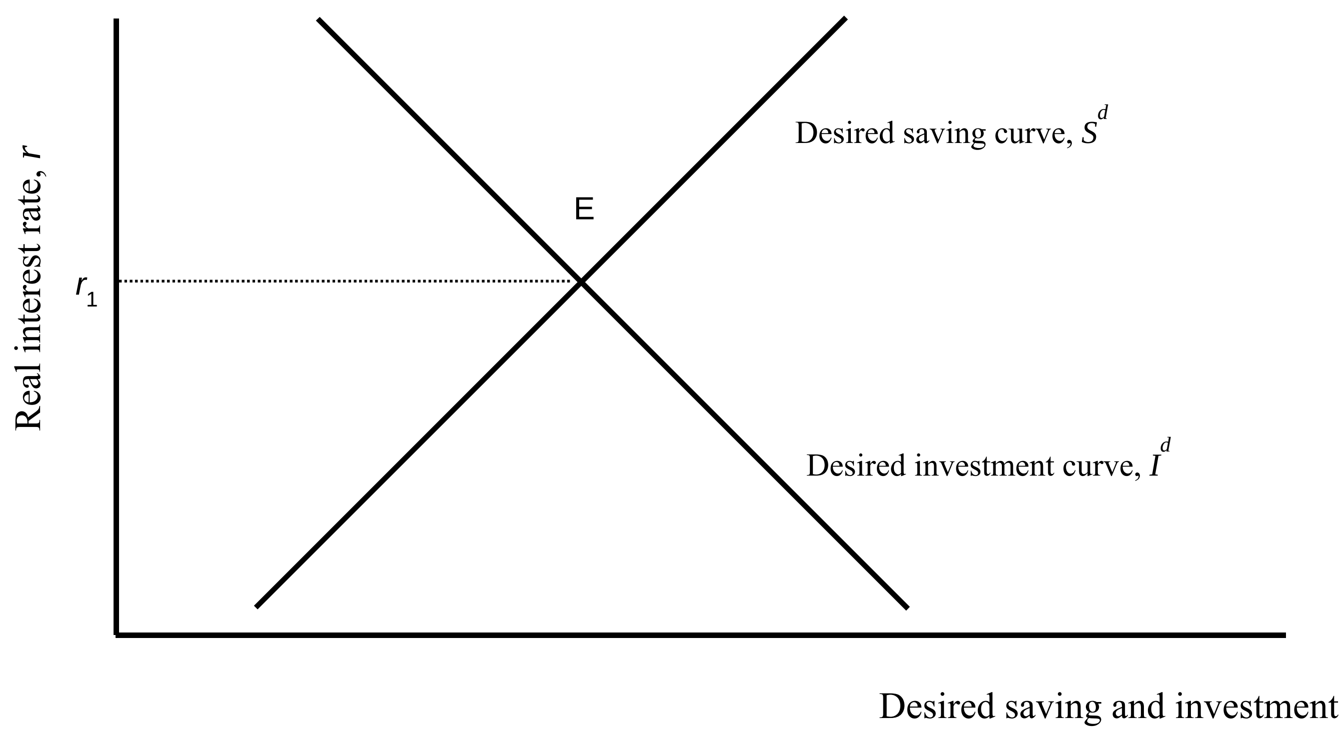


Figure 1 is relevant for a country closed to international capital flows, and it is also relevant for the global economy taken as a whole. In the global context, in the absence of economic frictions such as information asymmetry or restrictions on capital flows, there will be a single (risk-adjusted) real interest rate that clears the global market for saving and investment. Fundamental forces that change global desired saving and desired investment will shift the relevant curves, thus leading to a new equilibrium interest rate.

One example of a fundamental saving force is the famous "global saving glut" hypothesis put forth by former Fed Chairman (then Governor) Bernanke (2005). In that story, owing to the Asia financial crisis and to increased earnings by oil-producing nations, desired saving by many emerging market countries increased. In Figure 1, this would show up as a shift of the saving curve to the right – leading to lower long-run real interest rates, higher equilibrium investment and savings, and, as a by-product, increased capital inflows into countries like the United States. A second, related, example is the increased integration into the global economy of high-growth, high-saving, and financially underdeveloped economies like China. The combination of their high desired saving rate and lack of suitable domestic financial instruments for saving more than offsets the rewarding investment opportunities in these economies, thus leading to a lower equilibrium interest rate in the advanced economies.[3]

In addition to forces on the saving side, many forces can lead to shifts in the desired investment curve. Underlying this curve is the idea that firms choose capital and labor to maximize their profits. For the capital choice, a firm will increase the amount of capital it employs until the cost of one additional unit of capital just equals the expected benefit of one additional unit of capital. The cost of one additional unit of capital is the rental rate of capital, i.e., the real interest rate plus the depreciation rate on capital. The expected benefit of one additional unit of capital is the expected additional output from that unit, which we call the expected marginal product of capital (MPK). In addition, because the return to the additional unit of capital is risky, there is a risk premium that subtracts from the benefit. Putting the marginal cost and marginal benefit together yields the following relation:

$$r = E[MPK] - \delta - RP \qquad (1)$$

where δ is the depreciation rate, and RP is the risk premium on capital. Equation (1) shows that the real interest rate is tied to the expected MPK, the depreciation rate, and the risk premium. Decreases in the expected MPK, increases in the depreciation rate, and/or increases in the risk premium will be, all else equal, reflected in a decrease in the real interest rate. In a long-run context, we interpret each of the above variables in long-run terms.

Further, we show in the Appendix that, under standard assumptions, the marginal product of capital itself depends on the share of income that accrues to capital, and on total factor productivity (TFP) and the capital/labor ratio. Decreases in the first two variables, or an increase in the capital/labor ratio, will tend to decrease the marginal product of capital.

[3] Caballero *et al* (2008) and Mendoza *et al* (2009) develop formal models of this story. A third example of a global savings force is that, as a consequence of the Great Recession, there has been a persistent or even permanent increase in uncertainty, which could induce, for precautionary reasons, increased desired savings. This force would also shift the long-run desired savings curve to the right – also leading to lower long-run real interest rates.

In the standard framework that underlies most models used to study monetary and fiscal policy, such as DSGE models, the economy has a long-run "balanced growth equilibrium", in which all key macroeconomic variables (GDP, capital, consumption, etc.) grow at the same rate in the absence of shocks. The real interest rate in a balanced growth equilibrium is determined by just two forces: the long-run growth rate of TFP, and the rate of time preference of households. In an alternative framework that allows for more sophisticated treatments of demographics, the growth rate of employment or population also influences the real interest rate in the balanced growth equilibrium. These relations are presented in the Appendix.

We extend equation (1) in two ways. First, we extend it to include multiple sectors – in particular, we make a distinction between the capital goods sector and the consumption goods sector. In this context, MPK – the return to an additional unit of capital – needs to be appropriately measured in units of final consumption goods. The appropriate adjustment factor is the relative price of consumption goods to capital goods, captured below in Equation (2) by P. We call this long-run "multi-sector" MPK.

$$r = E[MPK * P] - \delta - RP \tag{2}$$

Second, we have discussed our saving-investment framework in a global setting. In such a setting, countries engage in international capital flows, and exports and imports of goods and services, with each other. In an open economy, the goods that a country produces differ from the goods a country consumes, in general. When considering investment returns, we need to convert the MPK measured in units of goods that are produced into units of goods that are consumed.[4] In terms of Equation (2) above, there is still an adjustment P, but it is now defined as the relative price of a country's output basket to the price of that country's consumption basket. We call this "open economy" MPK.

As mentioned above, we also summarize some recent research on estimates of steady-state real interest rates. This research uses extensions of the well-known Laubach and Williams (2003) framework, which starts from the fact that the steady-state real interest rate is inherently unobservable and must be estimated from observable data on GDP, interest rates, inflation, and other variables. To do such estimation, the framework includes two key features: a statistical decomposition of the key variables into long-run and cyclical components, and economic relationships such as a Phillips-curve and an equation linking the real interest rate to output growth.

The next section presents evidence on long-run and steady-state real interest rates from our own calculations and from two recent research papers on this subject, as well as on equilibrium savings and investment. The subsequent section presents evidence on the underlying variables suggested by our saving-investment framework.

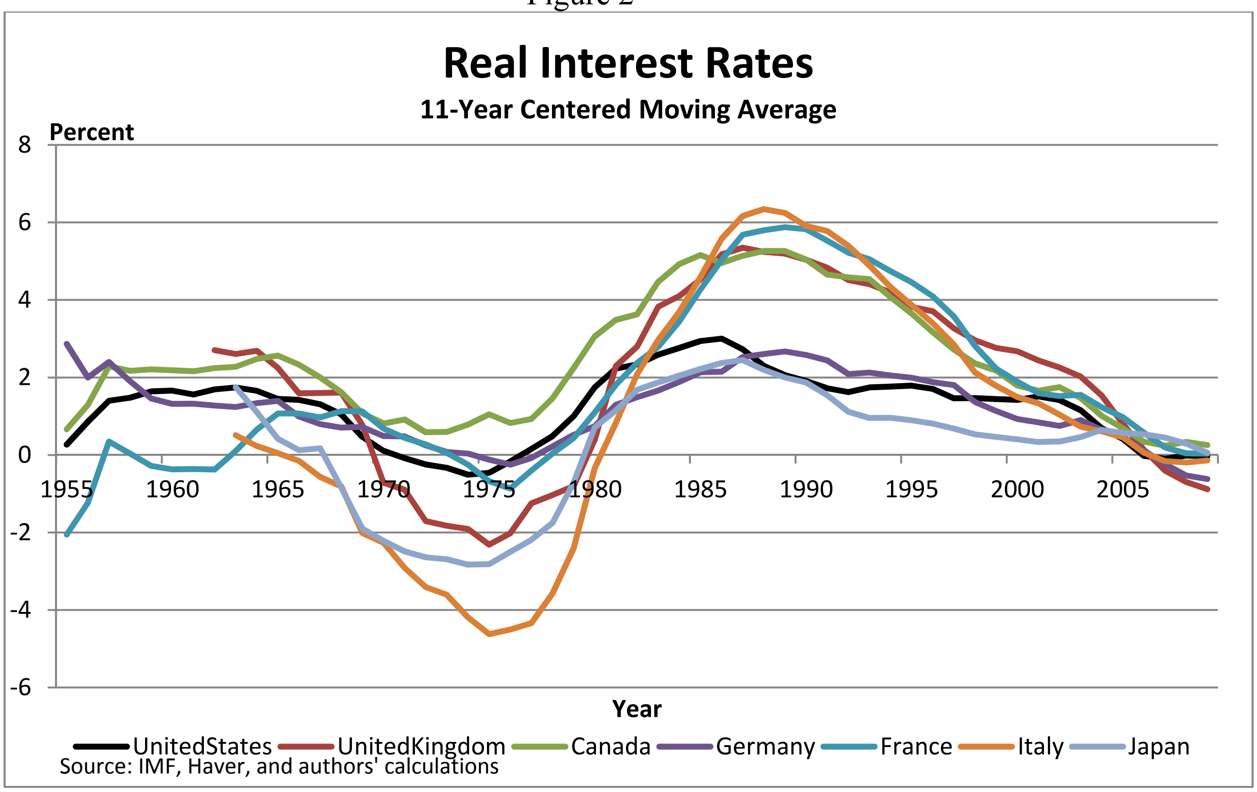
[4] A well-known arbitrage relationship links two countries' real interest rates to expected changes in their real exchange rate – the price of one country's basket of goods relative to the other country's (possibly different) basket of goods. However, it is also well-known that this relationship does not hold in the data (at least in the short-run).

4. Evidence on Long-Run and Steady-State Real Interest Rates and Equilibrium Savings and Investment

This section presents estimates of the long-run and steady-state real interest rates for up to 20 countries between 1955 and the present. The list of countries is given in the Appendix, but they include the largest economies in the world, at current exchange rates, as of 2014[5]. We broadly follow the approach in Hamilton *et al* (2015) to compute *ex ante* real interest rates. Where possible, we use the policy interest rate as our measure of the short-term interest rate, and we use the current inflation rate as our measure of the expected inflation rate to derive the short-term real interest rate. Further details are in the Appendix. To compute long-run real interest rates, we take 11-year centered moving averages.[6] Hereafter, we will refer to the 11-year centered moving average as the "long run".

Figure 2 presents long-run real interest rates for the G7 countries. Two patterns are apparent. First, G7 real interest rates are now quite close to each other, especially in recent years. Second, there have been three broad trends since the early 1960s: (a) a decline extending until the mid-1970s; (b) an increase until the late 1980s; (c) a decline since the late 1980s.[7]

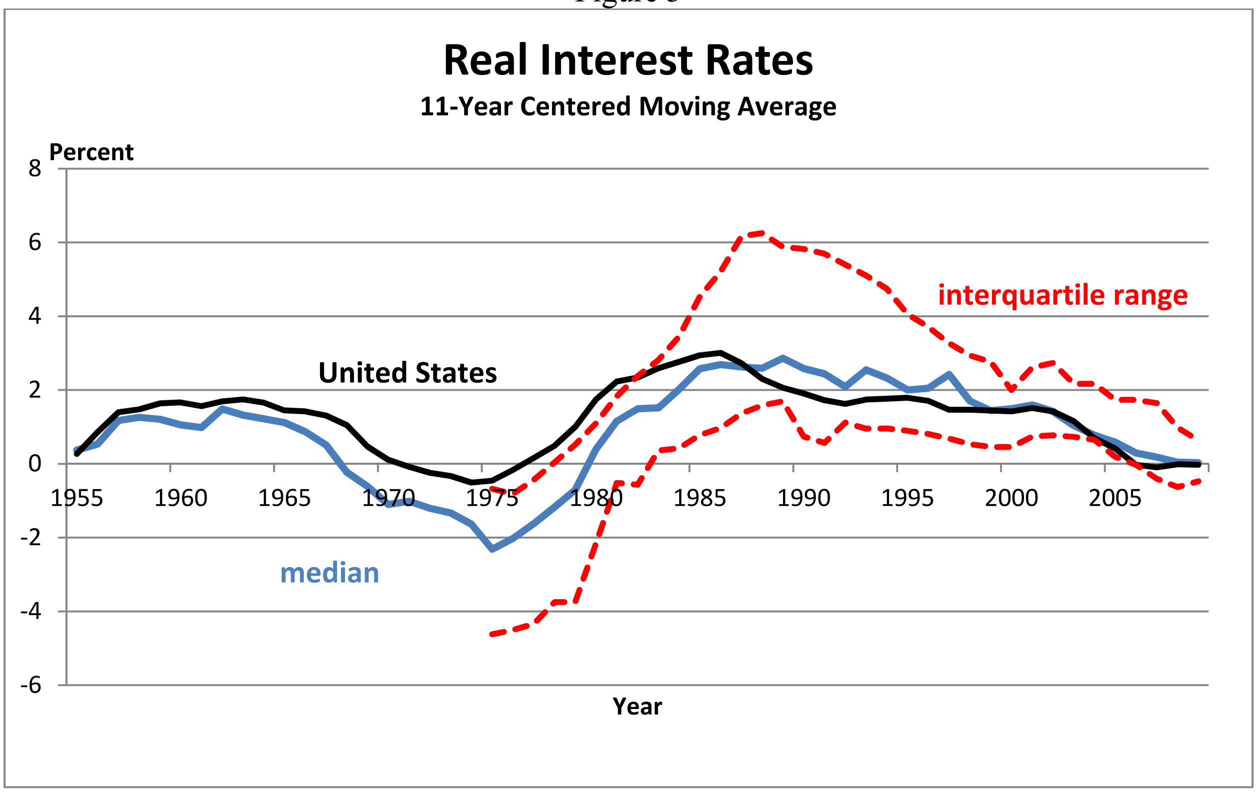
Figure 2



[5] Russia and Saudi Arabia are excluded owing to lack of a suitably long time series, leaving us with 20 of the 22 largest economies.
[6] We assume that 11 years is long enough for short and medium term shocks to die out. To the extent shocks are long-term or permanent, averages of past data may not be the best metric for future long-run real interest rates.
[7] Arellano and Perri (2014), Hamilton *et al* (2015), and Obstfeld and Tesar (2015) also document these trends.

Figure 3 shows the median of the long-run real interest rates across our full sample of countries for each year.[8] It also presents the inter-quartile range for the period 1975 to the present. The median long-run real interest rate follows the same three broad trends as the G7. In particular, the median tracks the U.S. long-run real interest rate path. The magnitude of the trend movements in the median is quite large – on the order of four percentage points from its low to its high. Finally, note the compression of the interquartile range over time. It has declined from about five percentage points to about one percentage point since the late 1980s. Real interest rates across countries have converged over time, which is consistent with the framework we discussed above in which financial integration has increased over time.

Figure 3



So far, we have focused on the long-run real interest rates in the United States and other large countries. We now turn to more sophisticated evidence on steady-state real interest rates for the United States based on extensions of the Laubach-Williams (2003, LW+) framework. As a reminder, the steady-state real rate is the short-term real interest rate that the economy will reach in the very long-run in the absence of any shocks. Two recent papers, by Johannsen and Mertens (2015) and Kiley (2015) apply LW+ frameworks to estimate steady-state real interest rates.

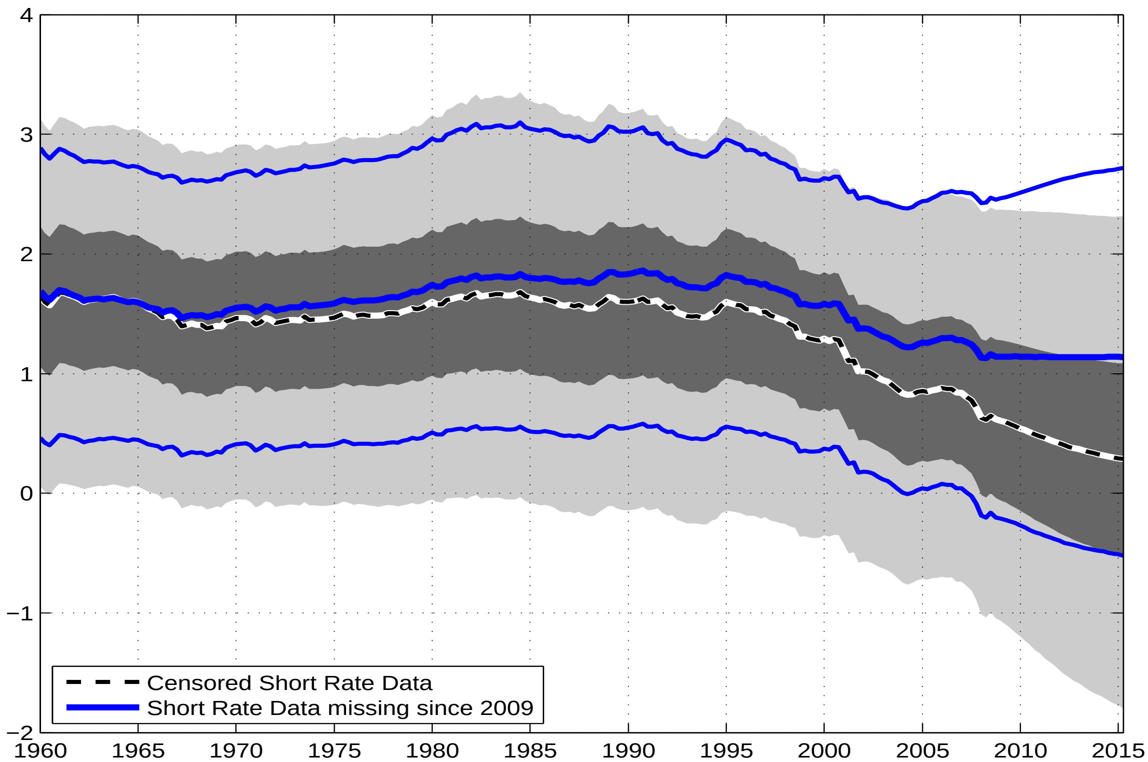
The key point of departure of Johannsen and Mertens (2015) is to explicitly take into account the ELB on interest rates and estimate a measure of the "shadow" interest rate, the interest rate that would prevail in the absence of the ELB. In particular, this shadow interest rate will

[8] The number of countries is not constant in each year. This applies to all subsequent charts with medians and interquartile ranges.

typically be negative when the economy is at the ELB. By contrast, Kiley (2015) allows for additional demand-type variables, such as measures of credit conditions, to help extract the steady-state real interest rate and the short-run real interest rate.

Figure 4 presents an update of the Johannsen-Mertens estimated steady-state real interest rate for the United States from 1960 to the present along with the 50 percent and 90 percent confidence sets.[9] The figure shows that the steady-state real interest rate was relatively flat from the 1960s to about 1990 and has fallen by almost one percentage point since then. The current estimate is about ½ percent. The recent data appear to be informative about the extent of any decline in the steady-state real rate; when the interest rate observations at the effective lower bound are treated as missing the estimated steady-state real rate is about 75bp higher. In addition, the figure suggests that there is considerably uncertainty about the level of the steady-state real interest rate. Kiley (2015) also finds a decline over time in the long-run real interest rate, but the decline is more gradual, from about 2 percent in the 1960s to about 1-1/4 percent today. The confidence set for Kiley's estimates are considerably narrower than in Johannsen and Mertens.

Figure 4
U.S. Steady-state real interest rate (from Johannsen and Mertens (2015b))



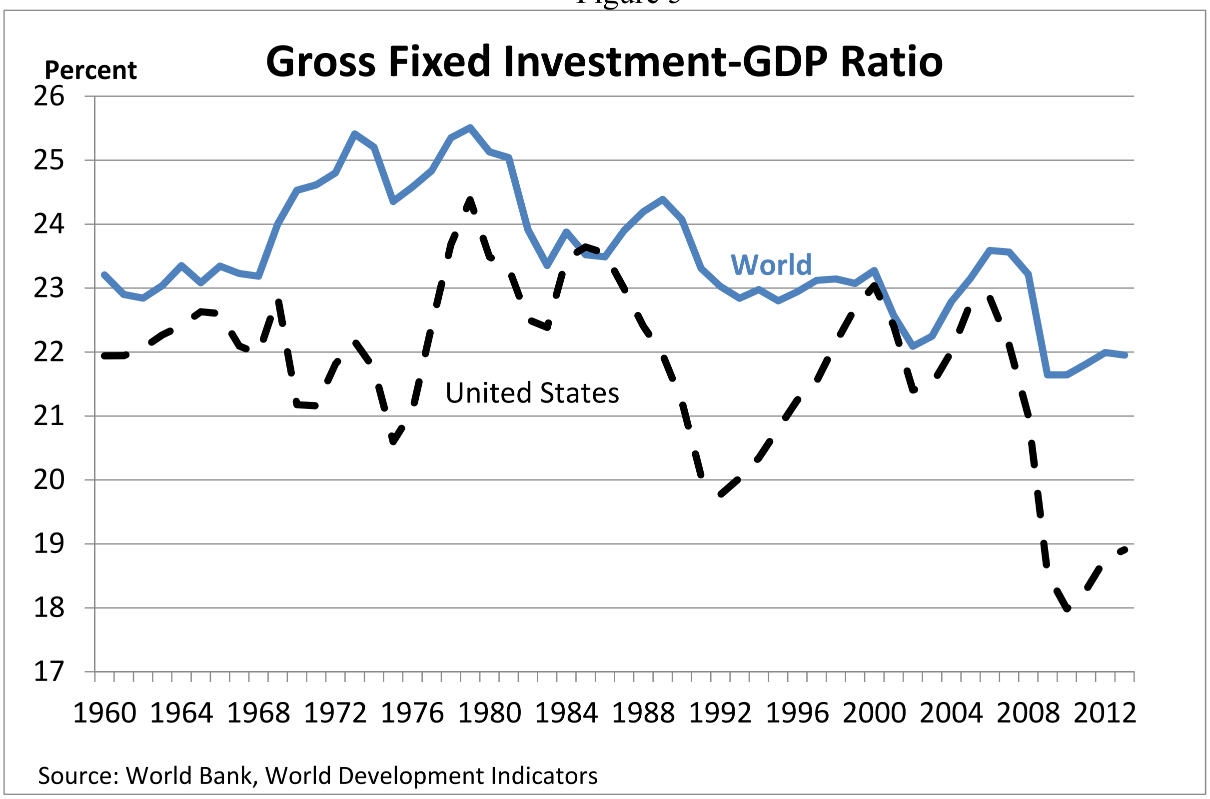
Our calculations of long-run (average) real interest rates employ a much simpler and starkly different methodology from Johannsen and Mertens (2015) and Kiley (2015), and the patterns of the estimated measures are quite different. Our U.S. measure shows more fluctuations,

[9] See Johannsen and Mertens (2015b)

especially in the 1970s and 1980s, than their measures, which could be connected to the rise and decline of high inflation in the United States.[10] That said, our simple measures and their sophisticated measures share a common feature: U.S. long-run or steady-state real interest rates are lower today than they were 10, 20, and 25 years ago.

We now turn from measures of interest rates to the other key part of our saving-investment framework, which is the equilibrium amount of saving and investment. Figure 5 shows the global gross fixed investment-GDP ratio from 1960 to the present.[11] For comparison, the United States ratio is included, too. The figure shows a broad increase in the global fixed investment-GDP ratio until about the late-1970s, and then a fairly steady decline since then. One exception to the trend is the early 2000s, during which the global fixed investment-GDP ratio increased, which is consistent with the global saving glut hypothesis. The United States has a similar pattern, although the recent decline is more pronounced. The overall declining trend since the early-1980s, coupled with the declining trend in long-run interest rates since the late-1980s,

Figure 5



[10] There are at least two possible scenarios involving connections to inflation. One scenario involves inflation expectations. If inflation expectations are more backwards-looking than our random walk assumption, then, in periods of rising inflation, our measure could over-estimate inflation expectations, and hence, under-estimate the real interest rate, and similarly, in periods of declining inflation, we would over-estimate the real interest rate. A second scenario involves the effects of prolonged periods of real rates that deviate from r* on inflation. For example, real interest rates were "too low", this could have caused high inflation. In both scenarios, the long-run average real rate we estimate would not be an accurate measure of the true long-run real interest rate.

[11] Gross fixed investment includes private and government fixed investment. The global ratio is constructed by calculating a weighted average of each country's (nominal) fixed investment-GDP ratio, where the weights are based on nominal GDP using current market exchange rates.

suggests the importance of a downward shift in the global investment demand curve. Next, we look for evidence on the sources of this downward shift.

5. Evidence on Fundamental Forces Underlying Long-run Real Interest Rates

The downward trend in global fixed investment as a share of GDP suggests that in investigating the underlying forces driving long-run real interest rates, we should focus on forces underlying investment demand. Equation 1 from our framework indicates that the expected marginal product of capital (MPK) and the depreciation rate on capital should move closely with the long-run real interest rate. We follow the approach of Caselli and Feyrer (2007) to compute MPK. Our data come primarily from the Penn World Tables, version 8.0, and from recent research by Monge *et al* (2015).[12] We will also look at the components of MPK.

We assume that over the long-run expected MPK equals actual MPK; we also assume that an 11-year horizon, as we assumed for real interest rates, is sufficient for the long-run. Figure 6a presents the long-run MPK for the United States, the median across our countries, and the interquartile range. The pattern for the median MPK is very clear: it dropped sharply by about five percentage points between the mid-1960s and the mid-1980s. Since then, it has remained at a level of about 11 percent. The observed decline, as well as the decline in TFP growth that we show later, is consistent with the textbook growth model in which diminishing returns to capital accumulation eventually set in. This story fits a number of countries, especially those that went through a period of rapid growth – owing to recovery from World War II or integration into the global economy – in the 1950s, 1960s, and 1970s. This decline does not closely track the trends in the long-run real interest rate presented in figures 2 and 3. Most of the interest rate decline was from the late 1980s forwards. The interquartile range has diminished over time; as with real interest rates, there has been convergence in long-run MPKs. However, the convergence in long-run MPKs is less than that of long-run real interest rates, which is not surprising given that it is easier to arbitrage away return differences with financial assets than with physical assets. Finally, U.S. long-run MPK shows a relatively small decline until the late 1970s, and a relatively small increase until the early 1990s, and has been flat since then. The first two sub-periods are consistent with long-run U.S. real interest rate movements.[13]

Figure 6b adjusts our primary long-run MPK measure to control for variation across countries in the relative price of consumption to capital goods, as discussed above in Equation (2). The figure shows a broadly similar pattern to the baseline measure in Figure 6a, but there is an increase in the median MPK of about 2 percentage points from the late 1980s to the present. Figure 6c adjusts our primary long-run MPK measure for an open economy setting, also as discussed above. Again the pattern is broadly consistent with that in Figure 6a.

[12] We thank Alex Monge and Juan Sánchez for kindly providing data on the natural resource rent share of GDP, enabling us to compute the capital income share of reproducible capital. Further details on the construction of MPK are in the Appendix.
[13] This is consistent with Gomme *et al* (2011).

Figure 6a

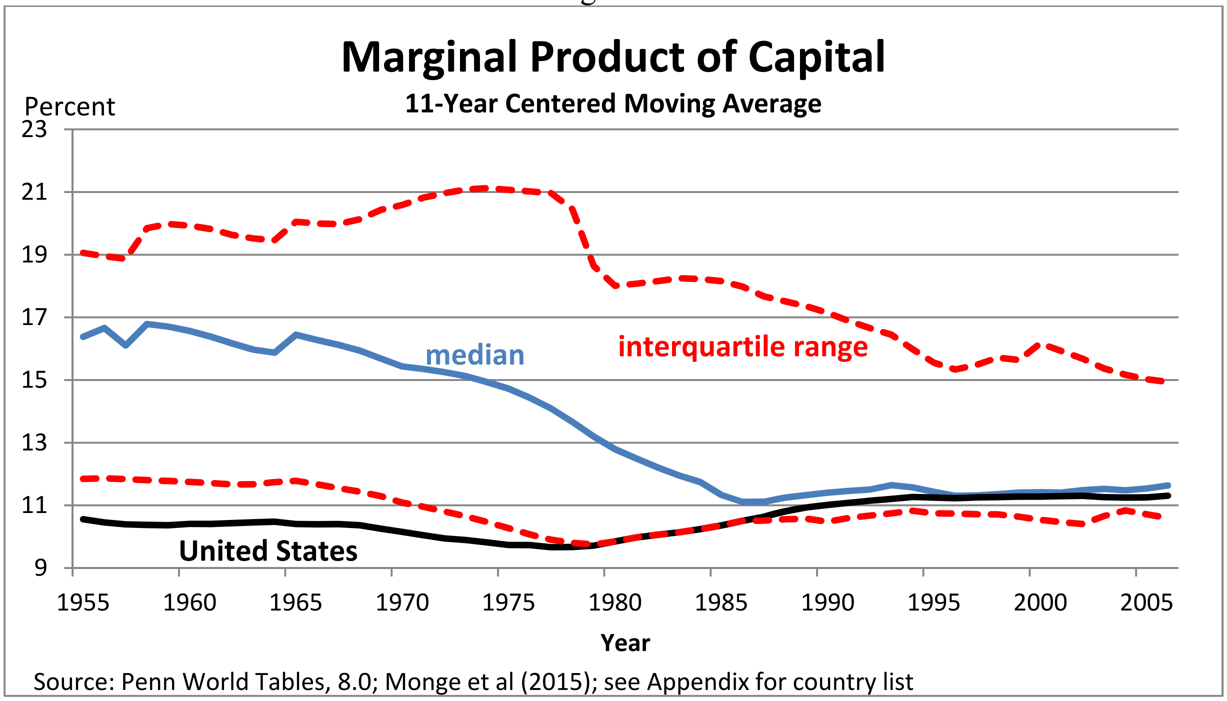


Figure 6b

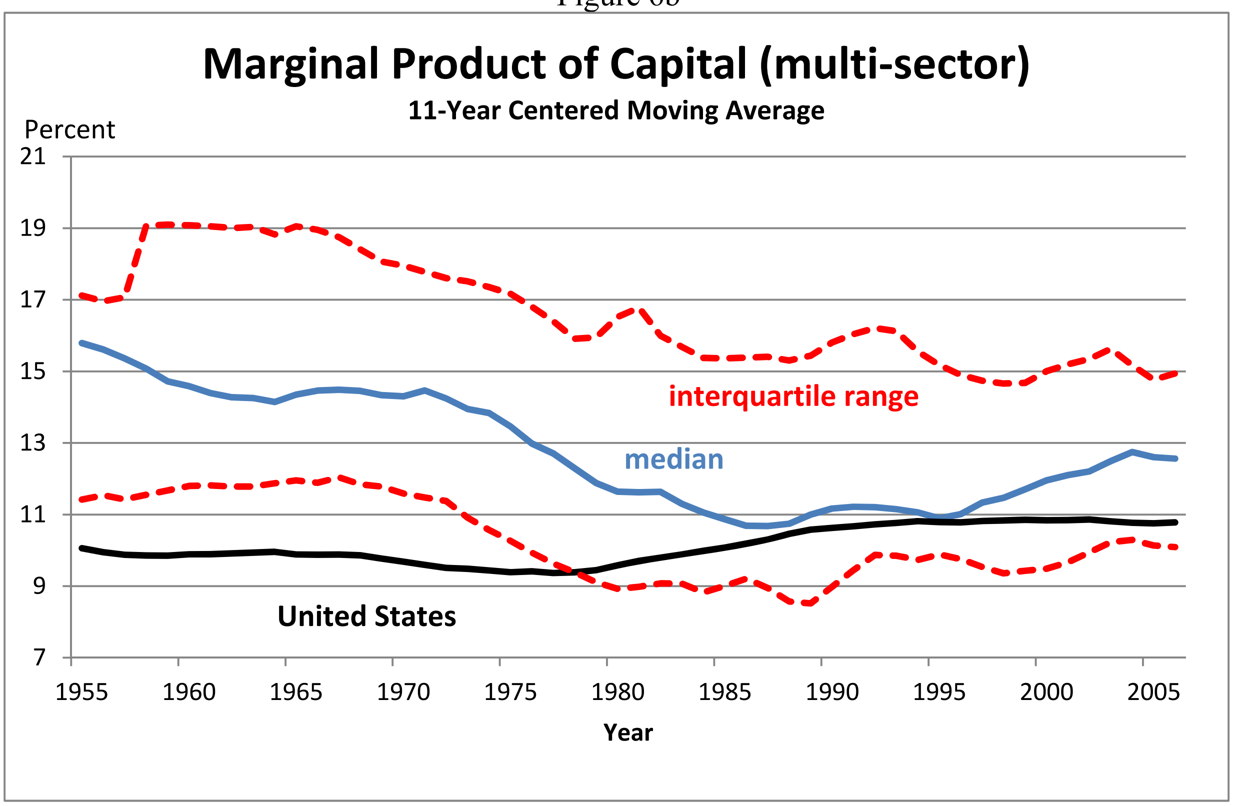


Figure 6c

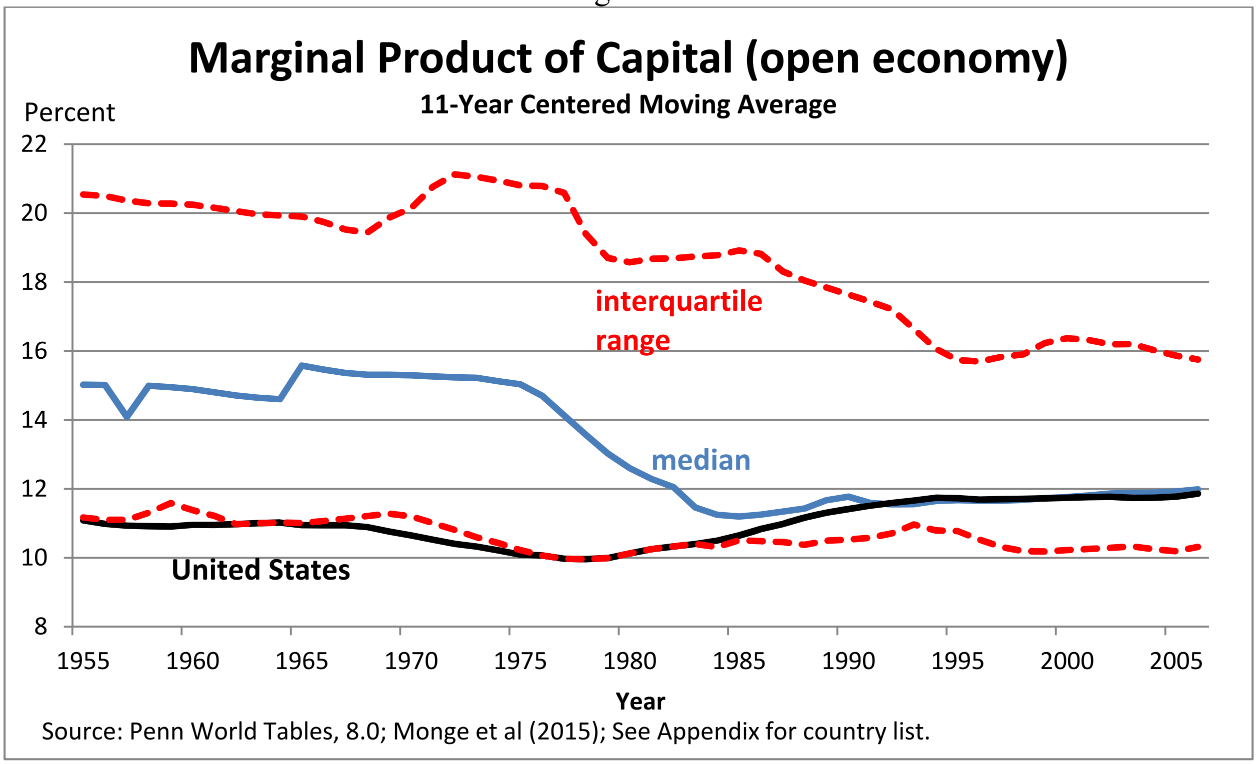


Figure 7

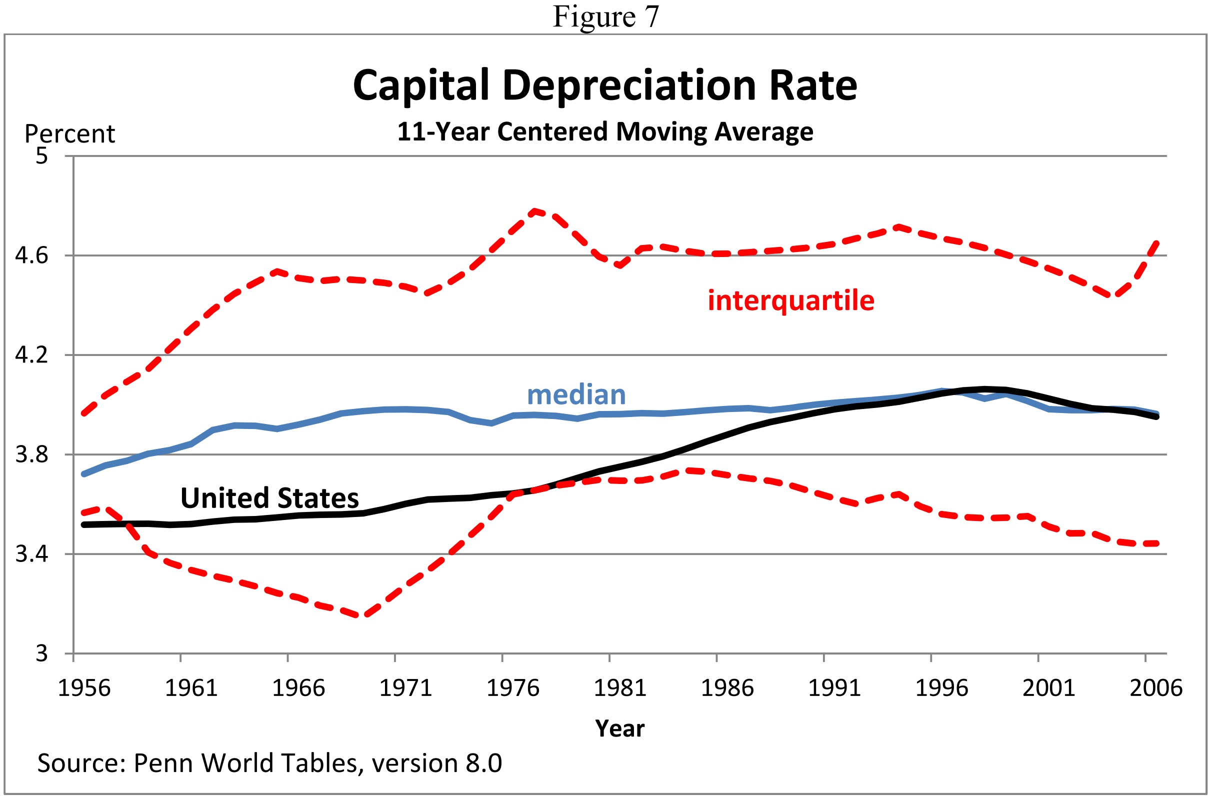


Figure 7 presents evidence on the long-run capital depreciation rate. There is a gradual trend upwards – consistent with a shift in the composition of capital away from structures towards equipment, machinery, and software. The median movements are small in relation to median movements in long-run real interest rates, but in the United States, the depreciation rate has risen by about ½ percentage point.

As discussed in Section 3, the forces underlying the marginal product of capital are TFP, the capital\labor ratio, and the capital share of income. Figure 8 shows the median, across countries, of the long-run TFP growth rate. For comparison, the long-run U.S. TFP growth rate is also shown. The figure shows that the median long-run TFP growth rate fell during the 1960s and 1970s from about 2 percent to less than 0.5 percent, and has been relatively flat since then. This pattern is consistent with the pattern in long-run MPK, and is also consistent with a story in which many countries exhibited high growth periods through the 1960s and early 1970s, but then diminishing returns to capital accumulation and to technology upgrading set in. The United States shows a similar pattern through the 1970s; however, long-run U.S. TFP growth increased in the 1980s and 1990s – with the latter decade associated with the increase in TFP growth owing to information technology (IT) investment – before declining since the early 2000s. Note that long-run U.S. TFP growth moves broadly with long-run U.S. MPK, and moves fairly closely with the long-run U.S. real interest rate.

Figure 8

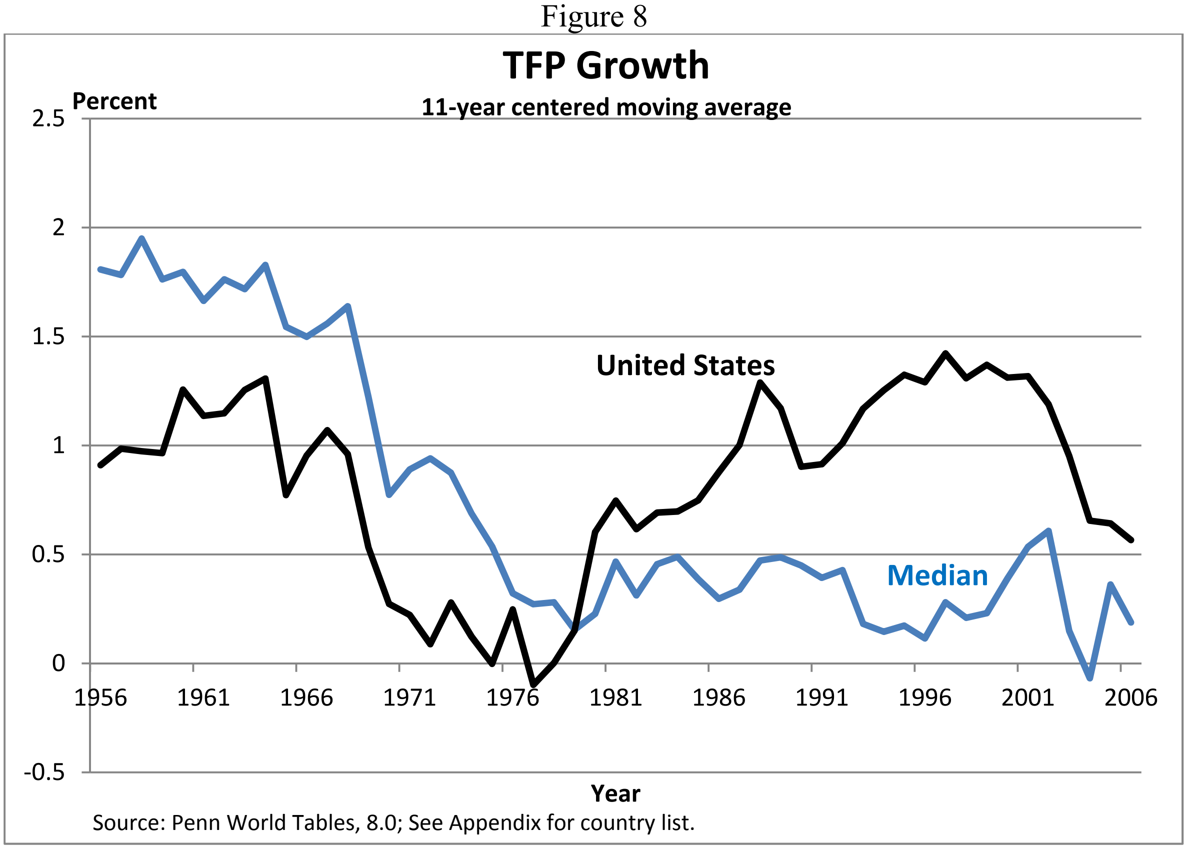


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Figure 9

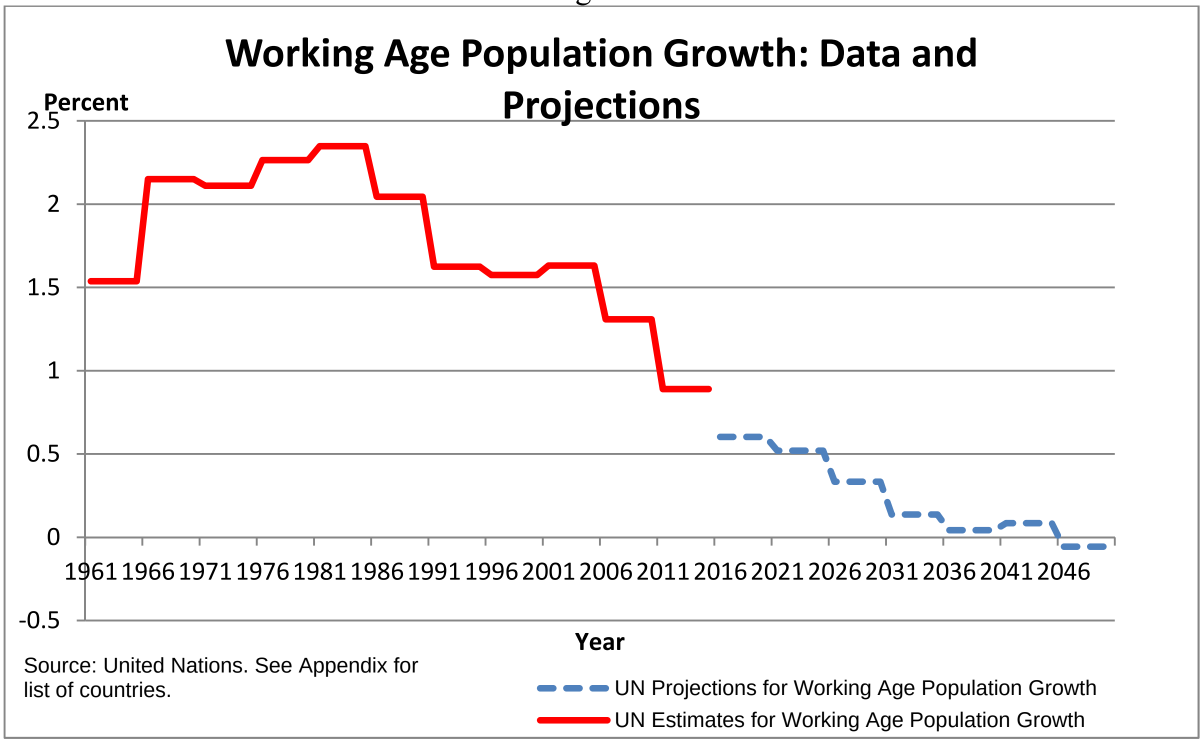
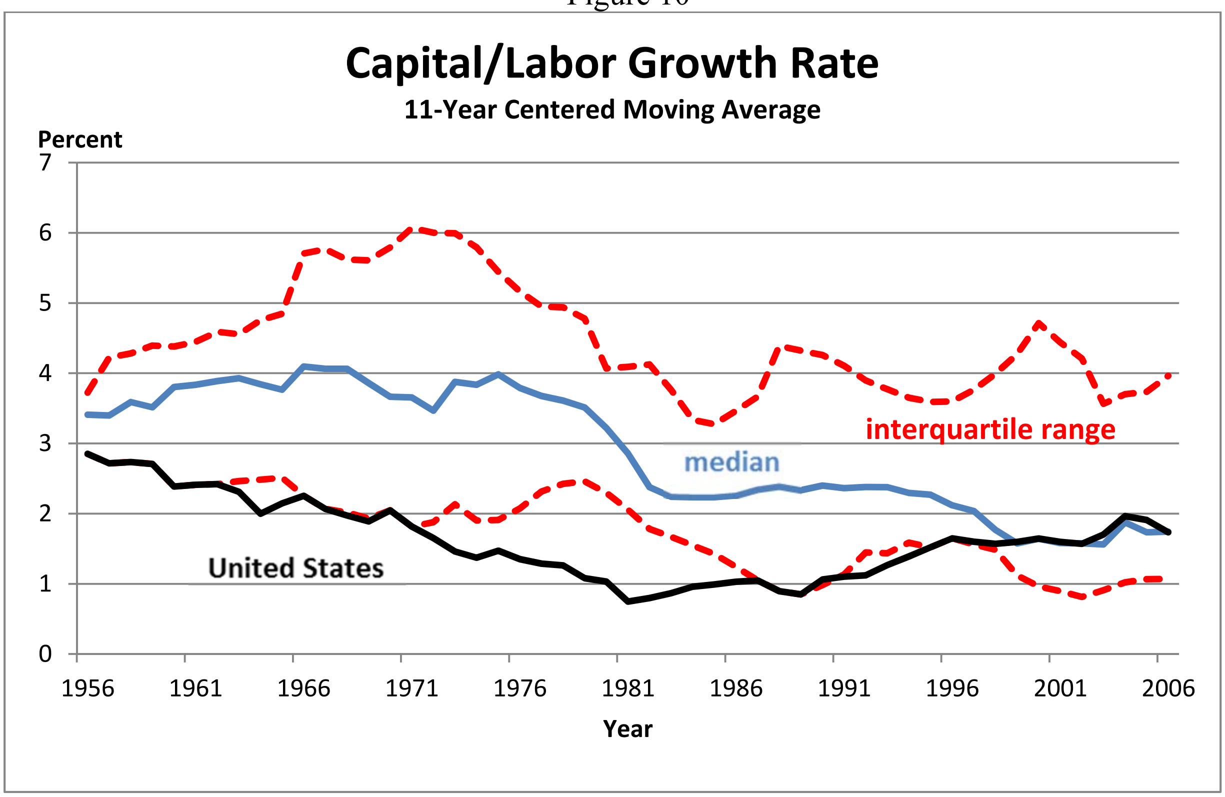


Figure 10



Using working-age population data from the United Nations, we compute the growth rate of the working-age population for our 20 countries taken together. The red solid line in Figure 9 shows that since the 1980s the growth rate of the working-age population has declined from about 2 ¼ percent to less than one percent today.

Figure 10 presents the capital/labor (K/L) growth rate for the United States, the median across our countries, and the interquartile range. The median K/L growth rate was high in the late 1950s through the late 1970s, and then has declined since then. The U.S. K/L growth rate tracked the median trend through the late 1970s, but has grown since then, likely owing to the IT boom.

Increases in K/L tend to push MPK down (owing to diminishing marginal returns), while increases in TFP (and the capital share of income) tend to push MPK up. The MPK figure 6a, together with the TFP growth and K/L growth figures, illustrate, in an accounting sense, the relative importance of each force over time. From the 1950s through the early-1980s, K/L growth was too high to be justified by TFP growth – which was high itself – alone. Consequently, MPK declined over time. The high K/L growth could well be the outcome of the rebuilding of the capital stock in many countries following World War II. In the years since the mid-1980s, K/L growth and TFP growth have been low, while the capital share of income has risen.[14] The overall impact is balanced in the sense that MPK has remained constant over this period.

5. Risk Premia

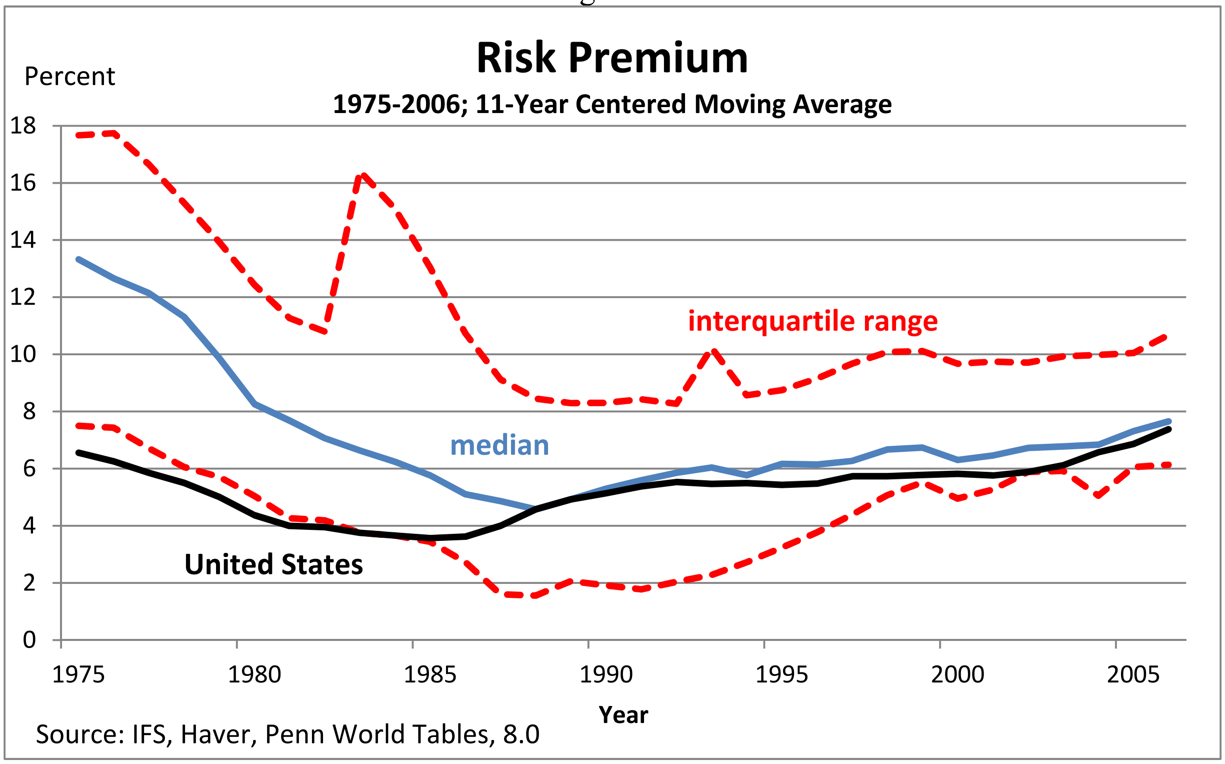
We revisit the median long-run real interest rate from Figure 3 and the median long-run MPK from Figure 6a. While there is a downward trend in both variables from the 1960s to the mid-1970s, and both variables are currently low relative to our 60 year time period, the two variables do not move together from the mid-1970s through the mid-2000s. In that period, the long-run real interest rate rose and then fell, while the long-run MPK was essentially flat. In our framework, these two facts can be reconciled via movements in the risk premium. We calculate the risk premium as a residual from Equation (1). The long-run risk premium for the United States, the median across our sample of countries, and the interquartile range, are presented in Figure 11. The risk-premium fell from the mid-1970s through the late 1980s, and then both the median and the U.S. risk premium rose about three-to-four percentage points since then.

What could account for the rise in the risk premium over the past quarter century? We offer three possibilities. One possibility is that with increased global goods and asset market integration, the risk of capital projects has increased – with global competition, the probability of success is lower, but conditional on success, the rewards are greater. A second possibility is that in the aftermath of the Great Recession and the global financial crisis, households' precautionary motive for saving has persistently, possibly even permanently, increased. This would drive down long-run real interest rates – and when juxtaposed against unchanging MPK and depreciation rates on capital – imply higher risk premia. A related and third possibility is that the

[14] We do not show data on the capital income share, but recent research has concluded that it has increased across a wide set of countries. See, for example, Karabarbounis and Neiman (2014).

demand for safe assets by households and firms has persistently increased over the past quarter century.[15]

Figure 11



6. Projections for TFP Growth and Population Growth.

For the United States, Fernald (2014) documents a slowdown in labor productivity growth beginning in 2004. The slowdown is a result of both slower TFP growth and less capital deepening. Going forward, Fernald projects annual labor productivity growth of 1.85 percent or about half of the average during the tech boom and during 1948-1973. The TFP growth component relevant for the long-run real interest rate (arising from a multi-sector model) is projected to be about 1 ¼ percent, also lower than before. This is 0.2 percentage points lower than TFP growth during the great moderation and almost ¾ percentage point lower than TFP growth during the tech boom. This lower TFP growth translates one-for-one into lower long-run real interest rates.[16]

The blue dashed line in Figure 9 gives the United Nations' projection of working-age population growth for our 20 countries taken together. The figure shows that the growth rate of the working-age population is projected to decline a full percentage point over the next 35 years, turning negative by 2050. All else equal, a decline in the long-run population growth rate will also lead to a decline in the long-run real interest rate.

[15] See Caballero and Farhi (2014) for a model of the "safety trap" and references that document the increased demand for safe assets.
[16] This assumes an intertemporal elasticity of substitution of 1. A lower intertemporal elasticity would imply larger long-run real interest rate movements.

7. Summary and Conclusion

The long-run average and steady-state real interest rates set reference points, or time-varying anchors, for calibrating monetary policy appropriately. However, these concepts are inherently unobservable. We compute long-run averages of real interest rates, and examine several forces suggested by a simple conceptual framework that are thought to drive real interest rates. We also report on recent research on the steady-state real interest rate for the United States. This investigation leads to the following summary and tentative conclusions:

1. Real interest rates since the 1960s have been characterized by three broad long-run trends. The latest trend is a decline across numerous countries since the 1980s. In addition, the Johannsen and Mertens (2015) and Kiley (2015) estimates suggest that the steady-state U.S. real interest rate has declined over essentially the same period. Currently, long-run average real interest rates are near their low for the 60-year period we examine. In addition, over the past quarter century there has been a good deal of convergence in long-run interest rates. These findings have three implications:
    a. Long before "secular stagnation" became popular, even long before the fizzling out of the IT boom, real interest rates were declining.
    b. Increasing financial integration may lead to even further convergence of real interest rates across countries.
    c. The data show there was a sustained increasing trend in long-run real interest rates prior to the 1980s. This suggests that the current long-run pattern could be reversed at some point in the future.
2. Long-run trends in real interest rates and global fixed investment suggest that forces leading to a weakening in global fixed investment demand are important. Our examination of the determinants of investment demand show that trends in the long-run marginal product of capital (MPK) track trends in the long-run real interest rate from 1960s to the mid-1970s, but not again until in recent years. (The United States is an exception in that long-run TFP growth, MPK, and real interest rates have broadly tracked each together for most of the 60-year time period.) Long-run trends in the marginal product of capital are consistent with long-run trends in total factor productivity growth and growth in the capital/labor ratio. Finally, in recent years, movements in long-run TFP growth, and trends in working-age population growth are consistent with a recent decline in long-run real interest rates.
3. Movements in the long-run risk premia help account for movements in the long-run real interest rate for most of the period following the mid-1970s. In particular, as interest rates rose during the late 1970s and through the 1980s, risk premia declined – and the opposite pattern has held for the past quarter century. There are natural reasons to expect higher risk premia in recent decades.
4. Going forward, U.S. TFP growth and global working-age population growth are projected to be lower. This coupled with existing low long-run MPK and real interest rates, and relatively high capital depreciation rates, suggest that long-run real interest rates may be low for some time.
5. With respect to monetary policy, our evidence suggests that the likelihood that the United States and other countries will hit the ELB has increased compared to prior to the crisis, at least for the foreseeable future.

Appendix

Derivation of Equations (1) and (2) in memo

See the technical appendix to this memo.

Marginal product of capital expression

Under standard Cobb-Douglas production, we can express the marginal product of capital (MPK) as:

$$MPK = \alpha A(\frac{K}{L})^{\alpha-1} = \frac{\alpha Y}{K}$$

where α is the capital share, A is total factor productivity (TFP), and K, L, and Y are capital, labor, and output, respectively.

Long-run real interest rate along "balanced growth path"

The above marginal product of capital equation applies in any given year. In assessing the forces that affect the long-run real rate, sometimes it is helpful to examine the steady-state or balanced growth implications.

I. In a neoclassical growth deterministic steady-state[17], long-run $r^* = MPK^* - \delta = \frac{1+g^*}{\beta} - 1$ where g^* is the long-run growth rate of TFP (in decimal form), and β is the rate of time preference, usually less than 1. Clearly, as the long-run growth rate of TFP declines, long-run r will decline.

II. In a deterministic overlapping generations (OLG) setting, long-run r is given by the following[18]: $r^* = \gamma(1+g^*)(1+n^*) - \delta$. γ is a constant that includes the capital income share and the preference discount factor, g^* and n^* are the long-run growth rate of TFP and the long-run growth rate of employment (in decimal form), respectively, and δ is the depreciation rate on capital. In this setting, as the long-run growth rate of employment declines, long-run r will decline. In addition, if the depreciation rate of capital increases, long-run r will decline.

Construction of Real Interest Rates

We follow the procedure of Hamilton *et al* (2015). We use annual interest rate data. Our nominal interest rate variable is typically the central bank policy rate at the end of the year. For Brazil, France, Indonesia, and Mexico, it is an annual average short-term market rate. For China it is an end-of-year deposit rate. The interest rate data came from Haver or the IMF's International Financial Statistics (IFS). We compute the inflation rate as the December-to-December consumer price inflation rate (for Australia it was the Q4/Q4 inflation rate). The price level data came from Haver or the IFS. For inflation expectations, since Atkeson and Ohanian (2001), it has been known that it is difficult to out-perform a random walk inflation model in out-of-sample forecasts. For this reason, we set expected inflation next year as the inflation rate this year. The real interest rate equals the difference between the nominal interest rate and the inflation rate expected for the next year.

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[17] We employ a framework with standard logarithmic preferences and a Cobb-Douglas production function.

[18] We employ a framework with logarithmic preferences and Cobb-Douglas production.

Construction of Marginal Product of Capital (MPK)

We follow the procedure of Caselli and Feyrer (2007), and mainly use data from the Penn World Tables (PWT) version 8.0 (Feenstra, Inklaar, and Timmer, 2013). Caselli and Feyrer show that with any constant returns to scale production function, MPK can be represented as $\alpha Y/K$ where α is the capital share, Y is GDP, and K is the capital stock. We use the PWT national accounts data for Y and K. K is a measure of reproducible capital only. We construct α = 1- labor share – natural resource rental share of GDP. Our measure of α differs across countries and is time varying for two reasons – the labor share varies over time, and the share of natural resource rents in GDP varies over time, as well. We obtain the labor shares from the PWT 8.0, and measures of the natural resource rents in GDP from Alex Monge and Juan . Sánchez, which draws from Monge et al (2015).

Sources of capital, depreciation rate, TFP, and working age population data

The capital, depreciation rate, and TFP data are from the Penn World Tables, version 8.0. We use the variables that are measured at constant national accounts prices. The working age population data are from the United Nations.

Country list

We study the 20 largest economies as measured in current dollar GDP, excluding Russia and Saudi Arabia owing to limited data. Our countries include: Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Mexico, Netherlands, Nigeria, South Korea, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States. Below is a table that lists the variables and years available for each country. (The risk premium series has the same availability as the real interest rate, and the capital growth series has the same availability as the MPK.)

Country	Real Interest Rate	MPK	Depreciation	TFP Growth Rate	Working Age Population Growth
Australia	1969-2014	1950-2011	1951-2011	1951-2011	1951-2011
Brazil	1980-2014	1950-2011	1951-2011	1951-2011	1951-2011
Canada	1950-2014	1950-2011	1951-2011	1951-2011	1951-2011
China	1985-2014	1952-2011	1953-2011	1953-2011	1953-2011
France	1950-2014	1950-2011	1951-2011	1951-2011	1951-2011
Germany	1950-2014	1950-2011	1951-2011	1971-2011	1961-2011
India	1963-2014	1950-2011	1951-2011	1961-2011	1961-2011
Indonesia	1970-2014	1960-2011	1961-2011	1961-2011	1961-2011
Italy	1958-2014	1950-2011	1951-2011	1951-2011	1951-2011
Japan	1958-2014	1950-2011	1951-2011	1951-2011	1951-2011
Mexico	1978-2014	1950-2011	1951-2011	1951-2011	1951-2011
Netherlands	1958-2014	1950-2011	1951-2011	1951-2011	1951-2011
Nigeria	1961-2014	1950-2011	1951-2011	#N/A	1961-2011
South Korea	1956-2014	1953-2011	1954-2011	1961-2011	1961-2011
Spain	1958-2014	1950-2011	1951-2011	1951-2011	1951-2011
Sweden	1950-2014	1950-2011	1951-2011	1951-2011	1951-2011
Switzerland	1950-2014	1950-2011	1951-2011	1951-2011	1951-2011
Turkey	1970-2014	1950-2011	1951-2011	1951-2011	1951-2011
United Kingdom	1957-2014	1950-2011	1951-2011	1951-2011	1951-2011
United States	1950-2014	1950-2011	1951-2011	1951-2011	1951-2011

References

Arellano, Cristina, and Fabrizio Perri (2014), "On Real Interest Rates", memo, Federal Reserve Bank of Minneapolis.

Atkeson. Andrew, and Lee E. Ohanian (2001), "Are Phillips Curves Useful for Forecasting Inflation?" Federal Reserve Bank of Minneapolis *Quarterly Review* (Winter): 2-11.

Bernanke, Ben (2005), "The Global Saving Glut and the U.S. Current Account Deficit." Speech at the Sandridge Lecture, Virginia Association of Economists, Richmond, VA, March 10.

Caballero, Ricardo J. and Emmanuel Farhi (2014), "The Safety Trap." NBER WP# 19927.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas (2008), "An Equilibrium Model of 'Global Imbalances' and Low Interest Rates." *American Economic Review* 98 (1): 358–93.

Caselli, Francesco and James Feyrer (2007), "The Marginal Product of Capital" *Quarterly Journal of Economics* 122 (May): 535-568.

Chung, Hess, Marco Del Negro, Thiago Ferreira, Cristina Fuentes-Albero, Marc Giannoni, Manuel P. Gonzalez-Astudillo, Luca Guerrieri, Matteo Iacoviello, Evan F. Koenig, Jean-Philippe Laforte, Matthias Paustian, Damjan Pfajfar, Andrea Raffo, Andrea Tambalotti (2015), "Estimates of Short-Run r* from DSGE Models." Memo, October.

Fernald, John (2014), "Productivity and Potential Output before, during, and after the Great Recession." *NBER Macro Annual 2014*: 1-51.

Gomme, Paul, B. Ravikumar, and Peter Rupert (2011), "The Return to Capital and the Business Cycle." *Review of Economic Dynamics* (14): 262-278.

Gust, Christopher, Benjamin K. Johannsen, David López-Salido, and Robert Tetlow (2015), "r*: Concepts, Measures, and Uses", Memo, October.

Hamilton, James, Ethan S. Harris, Jan Hatzius, and Kenneth D. West (2015), "The Equilibrium Real Funds Rate: Past, Present and Future." Manuscript.

Johannsen, Benjamin K. and Elmar Mertens (2015a), "The Shadow Rate of Interest, Macroeconomic Trends, and Time-Varying Uncertainity." Manuscript. Federal Reserve Board, July.

Johannsen, Benjamin K. and Elmar Mertens, (2015b) "Summary of Results 'Shadow Rates of Interest, Macroeconomic Trends, and Time-Varying Uncertainty'", Memo, Federal Reserve Board, October.

Karabarbounis, Loukas, and Brent Neiman (2014), "The Global Decline of the Labor Share." *Quarterly Journal of Economics* 129 (February): 61-103.

Kiley, Michael T. (2015), "What Can the Data Tell Us About the Equilibrium Real Interest Rate?" Finance and Economic Discussion Series 2015-077, Washington: Board of Governors of the Federal Reserve System.

Laubach, Thomas and John C. Williams (2003), "Measuring the Natural Rate of Interest." *The Review of Economics and Statistics* 85 (4): 1063-1070

López-Salido, David, Christopher Gust, Benjamin K. Johannsen, and Robert Tetlow (2015), "Monetary Policy at the Lower Bound with Imperfect Information about r*", Memo, October.

Mendoza, Enrique G., Vincenzo Quadrini, and José-Víctor Ríos-Rull (2009), "Financial Integration, Financial Development, and Global Imbalances." *Journal of Political Economy* 117 (3): 371-416.

Monge-Naranjo, Alexander, Juan M. Sánchez, and Raul Santaeulàlia-Llopis (2015), "Natural Resources and Global Misallocation." Federal Reserve Bank of St. Louis Working Paper 2015-013A.

Obstfeld, Maurice and Linda Tesar (2015), "Long-Term Interest Rates: A Survey." Manuscript. Washington: President's Council of Economic Advisors.

Technical Appendix: Derivation of Equations (1) and (2)

In this technical appendix, we derive equations (1) and (2) in the memo. We start with a one-sector closed economy with endowment, and then extend it to a production framework, which is our benchmark model. We also work out two extensions to the benchmark: a multi-good setting and a multi-country setting.

One-country and one-good model with endowment

The endowment Y_t is stochastic with finite support and a markov transition matrix over time. The representative consumer chooses consumption C_t and savings in risk-free assets B_t to maximize the lifetime discounted utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \tag{1}$$

subject to the budget constraint in each period t

$$C_t + B_t Q_t = Y_t + B_{t-1}.$$

where Q_t denotes the price of assets.

The first order conditions imply that the risk free interest rate R_t is given by

$$R_t = \frac{1}{Q_t} = \frac{U'(C_t)}{\beta E_t \left[U'(C_{t+1})\right]} = \frac{(C_t)^{-\sigma}}{\beta E_t \left[(C_{t+1})^{-\sigma}\right]} = \left\{\beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right]\right\}^{-1}.$$

Plugging in the feasibility conditions $B_t = 0$ and $C_t = Y_t$, we have

$$R_t = \frac{1}{Q_t} = \left\{\beta E_t \left[\left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma}\right]\right\}^{-1},$$

The expected output or consumption growth is positively correlated with the real interest rate. The paper by Hamilton *et al* (2015) basically employs the equation immediately above. They examine the relationship between the measured ex-ante real interest rate and output growth. They find this equation holds only weakly for historical data from the United States and other OECD countries.

One-country and one-good model with production: baseline

Production uses both capital K_t and inelastic labor supply L_t as inputs with a Cobb-Douglas form:

$$F(A_t, K_{t-1}, L_t) = A_t K_{t-1}^{1-\alpha} L_t^{\alpha},$$

where A_t is the stochastic productivity shock with finite support and a markov transition matrix over time.

The representative consumer maximizes utility given by equation (1) subject to the budget constraint

$$C_t + B_t Q_t + K_t - (1-\delta) K_{t-1} = F(A_t, K_{t-1}, L_t) + B_{t-1}.$$

The first order conditions are given by

$$R_t = \frac{1}{Q_t} = \frac{U'(C_t)}{\beta E_t\left[U'(C_{t+1})\right]},$$

$$U'(C_t) = \beta E_t\left[U'(C_{t+1})(R^K_{t+1} + 1 - \delta)\right],$$

where

$$R^K_{t+1} = F_K(A_{t+1}, K_t, L_{t+1}).$$

Manipulating the last two equations gives the equity premium equation:

$$r_t = E_t(R^K_{t+1}) - \delta - RP_t, \tag{2}$$

where $r_t = R_t - 1$ is the net risk free rate, and the risk premium RP_t is given by

$$RP_t = \text{cov}_t\left(\frac{-U'(C_{t+1})}{\beta E_t\left[U'(C_{t+1})\right]}, R^K_{t+1}\right) = \text{cov}_t\left(\frac{-C^{-\sigma}_{t+1}}{\beta E_t\left[C^{-\sigma}_{t+1}\right]}, R^K_{t+1}\right). \tag{3}$$

This equation links the expected returns of investment and the risk premium (the cov term) to the risk free rate. This equation also holds in an open economy in a simple framework where each country owns their own risky investment project.

One-country and two-good model with production

The first extension to the baseline model is to introduce a multi-sector setting. Assume that the economy has two sectors: consumption and investment goods, with frictionless goods and factor markets. Denote the price of consumption goods and investment goods by P_C and P_K, respectively. Then, Equation (2) above is modified to

$$r_t = E_t\left(\frac{P_C}{P_K} R^K_{t+1}\right) - \delta - RP_t. \tag{4}$$

Two-country and one-sector production model

The second extension is to introduce a multi-country setting. Assume that there are two countries with complete markets, so we lay out the social planner's problem for simplicity. The social planner maximizes the weighted utility, where λ_i denotes weights of country i,

$$E_0 \sum_{t=0}^{\infty} \beta^t(\lambda_1 U(C_t) + \lambda_2 U(C_{2t})), \tag{5}$$

subject to the resource constraints:

$$\sum_{n=1}^{2}\left[C_{nt} + K_{nt} - (1-\delta)K_{nt-1}\right] = \sum_{n=1}^{2} F(A_{nt}, K_{nt-1}, L_{nt}).$$

The first order conditions are summarized as

$$\lambda_1 U'(C_{1t}) = \lambda_2 U'(C_{2t}),$$

$$U'(C_{nt}) = \beta E_t\left[U'(C_{nt+1})(R^K_{nt+1} + 1 - \delta)\right].$$

Thus, the implied pricing equations are unchanged from the baseline model.

Two-country and two-tradable-good model with production

There are two tradeable goods X and Y, where the home produces good X, and the foreign produces good Y with the same production function

$$F(A, K, L) = AK^{1-\alpha}L^{\alpha}.$$

The composite good (used in both consumption and investment) in each country is an Armington aggregator over the home and foreign produced goods, with the elasticity captured by γ:

$$G(X_{1t}, Y_{1t}) = \left[\omega X_{1t}^{\gamma} + (1-\omega)Y_{1t}^{\gamma}\right]^{\frac{1}{\gamma}},$$

$$\tilde{G}(X_{2t}, Y_{2t}) = \left[(1-\omega)X_{2t}^{\gamma} + \omega Y_{2t}^{\gamma}\right]^{\frac{1}{\gamma}},$$

where X_{nt} and Y_{nt} denote the home and foreign produced goods demanded by country n. We allow for home bias in consumption when $\omega > 0.5$. $\tilde{G}$ is the mirror image of G.

The social planner maximizes utility given by Equation (5), subject to the resource constraints

$$C_{1t} + K_{1t} - (1-\delta)K_{1t-1} = G(X_{1t}, Y_{1t}),$$

$$C_{2t} + K_{2t} - (1-\delta)K_{2t-1} = \tilde{G}(X_{2t}, Y_{2t}),$$

$$X_{1t} + X_{2t} = X_t = F(A_{1t}, K_{1t-1}, L_{1t}),$$

$$Y_{1t} + Y_{2t} = Y_t = F(A_{2t}, K_{2t-1}, L_{2t}).$$

The first order conditions are given by

$$\lambda_1 U'(C_{1t})G_1(X_{1t}, Y_{1t}) = \lambda_2 U'(C_{2t})\tilde{G}_1(X_{2t}, Y_{2t}),$$

$$\lambda_1 U'(C_{1t})G_2(X_{1t}, Y_{1t}) = \lambda_2 U'(C_{2t})\tilde{G}_2(X_{2t}, Y_{2t}),$$

$$U'(C_{1t}) = \beta E_t \left[U'(C_{1t+1})\left(R^K_{1t+1}G_1(X_{1t+1}, Y_{1t+1}) + 1 - \delta\right)\right],$$

$$U'(C_{2t}) = \beta E_t \left[U'(C_{2t+1})\left(R^K_{2t+1}\tilde{G}_2(X_{2t+1}, Y_{2t+1}) + 1 - \delta\right)\right].$$

We can back out the equilibrium prices as follows. For the aggregate price, we have

$$P_{nt} = \lambda_n U'(C_{nt}).$$

Goods prices, which satisfy the Law of One Price, are given by

$$p_{Xt} = P_{1t}G_1(X_{1t}, Y_{1t}) = P_{2t}\tilde{G}_1(X_{2t}, Y_{2t}).$$

$$p_{Yt} = P_{1t}G_2(X_{1t}, Y_{1t}) = P_{2t}\tilde{G}_2(X_{2t}, Y_{2t}).$$

Thus, the first order conditions imply that G_1 equals the ratio of the price of the production basket to the price of the consumption basket.

The terms of trade for country 1 is given by the ratio of the imports over the exports:

$$TOT_{1t} = \frac{p_{Yt}}{p_{Xt}} = \frac{G_2(X_{1t}, Y_{1t})}{G_1(X_{1t}, Y_{1t})}.$$

The real exchange rate is given by

$$RER_t = \frac{P_{1t}}{P_{2t}}.$$

If preferences over the two goods are symmetric, the real exchange rate will be constant, i.e., PPP holds. If preferences are fully home biased, the exchange rate will mimic the terms of trade.

Now we look at the pricing equations. Given the differences in preferences, the home and foreign countries value the risk free bond differently.

$$R_{nt} = \frac{U'(C_{nt})}{\beta E_t \left[U'(C_{nt+1})\right]} = \frac{P_{nt}}{\beta E_t \left[P_{nt+1}\right]}. \tag{6}$$

Thus, we have

$$\frac{R_{1t}}{R_{2t}} = \frac{P_{1t}}{P_{2t}} \frac{E_t \left[P_{2t+1}\right]}{E_t \left[P_{1t+1}\right]} = RER_t \frac{E_t \left[P_{2t+1}\right]}{E_t \left[P_{1t+1}\right]}. \tag{7}$$

For risky investment, we have

$$r_{nt} = E_t \left[R^K_{nt+1} G_n(X_{nt+1}, Y_{nt+1})\right] - \delta - RP_{nt}, \tag{8}$$

where

$$RP_{nt} = \mathrm{cov}_t \left(\frac{-U'(C_{nt+1})}{\beta E_t \left[U'(C_{nt+1})\right]}, R^K_{nt+1} G_n(X_{nt+1}, Y_{nt+1}) \right). \tag{9}$$