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Carlos Ramírez*

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ABSTRACT

This paper proposes a novel link between the propagation of shocks within production networks and asset prices. It develops a dynamic network model in which the propagation of firm cash-flow shocks via inter-firm relationships affects the economy's equilibrium asset prices. When calibrated to match key features of customer–supplier networks in the United States, the model generates long-run risks, high and volatile risk premia, and a low and stable risk-free rate. Consistent with data from firms in manufacturing and service industries, the model predicts that central firms in the network command lower risk premiums than peripheral firms, and that firm-level return volatilities exhibit a high degree of co-movement.

Keywords: Inter-firm Relationships, Shock Propagation, Networks, Equilibrium Asset Prices.

JEL classification: G12, E32, L10.

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Inter-firm relationships, such as strategic alliances, joint ventures, research and development (R&D) partnerships, and customer–supplier relationships, are ubiquitous in modern economies. A growing body of empirical work emphasizes the importance of inter-firm relationships in the case of firms' distress and shows that they may serve as propagation mechanisms of negative shocks to individual firms.[1] As firm-level shocks may spread to a non-negligible fraction of firms in the economy, such a shock propagation can, in principle, generate aggregate fluctuations, altering equilibrium asset prices and aggregate risk premia.[2] Despite the importance of this shock propagation, its equilibrium asset pricing implications have been overlooked in the literature.[3] This paper fills this gap by developing a model that links equilibrium asset prices and aggregate risk premia to the propagation of cash-flow shocks across firms in a network economy.

When calibrated to match key characteristics of customer–supplier networks in the United States, I show that a dynamic network economy model in which cash-flow shocks propagate via inter-firm relationships and investors have Epstein-Zin-Weil preferences generates long-run risks and quantitatively replicates prime characteristics of asset market data. In particular, the model generates high and volatile risk premia, as well as a low and stable risk-free rate. Moreover, the model predicts that central firms in the network command lower risk premiums than peripheral firms, and that firm-level return volatilities exhibit a high degree of co-movement. These predictions are consistent with data from firms in manufacturing and service industries but cannot be accounted for by standard asset pricing models.

The model has two main features. First, firms' cash-flow growth prospects are determined via an exogenous network of long-term inter-firm relationships, as firm-level cash-flow shocks propagate via such relationships.[4] In the model, the propensity of inter-firm relationships to transmit shocks varies over time to capture changes in relationship-specific characteristics that make connected firms more vulnerable to negative spillovers. Changes in relationship-specific characteristics may

[1] See Hertzel et al. (2008), Jorion and Zhang (2009), Boone and Ivanov (2012), Carvalho, Nirei, and Saito (2014), Boyarchenko and Costello (2015), Todo, Nakajima, and Matous (2015), Boehm, Flaaen, and Pandalai-Nayar (2015) and Barrot and Sauvagnat (2016) among others.

[2] Using French firm-level data from 1990 to 2007, Di Giovanni, Levchenko, and Mejean (2014) provide empirical evidence of the importance of firm-specific shocks in generating aggregate fluctuations.

[3] The contemporaneous work of Herskovic (2015) is an exception.

[4] Long-term inter-firm relationships may allow connected firms to circumvent difficulties in contracting due to unforeseen contingencies, asymmetries of information, and specificity on firms' investments, e.g., Williamson (1979, 1983). The network is assumed to be exogenous because the focus of this paper is on the effect of shock propagation on equilibrium asset prices rather than on strategic network formation. For endogenous formation of production networks, see Oberfield (2013), Chaney (2014, 2016), and Lim (2016).

arise from variation in restrictions on firms' use of alternative inputs, e.g. Barrot and Sauvagnat (2016). Variation in such restrictions may emerge from changes—driven by innovation or industry regulation—in production technologies, complementarities among firms' activities, or market competition. As the propensity of inter-firm relationships to transmit shocks varies over time, the mechanism by which shocks propagate across firms also varies over time, introducing an endogenous time-varying correlation structure among firms' cash-flows. Second, investors have Epstein-Zin-Weil preferences—as in standard asset pricing models—and, thus, care about uncertainty regarding firms' long-term growth prospects.

In the calibrated model, low-frequency changes in the shock propagation mechanism endogenously generate persistence in firms' growth prospects due to the long-term nature of inter-firm relationships. In equilibrium, the persistence in firms' growth prospects drives a small and persistent component in expected aggregate consumption growth. Because investors have recursive preferences, the model accounts for sizeable risk premiums and a small and stable risk-free rate, as low-frequency movements in consumption growth induce large movements in marginal utility and stock prices. Intuitively, sizeable risk premiums arise because investors fear that extended periods of low economic growth coincide with low asset prices. Likewise, a small risk-free rate is driven by investors saving for long periods of low economic growth.

In the cross section, the calibrated model predicts that central firms in the network command lower risk premiums than peripheral firms. Central firms tend to benefit from the diversification of their customers and suppliers and, thus, mitigate contagion risk better than less central firms, commanding lower risk premiums. Consistent with data from firms in manufacturing and service industries, the model generates a realistic annual return spread of 3.8% between firms in the lowest tercile of centrality and firms in the highest tercile of centrality.[5] This return spread, which cannot be accounted for by standard asset pricing models, arises naturally in equilibrium as compensation for contagion risk.

In the time series, the calibrated model predicts high co-movement in firm-level return volatilities as changes in the shock propagation mechanism drive fluctuations in growth opportunities and uncertainty across firms. These cross-sectional fluctuations translate into simultaneous changes in

[5] Wu and Birge (2014) provide complementary evidence that manufacturing firms that are more central in the network earn lower returns.

stock returns, which generate a factor structure in the time series of firm-level returns and returns' volatilities.[6]

The small and persistent component in expected consumption growth generated by low-frequency movements in the shock propagation mechanism provides an equilibrium foundation for long-run risk models in the spirit of Bansal and Yaron (2004). Moreover, the model helps explain the cross section of expected returns as it provides a mapping between firms' importance in the network and their contagion risk. The model suggests that extending standard asset pricing models to take into account the way that shocks propagate within production networks can make significant progress toward generating a unifying framework that simultaneously captures dynamics of the aggregate and the cross section of stock returns.

This paper contributes to three strands of the literature. First, the paper develops a new theoretical framework that adds to a growing body of work focused on understanding the effects of economic linkages in asset pricing properties, e.g., Buraschi and Porchia (2012), Ahern (2013), and Herskovic (2015).[7] Unlike these papers, my model emphasizes relationships at the firm level to explore the asset pricing properties that stem from the propagation of shocks within production networks.

Second, this paper adds to a body of work that explores how granular shocks may lead to aggregate fluctuations in the presence of linkages among different sectors of the economy, e.g., Carvalho (2010), Gabaix (2011), Acemoglu et al. (2012); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Oberfield (2013), Carvalho and Gabaix (2013), Blume et al. (2013), Elliott, Golub, and Jackson (2014), Chaney (2014, 2016), and Lim (2016). This paper contributes to this literature by exploring the asset pricing implications of linkages at the firm level and studying how changes in the propagation of shocks within a network economy affect not only aggregate variables but also equilibrium asset prices and aggregate risk premia.

Third, this paper adds to recent research that examines the potential sources of long-run risks,

[6]The factor structure in returns' volatilities is aligned with recent empirical evidence documented by Duarte et al. (2014).

[7]Buraschi and Porchia (2012) show that more central firms in a market-based network have lower price dividend ratios and higher expected returns. Using the network of intersectoral trade, Ahern (2013) provides evidence that firms in more central industries have greater exposure to systematic risk. Herskovic (2015) focuses on efficiency gains that come from changes in the input-output network and how those changes are priced in equilibrium. My paper, on the other hand, focuses on how changes in the propagation of shocks within a fixed network alter equilibrium asset prices and risk premia.

e.g., Kaltenbrunner and Lochstoer (2010), Kung and Schmid (2015), Bidder and Dew-Becker (2016).[8] This paper contributes to this literature by showing that long-run risks may also arise from changes in the way that shocks propagate within sticky production networks.

The rest of the paper is organized as follows. Section I introduces the baseline model. Section II describes aggregate consumption growth in the baseline model. Section III derives expressions for the market return, the risk-free rate, the price of risk, and firms' equilibrium asset prices and expected returns. Section IV uses data on customer–supplier networks in the United States to calibrate the model. Section V shows that changes in the propagation mechanism of shocks in customer–supplier networks are quantitatively important to understand variations in stock returns in both the aggregate and the cross section. Section VI concludes.

I. Baseline Model

The baseline model embeds a single-good dynamic endowment economy, in the spirit of Lucas (1978), into a standard asset pricing model with investors with Epstein-Zin-Weil preferences. In an otherwise standard dynamic endowment economy, the outputs of the economy's productive units, henceforth firms, are determined by a network of long-term inter-firm relationships. The single-good endowment economy framework is assumed to facilitate exposition and can be extended to a production network setting in which every firm produces a different good and each good is necessary to produce other goods in the economy.[9]

A. The environment

Consider an economy with one perishable good and an infinite time horizon. Time is discrete and indexed by $t \in \{0, 1, 2, \cdots\}$. In each period, the single good is produced by n infinitely lived firms, with n being potentially large. The economy is populated by a large number of identical, infinitely lived individuals who are aggregated into a representative investor with Epstein-Zin-Weil

[8] Kaltenbrunner and Lochstoer (2010) shows that long-run risks endogenously arise in a standard production economy model, even when technology growth is i.i.d., because of consumption smoothing. Kung and Schmid (2015) shows that a model of endogenous innovation and R&D is able to generates long-run risks, while Bidder and Dew-Becker (2016) shows that long-run risks arise in an economy in which investors are pessimistic and not sure about the true model driving the economy.

[9] In such an environment, equilibrium asset prices are similar to the ones obtained here if the representative investor has preferences over a particular basket of goods, e.g., Herskovic (2015).

preferences who owns all assets in the economy. Firms' outputs, henceforth cash flows, are related via a network of inter-firm relationships. The production network is described by a graph consisting of a set of nodes, which represent firms, together with edges joining certain pairs of nodes, which represent inter-firm relationships. To fix notation, let $\mathcal{G}_n$ denote the production network among n firms. Because I focus on the effect of $\mathcal{G}_n$ on asset prices rather than on the strategic formation of inter-firm relationships, these relationships are assumed to be exogenously determined and fixed before $t = 0$.[10]

B. *The network of inter-firm relationships and firms' cash flows*

Firms' cash flows vary stochastically over time and depend not only on the production network $\mathcal{G}_n$ but also on the way that cash-flow shocks propagate within $\mathcal{G}_n$. In particular, relationships generate benefits—such as information and resource sharing, access to new markets, and easing of financial constraints via trade credit—which increase a firm's cash-flow growth rate. However, relationships also increase a firm's exposure to negative cash-flow shocks that affect other firms, as negative cash-flow shocks spread through probabilistic contagion via relationships. Such a trade off is captured by the following reduced-form equation:

$$\log\left(\frac{y_{i,t}}{Y_{t-1}}\right) \equiv \alpha_0 + \alpha_1 d_i - \alpha_2 \sqrt{n}\widetilde{\varepsilon}_{i,t} \ , \quad i \in \{1, \cdots, n\} \ , \tag{1}$$

where $y_{i,t}$ denotes firm i's cash flow at period t, and Y_{t-1} denotes the aggregate output of the economy at $t-1$. Parameters α_0, α_1, and α_2 are non-negative and equal across firms. Parameter d_i represents the number of direct relationships of firm i—which may differ across firms. The term $\sqrt{n}$ is included in equation (1) as a normalization factor to help characterize the equilibrium distribution of aggregate consumption growth later on. Uncertainty in $y_{i,t}$ is introduced by a Bernoulli random variable $\widetilde{\varepsilon}_{i,t}$, which equals one if firm i faces a negative cash-flow shock at period t and zero otherwise. Because

$$\log\left(\frac{y_{i,t}}{Y_{t-1}}\right) = \log\left(\frac{y_{i,t}}{y_{i,t-1}}\right) + \log\left(\frac{y_{i,t-1}}{Y_{t-1}}\right),$$

[10] See Demange and Wooders (2005), Goyal (2007), and Jackson (2008) for a detailed description of network formation models. For models of endogenous formation of production networks, see Oberfield (2013), Chaney (2014, 2016), and Lim (2016), among others.

parameter α_2 in equation (1) measures the decrease in a firm's cash-flow growth if a firm faces a negative cash-flow shock. Thus, transitory negative cash-flow shocks have an enduring effect as they depress not only firms' cash-flows today but also firms' growth prospects. Parameter α_1 captures the increase in a firm's cash-flow growth due to each direct relationship, whereas α_0 captures the part of firms' cash-flow growth that is unrelated to benefits or costs associated with inter-firm relationships.[11]

The distribution of $\widetilde{\varepsilon}_{i,t}$ is determined by the following stochastic process—which simplifies the modeling and abstracts from the temporal propagation of shocks. At the beginning of period t, each firm may face a negative cash-flow shock, independently of other firms, with probability $0 < q < 1$—which is equal across firms and time invariant. Firms affected by idiosyncratic cash-flow shocks have a probability of causing their direct partners to face a negative cash-flow shock as well, allowing negative shocks to potentially continue to spread from the newly affected firms.[12] In particular, a negative cash-flow shock to firm i at period t also affects firm j at period t, and, thus, $\widetilde{\varepsilon}_{i,t} = \widetilde{\varepsilon}_{j,t} = 1$ if two things happen: (1) there exists a sequence of relationships that connects firms i and j in $\mathcal{G}_n$ and (2) each relationship in that sequence is active in transmitting shocks at period t. For simplicity, in each period each relationship is either active in transmitting cash-flow shocks or not—independently of all other relationships. The relationship between firms i and j is active in transmitting shocks at period t with probability $\widetilde{p}_{ijt}$. The production network $\mathcal{G}_n$ is assumed to be undirected, and, thus, $\widetilde{p}_{ijt} = \widetilde{p}_{jit}$, $\forall (i,j) \in \mathcal{G}_n$, $\forall\ t$.[13]

[11] In the absence of relationships, α_0 equals the growth rate of the economy if $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$.

[12] Within the baseline model, only negative shocks are allowed to propagate in a probabilistic manner to focus on the effect on asset prices of the propagation of shocks in case of firms' distress. However, the baseline model can be easily extended to allow positive and negative cash-flow shocks to propagate through the network. To do so, define $\widetilde{\psi}_{i,t} \equiv \widetilde{\varepsilon}_{i,t} - 1/2$ so that cash-flow shocks can be positive and negative. Then, redefine equation (1) as

$$\begin{aligned} \log\left(\frac{y_{i,t}}{Y_{t-1}}\right) &= \alpha_0 + \alpha_1 d_i - \alpha_2 \sqrt{n}\widetilde{\psi}_{i,t} \\ &= \underbrace{\alpha_0 + \alpha_2\sqrt{n}/2}_{\hat{\alpha}_0} + \alpha_1 d_i - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t} \\ &= \hat{\alpha}_0 + \alpha_1 d_i - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t}\ , \end{aligned}$$

which is analogous to equation (1). The cross-sectional results in this paper continue to hold as long as the decrease in firms' cash-flow growth due to negative shocks is larger than the increase in firms' cash-flow growth due to positive shocks.

[13] This stochastic process can be thought of as a variation of either a reliability network or a bond percolation model in each period. In a typical reliability network model, the edges of a given network are independently removed with some probability. The remaining edges are assumed to transmit a message. A message from node i to j is transmitted as long as there is at least one path from i to j after edge removal—see Colbourn (1987) for more details. Similarly, in a bond percolation model, edges of a given network are removed at random with some probability. Edges that are not removed are assumed to percolate a liquid. The question in percolation is whether the liquid percolates

The value of $\widetilde{p}_{ijt}$ measures the propensity of the relation (i, j) to transmit cash-flow shocks from firm i (j) to j (i) at period t. At a fundamental level, the value of $\widetilde{p}_{ijt}$ captures relationship-specific characteristics that make firm i (j) more vulnerable to negative spillovers coming from firm j (i), which cannot be mitigated through contractual protections at period t. Intuitively, the higher the value of $\widetilde{p}_{ijt}$, the higher the likelihood that problems affecting firm i (j) also affect firm j (i) at period t. In the context of supply chains, $\widetilde{p}_{ijt}$ may capture restrictions on firm i's and j's use of alternative inputs at period t. The higher the value of $\widetilde{p}_{ijt}$, the higher the switching costs firms i or j may face at period t, and, thus, the higher the likelihood that a negative cash-flow shock to firm i (j) also affects firm j (i), provided that firm j (i) may not be able to restructure its production sufficiently fast to overcome firm i (j)'s negative cash-flow shock.

Probabilities $\{\widetilde{p}_{ijt}\}_{(i,j)\in\mathcal{G}_n}$ are drawn from a Beta distribution with parameters $\zeta_{1t} > 0$ and $\zeta_{2t} > 0$ at the beginning of period t. Parameters $\zeta_{it} > 0$, $i = \{1, 2\}$, which are drawn at the very beginning of period t, determine the shape of the distribution of propensities across relationships at period t. The model timeline at period t is depicted in figure 1.

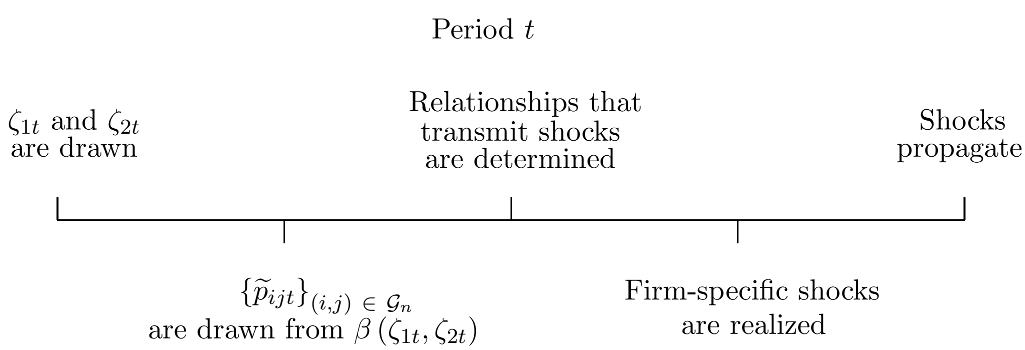

Figure 1. Model timeline in period t.

To sum up, equation (1) captures some of the potential consequences of long-term inter-firm relationships in a simple manner. While inter-firm relationships may increase firms' growth opportunities via efficiency gains, these relationships may also have additional consequences as they

from one node to another in the network—which is similar to the problem of transmitting a message in a reliability context. For more details see Grimmett (1989), Stauffer and Aharony (1994), and Newman (2010, Chapter 16.1). Blume et al. (2013) analyze a propagation mechanism similar to the one analyzed here. They focus, however, on strategic network formation issues in a static environment. They provide asymptotic bounds on the welfare of both optimal and stable networks and show that small amounts of "over-linking" may impose large losses in welfare to networks' participants.

increase a firm's exposure to negative cash-flow shocks that affect a broader set of firms in the economy. Although equation (1) is a reduced-form formulation, it can be recast so it is generated within an equilibrium context. For instance, Goyal and Moraga-González (2001) obtain similar dynamics for firms' profits in strategic environments where firms collaborate in an R&D network to decrease their production costs, but they also compete with their collaborators within the same homogeneous good market.

C. Shock propagation and the cross-sectional distribution of $\widetilde{\varepsilon}_{i,t}$

Given how shocks propagate, the joint distribution of the cross-sectional sequence $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^{n}$ is determined by $\mathcal{G}_n$, q, and the process driving the stochastic propensity matrix $\widetilde{p}_t \equiv [\widetilde{p}_{ijt}]_{(i,j)\,\in\,\mathcal{G}_n}$. Moreover, the marginal distribution of $\widetilde{\varepsilon}_{i,t}$, conditional on $\widetilde{p}_t$, depends on q, the network $\mathcal{G}_n$, and the location of firm i in $\mathcal{G}_n$. In other words,

$$\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\middle|\widetilde{p}_t\right) \;=\; f\left(q,\mathcal{G}_n,\text{location of firm } i \text{ in } \mathcal{G}_n\right)\ , \tag{2}$$

where $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=0\middle|\widetilde{p}_t\right) = 1-\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\middle|\widetilde{p}_t\right)$, and $f(\cdot)$ is a mapping characterized by the stochastic process described in section I.B—which endogenously generates a time-varying correlation structure among firms' cash flows as $\widetilde{p}_t$ varies over time.

Despite the fact that the mapping $f(\cdot)$ is hard to characterize for large n, its properties are easy to describe given the formulation of the stochastic process that generates it. First, in the absence of relationships, $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\middle|\widetilde{p}_t\right)=\mathbb{P}\left(\widetilde{\varepsilon}_{i,t}=1\right)=q\ ,\ \forall\ i$ and $\forall\ t$, so cash-flow growth rates are independent and identically distributed across firms over time. Second, if only one sequence of relationships exists between two firms, the longer the sequence, the smaller the correlation between their cash-flow growth rates. Thus, in network economies in which there is at most one sequence of relationships between any two firms, the more distant the two firms are, the less related their cash flows.[14]

[14]Having this feature—which is sometimes called correlation decay, e.g., Gamarnik (2013)—helps a great deal to obtain numerical solutions of the model relatively fast when n is large.

D. Temporal changes in shock propagation within the network economy

To capture temporal changes in relationship-specific characteristics, the shape parameters ζ_{it}, $i = \{1, 2\}$, are allowed to vary over time. Variation in the shape parameters may arise from changes in firms' production technologies, complementarities among firms' activities, or market competition. For simplicity, ζ_{it} can take two values, ζ_{iL} or ζ_{iH}, with $\zeta_{iL} < \zeta_{iH}$, $i = \{1, 2\}$, and the shape parameter vector $\zeta_t \equiv [\zeta_{1t}\ \zeta_{2t}]$ follows a four-state ergodic Markov process with transition matrix Ω and states $\zeta_{LL} \equiv [\zeta_{1L}\ \zeta_{2L}]$, $\zeta_{LH} \equiv [\zeta_{1L}\ \zeta_{2H}]$, $\zeta_{HL} \equiv [\zeta_{1H}\ \zeta_{2L}]$, and $\zeta_{HH} \equiv [\zeta_{1H}\ \zeta_{2H}]$.

II. Distribution of Consumption Growth

Two features of the model are important to understand aggregate consumption growth: (a) the topology of the production network $\mathcal{G}_n$ and (b) how shocks propagate across firms, captured by the propensity matrix $\widetilde{p}_t$ and its dynamics. In this section, I study how changes in these two features affect the distribution of aggregate consumption growth and, thus, alter the distribution of the pricing kernel. Let $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)$ and $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ denote the log consumption and output growth at $t+1$, respectively. In equilibrium, $\Delta\widetilde{c}_{t+1} = \widetilde{x}_{t+1}$. For tractability, consider $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$. Then, it follows from equation (1) that

$$\begin{aligned}
\Delta\widetilde{c}_{t+1} = \widetilde{x}_{t+1} &= \log\left(\prod_{i=1}^{n}\left(\frac{y_{i,t+1}}{Y_t}\right)^{1/n}\right) \\
&= \sum_{i=1}^{n}\frac{1}{n}\log\left(\frac{y_{i,t+1}}{Y_t}\right) \\
&= \alpha_0 + \alpha_1 \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} d_i\right)}_{\bar{d}} - \alpha_2\sqrt{n}\underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}\widetilde{\varepsilon}_{i,t+1}\right)}_{\widetilde{W}_{n,t+1}} \\
&= \alpha_0 + \alpha_1 \bar{d} - \alpha_2\sqrt{n}\,\widetilde{W}_{n,t+1}\,, \qquad (3)
\end{aligned}$$

where $\bar{d}$ denotes the average number of relationships per firm in the economy, whereas $\widetilde{W}_{n,t+1}$ denotes the average number of firms affected by negative cash-flow shocks at period $t+1$. It follows from equation (3) that the distribution of $\Delta\widetilde{c}_{t+1}$ is determined by the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$. Because the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ is affected by $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$, these two features also affect the distribution of $\Delta\widetilde{c}_{t+1}$.[15]

To appreciate the importance of $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$ in determining the distribution of $\Delta\widetilde{c}_{t+1}$, consider two cases. First, suppose there are no relationships. In this case, $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ is a sequence of i.i.d. Bernoulli random variables and $n\widetilde{W}_{n,t+1}$ follows a Binomial distribution. By the Central Limit Theorem (CLT), $\sqrt{n}\widetilde{W}_{n,t+1}$ is normally distributed as n grows large. Provided the absence of relationships, the realization of the matrix $\widetilde{p}_{t+1}$ is irrelevant to determining the distribution of $\Delta\widetilde{c}_{t+1}$, as the unconditional mean and variance of $\widetilde{W}_{n,t+1}$ are q and $\frac{q(1-q)}{n}$, respectively. Second, suppose that all firms have two relationships and that the propensity to transmit shocks of each relationship is p, which does not vary over time. Then, $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ is a sequence of dependent Bernoulli random variables and $n\widetilde{W}_{n,t+1}$ follows approximately a Binomial distribution if p is sufficiently small. In this case, the propensity of relationships to transmit shocks, p, affects the distribution of consumption growth, as the unconditional mean and variance of $\widetilde{W}_{n,t+1}$ are approximately π and $\frac{\pi(1-\pi)}{n}$, respectively; where $\pi \in [0, 1]$ solves the following equation:

$$\pi = q + (1-q)\pi p \left(\pi p + 2\left[p(1-\pi) + \pi(1-p)\right]\right).$$

Thus, in the presence of relationships, $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$ affect the distribution of consumption growth, as $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ is a sequence of dependent Bernoulli random variables. In this case, the conditions under which a CLT holds may not be satisfied as n grows large.[16] Relationships may generate convoluted interdependencies among firms' cash flows, which makes it difficult to characterize the distribution of $\Delta\widetilde{c}_{t+1}$. Figure 2 illustrates the previous point. Figure 2(a) depicts a star network in an economy with $n = 5$ firms, whereas figure 2(b) depicts the empirical probability density function of $\sqrt{n}\widetilde{W}_{n,t+1}$ for the star network depicted in figure 2(a). As figure 2(b) shows,

[15] The definition of Y_t implies that positive aggregate production requires positive production by each firm. To assume that $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$ is similar to assuming that Y_t is proportional to $\sum_{i=1}^{n} y_{i,t}$ if n is sufficiently large and all $y_{i,t} \neq 0$. The argument follows from applying a first order Taylor series expansion to $\log(Y_t)$ in which aggregate output, $Y_t \equiv \sum_{i=1}^{n} y_{i,t}$. A different way of justifying that $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$ is to consider that every firm produces a different perishable good and each good is necessary to produce other goods in the economy. In such an environment, one obtains asset pricing properties similar to the ones obtained in this paper if the representative investor has preferences over a Cobb-Douglas consumption aggregator of the form $C_t \equiv \prod_{i=1}^{n} c_{i,t}^{1/n}$, where $c_{i,t}$ represents consumption of the good produced by firm i at time t.

[16] For a large variety of network topologies, simulation shows that the distribution of $\Delta\widetilde{c}_{t+1}$ may differ from a normal distribution. In particular, if some elements of the matrix $\widetilde{p}_{t+1}$ are sufficiently close to one and $\mathcal{G}_n$ is locally connected—i.e., there is at least one sequence of relationships between any two firms in an arbitrarily large neighborhood around any given firm—then a non-negligible fraction of firms in the economy are almost surely affected by negative cash-flow shocks. Therefore, the distribution of $\Delta\widetilde{c}_{t+1}$ may exhibit thicker tails than a normal distribution would.

the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ may differ from a normal distribution if the elements of the matrix $\widetilde{p}_{t+1}$ are sufficiently close to 1. In particular, as some components in $\widetilde{p}_{t+1}$ tend toward one, the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ tends to be bimodal.

Despite the existence of relationships and the convoluted dependencies they may generate among firms' cash flows, the topology of $\mathcal{G}_n$ and values in the matrix $\widetilde{p}_{t+1}$ can be restricted so that $\Delta\widetilde{c}_{t+1}$ is normally distributed as n grows large (see Appendix A). In such a case, keeping track of temporal changes of the distribution of $\Delta\widetilde{c}_{t+1}$ is equivalent to keeping track of temporal changes in averages and standard deviations. In particular, if shocks tend to remain locally confined—i.e., no shock propagates over a large fraction of firms in the economy—$\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ becomes a sequence of weakly dependent random variables to which a CLT can be applied. Then, the dynamics of consumption growth can be recast as a version of Hamilton (1989)'s Markov-switching model.

III. Equilibrium Asset Prices

To see what the production network $\mathcal{G}_n$ and the dynamic of ζ_t imply for equilibrium asset prices, I embed the cash-flows correlation structure that is endogenously generated by the baseline model in a standard asset pricing framework. The representative investor has Epstein-Zin-Weil recursive preferences to account for asset pricing phenomena that are challenging to address with power utility preferences. The asset pricing restrictions on the gross return of firm i, $\widetilde{R}_{i,t+1}$, are

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{R}_{i,t+1}\right) \;=\; 1\;, \tag{4}$$

where $\widetilde{M}_{t+1} \equiv \left[\beta\left(e^{\Delta\widetilde{c}_{t+1}}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}$ represents the pricing kernel at $t+1$ and $\widetilde{R}_{a,t+1}$ denotes the gross return on aggregate wealth—an asset that delivers aggregate consumption as its dividend each period. Parameter $\rho > 0$, $\rho \neq 1$, represents the inverse of the inter-temporal elasticity of substitution (IES), $\gamma > 0$ is the coefficient of relative risk aversion for static gambles, and $\beta > 0$ measures the subjective discount factor under certainty.[17]

To solve the model, I look for equilibrium asset prices so that price–dividend ratios are stationary,

[17] If $\gamma = \rho$, these recursive preferences collapse to the standard case of Von Neumann-Morgenstern (VNM) time-additive expected utility. The functional form of the pricing kernel when $\rho = 1$ is different from the one shown above. See Weil (1989, Appendix A) for details. I use the standard terminology to describe γ and ρ. However, Garcia, Renault, and Semenov (2006) and Hansen et al. (2007) indicate that this interpretation may not be correct if $\rho \neq \gamma$.

as in Mehra and Prescott (1985), Weil (1989), and Kandel and Stambaugh (1991), among many others. Because equilibrium values are time-invariant functions of the state of the economy—which is determined by the state of the parameter vector ζ_t—the index t can be eliminated. Hereinafter, c denotes the current level of aggregate consumption, y denotes the current level of aggregate output, and $s \in \{\text{LL}, \text{LH}, \text{HL}, \text{HH}\}$ denotes the current state of parameter vector ζ.

I first solve for the price of aggregate wealth and the risk-free rate. These expressions are then used to solve for equilibrium asset prices and expected excess returns in the cross section.

PROPOSITION 1 (Price of Aggregate Wealth): *Let $P_a(c,s)$ denote the current price of aggregate wealth. $P_a(c,s) = w_s^a c$, where w_s^a is the solution of the following nonlinear system of equations,*

$$w_s^a = \beta \left(\sum_{s' \in \{LL,LH,HL,HH\}} \omega_{s,s'} \mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}} \middle| s' \right) \left(w_{s'}^a + 1 \right)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma}}, \tag{5}$$

where $\mathbb{E}\left(\cdot \middle| s'\right)$ denotes the conditional expectation operator if the state of ζ is s' and $\omega_{s,s'}$ represents the (s,s') element of Ω.

I restrict my analysis to the set of model primitives in which the existence of a non-negative solution of the system of equations (5) is ensured.[18] The expected period gross return of aggregate wealth in the current state is then

$$\mathbb{E}\left(R_a | s\right) = \sum_{s' \in \{\text{LL,LH,HL,HH}\}} \omega_{s,s'} \frac{w_{s'}^a + 1}{w_s^a} \mathbb{E}\left(e^{\Delta\widetilde{c}_{t+1}} \middle| s' \right). \tag{6}$$

It follows from equations (5) and (6) that the price and expected period return of aggregate wealth are driven by: (a) the topology of the production network $\mathcal{G}_n$, and (b) the dynamics of ζ. As noticed in section II, the topology of $\mathcal{G}_n$ and $\widetilde{p}_{t+1}$ drive the distribution of $\Delta\widetilde{c}_{t+1}$. Given a network $\mathcal{G}_n$, ζ shapes the distribution of aggregate consumption growth, as ζ determines $\widetilde{p}_{t+1}$. In particular,

[18] Provided that $e^{\Delta\widetilde{c}_t}$ is positive for all t, parameters ρ and γ need to be restricted so that the function $h(\cdot)$ defined as

$$h\left(w_i^a\right) \equiv \beta \left(\sum_{j \in \{\text{LL,LH,HL,HH}\}} \omega_{i,j} \mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}} \middle| j \right) \left(w_j^a + 1 \right)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma}}$$

is continuous. If $h(\cdot)$ is continuous, the system of equations (5) has a solution by Brouwer's Fixed Point Theorem. Further restrictions in the set of parameter values can be imposed such that the solution of the system of equations is unique.

changes in ζ convey changes in the cross-sectional distribution of $\{\widetilde{p}_{ij}\}_{(i,j)\,\in\,\mathcal{G}_n}$ which determines how shock propagate across firms. The dynamics of ζ—parameterized by Ω—affect the price and the expected period return of aggregate wealth, as Ω determines (a) how frequently the economy is in a state in which relationships are more prone, on average, to transmit negative cash-flow shocks and (b) how persistent are changes in the shock propagation mechanism.

I next consider the risk-free asset, which pays one unit of the consumption good during the next period with certainty.

PROPOSITION 2 (Risk-free Rate): *Let $R_f(s)$ denote the period gross return of the risk-free asset in the current state. $R_f(s)$ solves*

$$\frac{1}{R_f(s)} \;=\; \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \{LL,LH,HL,HH\}} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1}} \middle| s' \right) \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \right) , \tag{7}$$

where w^a_s are the solutions of the system of equations (5).

It follows from equation (7) that the equilibrium risk-free rate is also driven by the topology of $\mathcal{G}_n$ and the dynamics of ζ, as these two features affect the distribution of aggregate consumption growth and prices of aggregate wealth.

Using the previous expressions, I now study what the production network $\mathcal{G}_n$ and dynamics of ζ imply for the cross section of asset prices and risk premiums. The following proposition determines the ex-dividend stock price of firm i and its expected period return.

PROPOSITION 3 (Firms' Stock Prices and Expected Period Returns): *Let $P_i(y,s)$ denote the current ex-dividend stock price of an asset that delivers firm i's cash flows as its dividend each period. For large n, $P_i(y,s) \approx v_i(s)y$, where $v_i(s)$ is the solution of the following linear system of equations*

$$\begin{aligned} v_i(s) \;&=\; \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \{LL,LH,HL,HH\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{\widetilde{x}_{t+1} - \gamma \Delta \widetilde{c}_{t+1}} \middle| s' \right) v_i(s') \right) \\ &\quad+\; \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0 + \alpha_1 d_i} \left(\sum_{s' \in \{LL,LH,HL,HH\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1}} \middle| s' \right) \left[1 - \pi_i(s') \right] \right) , \end{aligned} \tag{8}$$

where $\pi_i(s') \equiv \mathbb{E}\left(\widetilde{\varepsilon}_{i,t+1} \middle| s'\right) = \mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1} = 1 \middle| s'\right)$. Moreover, the expected one period gross return of *firm i is given by*

$$\mathbb{E}\left(\widetilde{R}_{i,t+1}\big|s\right) = \frac{1}{v_i(s)}\left(\sum_{s'\in\{LL,LH,HL,HH\}}\omega_{s,s'}\left\{v_i(s')\mathbb{E}\left(e^{\widetilde{x}_{t+1}}\big|s'\right)+e^{\alpha_0+\alpha_1 d_i}\left(1-\pi_i(s')\right)\right\}\right). \quad (9)$$

To appreciate the importance of the location of firm i in $\mathcal{G}_n$ on equilibrium asset prices and expected returns, suppose all firms have the same number of direct relationships. Then, $d_i = \bar{d}$ and $\pi_i = \bar{\pi} \geq q$ for all i. It follows from the second term in the right hand side of (8) that all firms have the same ex-dividend stock price. As equation (8) shows, differences in prices across firms arise solely from differences in the location of firms in $\mathcal{G}_n$. In particular, differences in prices across firms are driven not only by the number of direct relationships of a firm, captured by d_i, but also by the set of firms to which a firm is connected, captured by π_i. The same applies for the cross section of expected returns. Differences in expected returns across firms arise solely from differences across firms' locations. To understand the cross section of firms' risk premiums, equation (4) can be rewritten as a beta pricing model,

$$\mathbb{E}\left(\widetilde{R}_{i,t+1}\big|s\right) - R_f(s) = \underbrace{\left(\frac{\mathrm{Cov}\left(\widetilde{R}_{i,t+1},\widetilde{M}_{t+1}\big|s\right)}{\mathrm{Var}\left(\widetilde{M}_{t+1}\big|s\right)}\right)}_{\beta_{i,\widetilde{M}}(s)}\underbrace{\left(\frac{-\mathrm{Var}\left(\widetilde{M}_{t+1}\big|s\right)}{\mathbb{E}\left(\widetilde{M}_{t+1}\big|s\right)}\right)}_{\lambda_{\widetilde{M}}(s)}, \quad (10)$$

where $\beta_{i,\widetilde{M}}(s)$ and $\lambda_{\widetilde{M}}(s)$ denote firm i's quantity of risk and the conditional price of risk in state s, respectively. The following proposition determines the conditional price of risk, $\lambda_{\widetilde{M}}(s)$.

PROPOSITION 4 (Conditional Price of Risk: $\lambda_{\widetilde{M}}(s)$): *The conditional price of risk in state s, $\lambda_{\widetilde{M}}(s)$, equals*

$$\lambda_{\widetilde{M}}(s) = \frac{1}{R_f(s)} - R_f(s)\left(\beta^{2\left(\frac{1-\gamma}{1-\rho}\right)}\sum_{s'\in\{LL,LH,HL,HH\}}\omega_{s,s'}\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)}\mathbb{E}\left(e^{-2\gamma\Delta\widetilde{c}_{t+1}}\big|s'\right)\right), \quad (11)$$

where $R_f(s)$ denotes the period gross return of the risk-free asset in state s.

As equation (11) shows, the price of risk is time varying. Changes in ζ introduce changes in the cross-sectional distribution of $\{\widetilde{p}_{ij}\}_{(i,j)\in\mathcal{G}_n}$, which, in turn, generate changes in the distribution of aggregate consumption growth, in the price of aggregate wealth, and in the risk-free rate, all of which manifest in changes of the price of risk. To compute firms' quantities of risk, equation (10) can be rearranged as

$$\beta_{i,\widetilde{M}}(s) = \frac{\mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s\right) - R_f(s)}{\lambda_{\widetilde{M}}(s)} \tag{12}$$

so that $\beta_{i,\widetilde{M}}(s)$ can be computed from equations (7), (9), and (11).

IV. Calibration

So far, the model illustrates how changes in the propagation mechanism of shocks within a production network alters equilibrium asset prices and expected returns. I now calibrate the model to match several features of customer–supplier networks in the United States and explore its ability to replicate characteristics of asset returns. Section IV.A describes the data and the strategy employed to calibrate the network $\mathcal{G}_n$ as well as the dynamics of ζ. Section IV.B describes the selection of the rest of the parameters in the model.

A. Description of Data, Customer–Supplier Networks, and Dynamics of ζ_t

I use annual data on customer–supplier relationships among public U.S. firms to pin down the topology of $\mathcal{G}_n$. The Statement of Financial Accounting Standards (SFAS) No.131 requires firms to report the existence of customers who represent more than 10% of their annual sales. This information is available on the COMPUSTAT Segment files. However, these files tend to list only abbreviations of customers' names. I then use the Cohen and Frazzini (2008) database on customer–supplier relationships—a subset of the COMPUSTAT Segment database—in which firms' principal customers are uniquely identified. Their dataset consists of 6,425 different public firms, considers common stocks, and represents 26,781 unique annual customer–supplier relationships from 1980 to 2005. Customer–supplier relationships last about three years on average. The distribution of firms' sizes resembles the size distribution of the CRSP universe over the sample period, but the size distribution of firms' principal customers is tilted toward large companies, as firms are only required to report customers that represent more than 10% of their annual sales. Table II reports the distribution of firms across major industry groups for which monthly return data are available from 1980 to 2004. As table II shows, almost 70% of companies in the dataset are either manufacturing or service firms.[19]

Using the Cohen and Frazzini (2008) database, I construct networks at an annual frequency over the sample period. For the network of year t, nodes represent public firms that report customer–supplier agreements at year t and links represent customer–supplier relationships during that year. The idea is to select a topology for the benchmark economy that matches several features of the time series of U.S. customer–supplier networks. For illustration, figures 4, 5, and 6 depict the U.S. customer–supplier networks from 1980 to 1997. As these figures show, U.S. customer–supplier networks are highly asymmetric in the sense that only a few firms are connected to many others, while most firms have either one or at most two connections. In fact, the degree distributions of these networks, which measure the frequency of firms with a given number of direct relationships, are highly right-skewed and their upper tails can be approximated via power law distributions. Table III shows averages and standard deviations for key characteristics of the time series of U.S. customer–supplier networks. As Table III shows, the high asymmetry of U.S. customer–supplier networks is fairly persistent over the sample period, as measured by the ratio

$$\frac{\text{SD(exponent of power law distribution fitted to degree distribution)}}{\text{Mean(exponent of power law distribution fitted to degree distribution)}} = 0.07.$$

I select the topology of the benchmark economy by manually constructing a network whose topology simultaneously matches several of the averages reported in Table III. In particular, the topology of the benchmark economy matches the average number of firms, and the average empirical degree distribution of the time series of U.S. customer–supplier networks. The selected topology also captures the clustering pattern exhibited by these networks. Using two consecutive depth-first searches, I compute the size of the five largest connected components in each customer–supplier network. A connected component is a subset of the network in which any two firms are connected to each other by sequences of relationships, and which is connected to no additional firms in the network. The selected topology matches the average size of each of the five largest connected

[19] This data is available at http://www.econ.yale.edu/∼ af227/. According to the U.S. Department of Labor, manufacturing firms (division D) include companies engaged in mechanical or chemical transformation of material or substances into new products, whereas service firms (division I) are companies that usually provide a wide variety of services for individuals, businesses, and governments, such as hotels, automobile repair, and health services.

components over the sample period.[20] I also restrict the topology of $\mathcal{G}_n$ to have no cycles so that firms' probabilities of facing negative shocks in each state of the economy—expressions $\{\pi_i(s)\}_{i=1}^n$ in equations (8) and (9)—are easy to compute.[21] This restriction seems to be innocuous, because cycles are not frequent in the dataset. Figure 7 depicts the degree distribution of the benchmark production network.

To pin down the parameters that define the dynamics of ζ, I need proxies for the cross-sectional distributions of propensities of relationships to transmit shocks. However, the propensity of the relationship between firm i and j at year t, $\widetilde{p}_{ijt}$, is unobservable. In the context of supply chains, nonetheless, evidence documented by Barrot and Sauvagnat (2016) suggest that $\widetilde{p}_{ijt}$ is related with firms i and j's input specificities. As firms' input specificities are likely to depend on the percentage of sales that a customer represents for its supplier as well as firms' industry concentration, I proxy for $\widetilde{p}_{ijt}$ as

$$\widetilde{p}_{ijt} \approx \text{\% of sales that } j \text{ represents for } i \text{ at } t \times \text{Concentration score in } j\text{'s industry at } t. \quad (13)$$

Thus, the higher the percentage customer j represents for supplier i, the higher the likelihood that shocks affecting j also affect i. In addition, if customer j's industry is highly concentrated then supplier i is likely to face problems finding another customer in case j faces distress, other things being equal. I obtain the percentage of sales that a customer represents for its supplier at a given year from the Cohen and Frazzini (2008)'s dataset, and industry concentration scores from the Hoberg and Phillips (2010)'s fitted SIC-based concentration data.

For illustration, figures 8, 9 and 10 depict the time series of cross-sectional distributions of propensities from 1980 to 1997; namely, $\left\{\{\widetilde{p}_{ijt}\}_{(i,j)\in\mathcal{G}_n}\right\}_{t=1980}^{1997}$. To determine the parameters that define the dynamics of ζ, I fit a Beta distribution to each cross-sectional distribution of propensities. The fitted Beta distributions are depicted with dots in figures 8, 9 and 10. From this procedure, I obtain a time series of estimates for ζ, $\{\zeta_t^*\}_{t=1980}^{2005}$, which are depicted in figure 11. I then fit a vector autoregressive (VAR) process to the time series of estimates $\{\zeta_t^*\}_{t=1980}^{2005}$. After doing so, I discretize the fitted VAR into a four-states Markov chain using Gospodinov and Lkhagvasuren

[20] The construction of the network emulates a variation of a preferential attachment model and it is similar to a static version of the model proposed by Atalay et al. (2011).

[21] A cycle consists of a sequence of firm relationships starting and ending at the same firm, with each pair of consecutive firms in the sequence directly connected to each other in the network.

(2014)'s method and obtain $\zeta^*_{1\text{L}} = 0.90$, $\zeta^*_{1\text{H}} = 1.19$, $\zeta^*_{2\text{L}} = 52.41$, $\zeta^*_{2\text{H}} = 72.70$, and

$$\Omega^* = \begin{bmatrix} 0.61 & 0.16 & 0.16 & 0.04 \\ 0.16 & 0.63 & 0.04 & 0.17 \\ 0.17 & 0.04 & 0.63 & 0.16 \\ 0.04 & 0.16 & 0.16 & 0.61 \end{bmatrix},$$

with a stationary distribution given by $\mathbb{P}(\zeta^*_{\text{LH}}) = \mathbb{P}(\zeta^*_{\text{HL}}) = \mathbb{P}(\zeta^*_{\text{LL}}) = \mathbb{P}(\zeta^*_{\text{HH}}) = 0.25$.

Figures 12, 13 and 14 depict the centrality of a relationship as a function of its propensity to transmit shocks for each customer–supplier network from 1980 to 1997. As these figures suggest, there is a slight negative relationship between the propensity of relationships to transmit shocks and relationships centrality, as the relationships of peripheral firms tend to exhibit higher propensities than the relationships of central firms.

It is important to note one important caveat regarding the selection of the network topology using this database. Because firms need to be sufficiently large to be publicly traded and to represent at least 10% of the annual sales of a publicly traded company, many U.S. firms and their relationships are overlooked. As a consequence, one may be able to construct, in the most favorable case, a network that resembles a small part of the aggregate U.S. economy. To partially ensure that the topology of the benchmark economy provides a fair representation of the network that underlies the aggregate U.S. economy, I compare the topology of the benchmark economy with the topology of networks constructed from BEA input–output tables. As table VII shows, the network in the benchmark economy does a good job representing some features of the U.S. input–output network and, in doing so, potentially provides a reasonable representation of the aggregate U.S. economy.[22]

[22] It is an empirical issue whether a network uncovered using BEA input–output tables provides a sensible representation of the network structure that underlies the U.S. economy—I leave this for future research. Another way to uncover the underlying network using the framework in this paper is to use probabilistic graphical models—which are commonly used to represent statistical relationships in large and complex systems—as my baseline model predicts certain behavior of return covariances across stocks. For instance, one may calibrate the network using a graphical lasso estimator (GLASSO) to match observed return covariances. In doing so, one estimates an undirected and temporally invariant network by estimating a sparse inverse covariance matrix using a lasso (L1) penalty as in Friedman, Hastie, and Tibshirani (2008). The basic estimation strategy assumes that observations have a multivariate Gaussian distribution with mean μ and covariance matrix Σ. If the ijth component of Σ^{-1} is zero, then variables i and j are conditionally independent, given the rest of the variables, which is graphically represented as the lack of an edge between variables i and j in $\mathcal{G}_n$. The normality assumption can be relaxed as in Liu et al. (2012).

B. *Selecting the rest of the parameter values*

For the sake of illustration, the rest of the parameters can be separated into three groups. Table IV reports the key parameter values in the calibrated model. Parameters in the first group define the preferences of the representative investor, which I select in line with Bansal and Yaron (2004). Thus, $\beta = 0.997$, $\gamma = 10$ and $\rho = 0.65$ (IES ≈ 1.5).

Parameters in the second group define the dynamics of firms' cash flows. I use annual data on earnings per share from COMPUSTAT to proxy for firms' cash flows. I restrict my focus to firms mentioned in the customer–supplier database, as the number of direct relationships of a firm is known only for such firms. To determine parameters α_0 and α_1, I run cross-sectional OLS regressions specified by equation (1) at an annual frequency. To run such cross-sectional regressions, I need to determine whether firm i faces a negative cash-flow shock in any given year. To do so, I exploit the temporal variation of firms' cash flows and run time series regressions at the firm level, correcting for the existence of time trends. In particular, I run the following n time series regressions,

$$\log\left(\frac{y_{i,t}}{Y_{t-1}}\right) = \beta_0 + \beta_1 * t + \epsilon_t, \tag{14}$$

and consider that firm i faces a negative shock at year t if $\log\left(\frac{y_{i,t}}{Y_{t-1}}\right)$ is below the value predicted by equation (14) for more than one standard deviation of the residuals computed from equation (14). This procedure allows me to potentially identify the years in which firms faced negative cash-flow shocks and compute annual estimates for α_0 and α_1, which are depicted in figure 16. I set $\alpha_0 = 0.27$ and $\alpha_1 = 0.05$, which correspond to the averages of annual estimates.[23] For simplicity, I set $\alpha_2 = 0.0626$ and $q = 0.129$ so that the unconditional mean and volatility of dividend growth generated by the calibrated model are similar to the ones found in the data.[24] Appendix C describes the method used to simulate the model.

Parameters in the third group define the difference between aggregate output and consumption

[23]Estimates of α_0 and α_1 are statistically significant for most of the years in the sample.

[24]To determine the benchmark values of α_0 and α_1 at a monthly frequency, I assume that $y_{i,\text{year}} = 12 \times y_{i,\text{month}}$, with $i \in \{1, \cdots, n\}$. Provided that the data on firms' cash flows are at an annual frequency, this assumption facilitates the computation of parameters α_0 and α_1 at a monthly frequency because $Y_{\text{year}} = 12 \times Y_{\text{month}}$ so that $\log\left(\frac{y_{i,\text{year}+1}}{Y_{\text{year}}}\right) = \log\left(\frac{y_{i,\text{month}+1}}{Y_{\text{month}}}\right)$.

growth. Within the baseline model, output growth equals consumption growth at equilibrium. To provide a more realistic description of dividends and improve the fit of the calibrated model to data, I augment the baseline model so that consumption and dividends are two different processes. Similar to many others, including Cecchetti, Lam, and Mark (1993), Abel (1999), Campbell (1999, 2003), and Bansal and Yaron (2004), I assume that dividend and consumption growth jointly satisfy,

$$\widetilde{x}_{t+1} = \bar{\mu} + \tau \Delta \widetilde{c}_{t+1} + \sigma_x \widetilde{\xi}_{t+1}. \tag{15}$$

Parameters $\bar{\mu}$ and τ are constant and $\widetilde{\xi}_{t+1}$ is an i.i.d. normal with zero mean and unit variance. Thus, the representative investor is implicitly assumed to have access to labor income in the augmented model. For simplicity, $\widetilde{\xi}_{t+1}$ is independent of both $\Delta \widetilde{c}_{t+1}$ and variables $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$. As in Abel (1999), parameter τ represents the leverage ratio on equity. If $\bar{\mu} = \sigma_x = 0$ and $\tau = 1$, then the market portfolio is a claim to total wealth and the baseline model is recovered. I follow Bansal and Yaron (2004) and set $\tau = 3$. I set $\bar{\mu} = -0.019/12$ so that the difference between unconditional means of consumption and dividend growth generated by the calibrated model is similar to the one found in the data. Finally, I set $\sigma_x = 0.05/\sqrt{12}$ so that the difference between unconditional volatilities of dividend and consumption growth generated by the calibrated model is similar to the one found in the data.[25]

V. Implications of the Calibrated Model

This section quantitatively evaluates the ability of the calibrated model to rationalize dynamics of stock returns. It shows that changes in the propagation of shocks within production networks that resemble U.S. customer–supplier networks are quantitatively important to understanding variations in stock returns in both the aggregate and the cross section. Section V.A shows that the model generates long-run risks in consumption, high and volatile risk premia and a low and stable risk-free rate. Section V.B shows that the model generates a realistic return spread between central and peripheral firms in the network, which cannot be accounted for by standard asset pricing

[25] Despite the fact that aggregate output and consumption are two different processes within the augmented model, both of these processes are still determined by the propagation of shocks within the network economy. In particular, the distribution of $\widetilde{x}_{t+1}$ is fully determined by the propagation of shocks, as equation (3) shows, whereas the distribution of $\Delta \widetilde{c}_{t+1}$ is also determined by the propagation of shocks, as equation (15) states.

models. Section V.C shows that the model also generates a high degree of common time variation in firm-level return volatilities, which is aligned with recent empirical evidence.

A. Customer–Supplier Networks and Long-Run Risks

Table V exhibits moments generated under the benchmark parameterization. By construction, the benchmark parameterization delivers annual averages and volatilities of consumption and dividend growth similar to those found in the data. It also delivers an average market return of 12%, an annual volatility of the market return of 18.92%, an average risk-free rate of 2.16%, an annual volatility of the risk-free rate of 0.7%, an annual equity premium of 10%, and an average Sharpe ratio of 0.52. With the exception of the volatility of the risk-free rate and Sharpe ratio, all values are aligned with those found in the data.

Besides matching the above moments, the calibrated model generates a persistent component in expected consumption growth and stochastic consumption volatility similar to those assumed by the long-run risks (LRR) model of Bansal and Yaron (2004). As Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) show, these two features, together with Epstein-Zin-Weil preferences, help to quantitatively explain an array of important asset market phenomena.[26] Table VI reports summary statistics of several similarity measures of time series generated with either the calibrated model or the LRR model. To compute averages and standard deviations of these similarity measures, I sample from the calibrated model and the LRR model to construct two distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$, and one for the conditional volatility of consumption growth, $\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$. Reported values are based on 300 simulated economies over 620 periods. The first 100 periods are disregarded to eliminate bias coming from the initial condition. As table VI suggests, both models generate similar time series for conditional expected consumption growth and conditional consumption volatility.

It is important to appreciate that the persistent component in expected consumption growth and stochastic consumption volatility are endogenously generated within my model rather than

[26]Since Bansal and Yaron (2004), several authors have used the long-run risk framework to explain an array of market phenomena. For instance, Kiku (2006) provides an explanation of the value premium within the long-run risks framework. Drechsler and Yaron (2011) show that a calibrated long-run risks model generates a variance premium with time variation and return predictability that is consistent with the data. Bansal and Shaliastovich (2013) develop a long-run risks model that accounts for bond return predictability and violations of uncovered interest parity in currency markets.

exogenously imposed, as in many asset pricing models. The calibrated model generates these two features because of two reasons: (1) inter-firm relationships are long-term, and (2) the four parameter vectors estimated from the data, ζ^*_{LL}, ζ^*_{LH}, ζ^*_{HL}, and ζ^*_{HH}, generate similar cross-sectional distributions of $\{\widetilde{p}_{ijt}\}_{(i,j)\in\mathcal{G}_n}$, as figure 15 shows. As a consequence, the propagation mechanism of shocks across firms is fairly stable over time.

While the model endogeneously generates long-run risk, it does not provides a complete micro-foundation of long-run risks because inter-firm relationships are exogenously determined. Nonetheless, the model provides a novel link between equilibrium asset prices and the propagation of firm level shocks within customer–supplier networks that is consistent with the existence of long-run risks. In doing so, the model provides a new perspective on the potential sources of long-run risks and suggests that small changes in the propagation mechanism of shocks in fairly sticky production networks are quantitatively relevant to understanding asset market phenomena.

B. Firms' Centrality and the Cross Section of Risk Premiums

Besides helping to explain aggregate asset market phenomena, the model helps to understand the cross section of expected returns because it provides a mapping between firms' quantities of priced risk and firms' importance in the network. To measure the importance of a firm in the network, I define the centrality of firm i at period t as the average number of firms that can be affected by a shock to firm i at period t. This measure captures the relative importance of firm i in transmitting shocks to other firms in the economy at period t. Provided that the cross-sectional distribution of $\{\widetilde{p}_{ij}\}_{(i,j)\in\mathcal{G}_n}$ changes over time, firms' centrality scores also change over time.

To quantitatively assess the effect of a firm's importance in the network on a firm's return–risk trade off, I simulate the benchmark economy at a monthly frequency and construct portfolios based on centrality. Firms are assigned into centrality terciles once per year, and the value-weighted portfolios are not rebalanced for the next 12 months. Using simulated data, a portfolio that is long in the low centrality tercile portfolio and short the high centrality tercile portfolio generates a statistically significant annual return of 3.8% (0.32% per month). Such a return is computed using 200 simulated economies over 1100 monthly observations. I disregard the first 100 observations in each simulation to eliminate the potential bias coming from the initial condition.

The above result is explained by the fact that relationships of peripheral firms tend to exhibit higher propensities to transmit shocks than relationships of central firms. As a consequence, peripheral firms tend to have higher exposure to negative shocks that affect their partners. Such a contagion risk overweights the potential benefits that peripheral firms receive from their few relationships and, thus, peripheral firms command higher risk premiums than central firms on average. However, central firms benefit from the diversification of their customers and suppliers because their relationships exhibit, on average, small propensities to transmit shocks and, thus, their contagion risk is overweighted by the benefits generated by their many relationships.

Table VIII shows that the calibrated model generates a realistic spread between low and high centrality portfolios as the average annual return difference between low and high centrality portfolios in the Cohen and Frazzini (2008) database is 4.1% (0.34% per month). Table VIII reports monthly average returns, alphas—from the CAPM, Fama and French (1993) three–factor model, Carhart (1997) four–factor model, and Fama and French (2015) five–factor model—and loadings from the five–factor model of Fama and French (2015) for three portfolios of stocks sorted by annual centrality as well as the portfolio that is long the low centrality tercile and short the high centrality tercile. Firms are assigned into centrality terciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample is from June 1981 to December 2004. The last five columns report betas for the five–factor model of each centrality tercile.

As table VIII suggests, there is a strong negative relation in the data between firms' centrality and future returns that cannot be captured by commonly used asset pricing models. Firms in the low centrality tercile command an average monthly return of 2.06%, whereas firms in the high centrality tercile command an average monthly return of 1.71%. The 0.34% monthly difference in returns between these two portfolios is economically and statistically significant and appears naturally in an equilibrium context, like the one illustrated in this paper, as a compensation for contagion risk.

To better understand the sources of the difference in future returns between firms with different centrality scores, I focus on manufacturing and service firms because they jointly represent almost 70% of the firms in the dataset. Table IX considers only manufacturing firms, table X considers only service firms, and table XI considers both manufacturing and service firms. As in table VIII, tables IX, X, and XI report raw returns, alphas, and betas of each centrality tercile and the portfolio that is long the low centrality tercile and short the high centrality tercile. They also report alphas from the CAPM, and the three–, four– and five–factor models. Consistent with empirical evidence documented by Wu and Birge (2014), these tables suggest that central firms in manufacturing and service industries may benefit from the diversification of customers and suppliers. Such diversification helps central firms to mitigate contagion risk better than less central firms and, thus, command lower risk premiums.

C. Factor Structure on Firm-Level Return Volatility

The calibrated model also generates a high degree of common time variation in return volatilities at the firm level, which is aligned with recent empirical evidence, e.g., Herskovic et al. (2014) and, Duarte et al. (2014). To facilitate comparison with evidence documented by Herskovic et al. (2014), figure 17 illustrates the annual total return volatility at the firm-level averaged within start-of-year size quintiles. As figure 17 shows, firms of all sizes exhibit similar time series volatility patterns in the calibrated model. On average, the first principal component of the cross section of annual return volatility accounts for 99% of the variance. Within the model, the existence of this factor structure is not surprising, as fluctuations in the cross-sectional distribution of $\{\widetilde{p}_{ijt}\}_{(i,j)\,\in\,\mathcal{G}_n}$ drive changes in growth opportunities and uncertainty across firms, which translate into changes in prices and returns at the firm-level. Provided that stock returns respond to a common factor—given by the state of ζ—firm-level return volatilities inherit a factor structure.[27]

VI. Conclusion

This paper develops a dynamic equilibrium model to study the asset pricing properties that stem from the propagation of shocks across firms in a network economy. The fundamental insight is that extending standard asset pricing models to take into account the way that shocks propagate within fairly sticky production networks can make significant progress toward generating a unifying

[27]Recent empirical evidence also suggests the existence of common time variation in firm-level idiosyncratic volatilities, e.g., Herskovic et al. (2014) and Duarte et al. (2014). In unreported results, I explore the extent to which idiosyncratic volatilities exhibit a factor structure within the calibrated model. After removing the market as a common factor of return volatilities, the high degree of common time variation in firm-level return volatilities tends to disappear. On average, the first principal component of the cross section of annual idiosyncratic volatility accounts only for 3% of the variance (see figure 17(b)).

framework that simultaneously captures dynamics of the aggregate and the cross section of stock returns.

A calibrated model that matches key features of customer–supplier networks in the United States generates long-run risks, high and volatile risk premiums, and a low and stable risk-free rate. In the model, low-frequency changes in the shock propagation mechanism endogenously generate persistence in firms' growth prospects, which, in turn, drives a small and persistent component in expected aggregate consumption growth. With recursive preferences, sizeable risk premiums arise because investors fear that extended periods of low economic growth coincide with low asset prices. Similarly, a small risk-free rate is driven by investors saving for long periods of low economic growth.

The model also helps in understanding the cross section of expected returns, as it provides a mapping between firms' quantities of priced risk and firms' importance in the network. In the calibrated economy, firms that are more central in the network command lower risk premiums than firms that are less central: central firms tend to benefit from the diversification of their customers and suppliers and, thus, they mitigate contagion risk better than less central firms. In the time series, firm-level return volatilities exhibit a high degree of co-movement. These two predictions are consistent with data from firms in manufacturing and service industries and recent empirical evidence but cannot be accounted for by standard asset pricing models.

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Appendix A: Asymptotic Normality of Consumption Growth

To fix notation, let $\mathcal{G}_{n+1}$ denote the network $\mathcal{G}_n$, to which I add one new firm and all the relationships the entrant firm may create with incumbent firms in $\mathcal{G}_n$. The following proposition imposes sufficient conditions on: (a) the sequence of production networks $\{\mathcal{G}_n\}_{n\geq 1}$ and (b) the values within the propensity matrix, $\widetilde{p}_{t+1}$, so that $\Delta\widetilde{c}_{t+1}$ is normally distributed as n grows large.

PROPOSITION 5 (Asymptotic Normality of $\Delta\widetilde{c}_{t+1}$): *Given* $0 < q < 1$ *and a sequence of production networks,* $\{\mathcal{G}_n\}_{n\geq 1}$, *define the threshold probability* p_c^q *and the set* $\mathcal{C}_n$ *as*

$$
\begin{aligned}
p_c^q &\equiv \sup_{p\in(0,1)} \Big\{ p : \textit{If every relationship in } \mathcal{G}_n \textit{ has propensity } p, \textit{ then } \lim_{n\to\infty} P_q^{\alpha}(\mathcal{G}_n) = 0 \Big\} \\
\mathcal{C}_n &\equiv \{ G \textit{ is a connected component of } \mathcal{G}_n : \textit{Number of nodes in } G = O(n) \}
\end{aligned}
$$

where $P_q^{\alpha}(\mathcal{G}_n)$ *denotes the probability that a shock to any given firm in* $\mathcal{G}_n$ *affects at least* αn *firms via shock propagation, with* $\alpha > 0$*. The graph* G *is said to be a connected component of* $\mathcal{G}_n$ *if* G *is a subset of* $\mathcal{G}_n$ *in which any two firms are connected to each other by sequences of relationships and which is connected to no additional firms in* $\mathcal{G}_n$*. Notation* $x = O(n)$ *indicates that* x *grows, at most linearly, with* n*. If*

$$
\lim_{n\to\infty} \left\{ \max_{(i,j)\,\in\,\mathcal{C}_n} \widetilde{p}_{ij,t+1} \right\} \quad < \quad p_c^q,
$$

then $\sqrt{n}\widetilde{W}_{n,t+1}$ *and* $\Delta\widetilde{c}_{t+1}$ *are normally distributed as* n *grows large.*

Under the conditions of Proposition 5, the distribution of $\Delta\widetilde{c}_{t+1}$ can be characterized in terms of its mean and variance. Because the network topology is fixed, the dynamics of the mean and variance of consumption growth are fully determined by the dynamics of the propensity matrix $\widetilde{p}_{t+1}$. Provided that the dynamic of $\widetilde{p}_{t+1}$ is determined by ζ_{t+1}, the economy follows a Markov process with a continuum of values for aggregate consumption and its growth rate, $\Delta\widetilde{c}_{t+1}$, but only four values for the first two moments of the distribution of consumption growth.

The following corollaries provide a more detailed characterization of those large network economies in which consumption growth is normally distributed. Corollary 1 focuses on large networks in which all firms have the same number of direct relationships.

COROLLARY 1 (Symmetric Production Networks): *Suppose* $\widetilde{p}_{ij,t+1} = p_{t+1} > 0$, $\forall (i,j) \in \mathcal{G}_n$. *Given a sequence of production networks,* $\{\mathcal{G}_n\}_{n\geq 1}$, *with limiting topology* $\mathcal{G}_\infty$,[28]

- $p_c^q = 1 - 2\sin\left(\frac{\pi}{18}\right) \approx 0.65$ *if* $\mathcal{G}_\infty$ *is the two-dimensional honeycomb lattice.*
- $p_c^q = \frac{1}{2}$ *if* $\mathcal{G}_\infty$ *is the two-dimensional square lattice.*
- $p_c^q = 2\sin\left(\frac{\pi}{18}\right) \approx 0.34$ *if* $\mathcal{G}_\infty$ *is the two-dimensional triangular lattice.*
- $p_c^q = \frac{1}{z-1}$ *if* $\mathcal{G}_\infty$ *is the Bethe lattice with* z *neighbors per each firm.*

Figure 3 illustrates each of the network economies considered in corollary 1. Corollary 2 focuses on large networks in which the number of direct relationships differs across firms.

COROLLARY 2 (Asymmetric Production Networks): *Suppose* $\widetilde{p}_{ij,t+1} = p_{t+1} > 0$, $\forall (i,j) \in \mathcal{G}_n$. *Given a sequence of production networks,* $\{\mathcal{G}_n\}_{n\geq 1}$,

- $p_c^q = \frac{1}{\textit{branching number of } \mathcal{G}_\infty}$ *if* $\mathcal{G}_\infty$ *is a tree. The branching number of a tree is the average number of relationships per firm in a tree.*[29]
- $p_c^q = \frac{1}{e_M}$ *if* $\mathcal{G}_n$ *is sparse and locally treelike.* $\mathcal{G}_n$ *is said to be sparse if the number of relationships in* $\mathcal{G}_n$ *increases linearly with* n, *as* n *increases.* $\mathcal{G}_n$ *is said to be locally treelike if an arbitrarily large neighborhood around any given firm takes the form of a tree. For finite* n, *parameter* e_M *is the leading eigenvalue of the matrix*

$$M_n = \begin{pmatrix} A_n & \mathbb{I}_n - D_n \\ \mathbb{I}_n & 0 \end{pmatrix} \tag{1}$$

where A_n *is the adjacency matrix of* $\mathcal{G}_n$, *i.e. the* $n \times n$ *matrix in which* $A_{ij} = 1$ *if there is a direct relationship between firms* i *and* j *and zero otherwise.* $\mathbb{I}_n$ *is the* $n \times n$ *identity matrix, and* D_n *is the diagonal matrix that contains the number of relationships per firm along the diagonal.*

Appendix B: Proofs

This section contains the proofs of the propositions and corollaries in the body of the paper and Appendix A. The following computations consider three assumptions:

ASSUMPTION 1: *Firm* i*'s output at period* $t+1$, $y_{i,t+1}$, *follows*

$$\log\left(\frac{y_{i,t+1}}{Y_t}\right) \equiv \alpha_0 + \alpha_1 d_i - \alpha_2 \sqrt{n}\widetilde{\varepsilon}_{i,t+1} , \tag{1}$$

where Y_t *denotes the aggregate output of the economy at* t *and* $\widetilde{\varepsilon}_{i,t+1}$ *denotes a Bernoulli random variable that equals*

[28] A lattice is a graph whose drawing can be embedded in $\mathbb{R}^n$. The two-dimensional honeycomb lattice is a graph in 2D that resembles a honeycomb. The two-dimensional square lattice is a graph that resembles the $\mathbb{Z}^2$ grid. The two-dimensional triangular lattice is a graph in 2D in which each node has six neighbors.

[29] A tree is a network in which any two nodes are connected by exactly one sequence of edges. A forest is a network whose components are trees.

one if firm i faces a negative shock at t+1 and zero otherwise. Parameter d_i denotes the number of direct relationships of firm i. Parameters α_0, α_1 and α_2 are non-negative real numbers.

ASSUMPTION 2: *The aggregate output of the economy at t, Y_t, is defined as*

$$Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}. \tag{2}$$

ASSUMPTION 3: *Let $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ be the log output growth rate of the economy at $t+1$ and let $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)$ be the log aggregate consumption growth rate at $t+1$. The processes for $\widetilde{x}_{t+1}$ and $\Delta\widetilde{c}_{t+1}$ jointly satisfy*

$$\widetilde{x}_{t+1} = \bar{\mu} + \tau\Delta\widetilde{c}_{t+1} + \sigma_x\widetilde{\xi}_{t+1}, \tag{3}$$

where $\bar{\mu}$ and τ are constants, $\sigma_x > 0$, and $\widetilde{\xi}_{t+1} \xrightarrow{d}$ i.i.d. $\mathcal{N}(0,1)$. Variable $\widetilde{\xi}_{t+1}$ is independent of $\Delta\widetilde{c}_{t+1}$ and all the variables in the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^n$.

Let s_t denote the state of the parameter vector ζ_t. Because the network topology is fixed, s_t determines the distributions of aggregate output and consumption growth at t. Provided that ζ_t follows a Markov process, the distributions of aggregate output and consumption growth vary over time and the dynamics of the moments of these distributions satisfy the Markov property.

Sketch of proof of Proposition 5 and Corollaries 1 and 2. Given a sequence of network topologies $\{\mathcal{G}_n\}_{n\geq 1}$ and the realization of the matrix $\widetilde{p}_t$, the goal is to find the conditions under which $\sqrt{n}\widetilde{W}_{n,t}$ is normally distributed as n grows large. Without loss of generality, fix period t so that subscript t on the sequence $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^n$ and on the matrix $\widetilde{p}_t$ can be eliminated. If the sequence of Bernoulli random variables $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is independent, the Lindeberg-Lévy central limit theorem implies that $\sqrt{n}\widetilde{W}_n$ is normally distributed as n grows large. Consequently, if $\widetilde{p}$ is a matrix of zeros then $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is a sequence of independent random variables and $\sqrt{n}\widetilde{W}_n$ is asymptotically normally distributed. In the presence of inter-firm relationships, however, some elements in the matrix $\widetilde{p}$ are different from zero. In particular, $\widetilde{\varepsilon}_i$ and $\widetilde{\varepsilon}_j$ are correlated if there exists a sequence of relationships between firms i and j in $\mathcal{G}_n$. In this case, $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is a sequence of dependent random variables and the conditions under which the Lindeberg-Lévy central limit theorem hold are not satisfied.

Despite the fact that the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ may be dependent, $\sqrt{n}\widetilde{W}_n$ may still be asymptotically normally distributed if the dependence among variables in $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is sufficiently weak. In particular, if the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is α-mixing and stationary, $\sqrt{n}\widetilde{W}_n$ follows a normal distribution as n grows large—see Billingsley (1995, Theorem 27.4). Generally speaking, the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is said to be α-mixing if $\widetilde{\varepsilon}_k$ and $\widetilde{\varepsilon}_{k+n}$ are approximately independent for large n and $k \geq 1$.[30] The sequence is said to be stationary if the distribution of the subsequence $\{\widetilde{\varepsilon}_l, \widetilde{\varepsilon}_{l+1}, \cdots, \widetilde{\varepsilon}_{l+j}\}$

[30] To be more specific, let α_n be a non-negative number such that

$$\left|\mathbb{P}(A\cap B) - \mathbb{P}(A)\mathbb{P}(B)\right| \leq \alpha_n \tag{4}$$

with $A \in \sigma(\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k)$, $B \in \sigma(\widetilde{\varepsilon}_{k+n}, \widetilde{\varepsilon}_{k+n+1}, \cdots)$, $k \geq 1$ and $n \geq 1$; where $\sigma(\cdot)$ denotes the σ-algebra defined on the power set of $\{0,1\}^n \equiv \{0,1\} \times \cdots \times \{0,1\}$. The sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is said to be α-mixing if $\alpha_n \to 0$ as n grows large.

does not depend on the subscript l. A special case of the above result occurs if there exists an ordering of the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ such that the dependence between variables $\widetilde{\varepsilon}_k$ and $\widetilde{\varepsilon}_j$ decreases as the distance between them in such an ordering increases. In particular, if there exists such an ordering and a positive constant m such that the subsequences $\{\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k\}$ and $\{\widetilde{\varepsilon}_{1,k+s}, \cdots, \widetilde{\varepsilon}_{k+s+l}\}$ are independent whenever $s > m$, the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is said to be m-dependent in which case $\sqrt{n}\widetilde{W}_n$ follows a normal distribution as n grows large.[31]

In what follows, I apply an idea similar to that behind the notion of m-dependent random variables to sketch the proof of the asymptotic normality of $\sqrt{n}W_n$. In particular, I impose sufficient conditions on the sequence of networks $\{\mathcal{G}_n\}_{n\geq 1}$ and on the matrix $\widetilde{p}$ so that negative shocks to individual firms tend to remain locally confined as n grows large. In case the shocks remain locally confined, shocks would almost surely spread only over sets of firms of finite size—with all sets independent among each other—whose size would become negligible compared to the size of the economy as n grows large. As a consequence, one would almost surely be able to find an index ordering $\mathcal{I}$ and a positive constant $\widehat{m}$ under which the sequence $\{\widetilde{\varepsilon}_i\}_{i\in\mathcal{I}}$ would be $\widehat{m}$-dependent and, thus, $\sqrt{n}W_n$ would follow a normal distribution as n grows large.

For the sake of illustration, suppose that $\forall\ (i,j) \in \mathcal{G}_n,\ p_{ij} = p_0 > 0$, with $i \neq j$. Given $0 < q < 1$ and a sequence of networks, $\{\mathcal{G}_n\}_{n\geq 1}$, define the threshold probability p_c^q as

$$p_c^q \quad \equiv \quad \sup_{p\in(0,1)} \left\{p : \lim_{n\to\infty} P_q^\alpha(\mathcal{G}_n) = 0\right\}, \tag{5}$$

where $P_q^\alpha(\mathcal{G}_n)$ denotes the probability that a shock to any individual firm in $\mathcal{G}_n$ at least affects other αn firms via shock propagation, with $\alpha > 0$. Determining the conditions under which a CLT applies to $\sqrt{n}W_n$ is related to determining the threshold probability p_c^q. If every relationship in $\mathcal{G}_n$ has a propensity p_0 and $p_0 < p_c^q$, then shocks would remain locally confined because the number of firms affected by the shock would become negligible compared to the size of the economy as n grows large. Then, one would almost surely be able to find an index ordering $\mathcal{I}$ and a positive constant $\widehat{m}$ under which the sequence $\{\widetilde{\varepsilon}_i\}_{i\in\mathcal{I}}$ would be $\widehat{m}$-dependent.

The condition $p_0 < p_c^q$ may be stronger than necessary to prove the asymptotic normality of $\sqrt{n}W_n$. Imposing such a condition, however, greatly facilitates the proof, as the determination of the threshold p_c^q has been extensively studied in percolation theory, e.g. Grimmett (1989) and Stauffer and Aharony (1994). In percolation, p_c^q is sometimes called the critical probability or critical phenomenon because it indicates the arrival of an infinite, connected component as n grows large. A connected component of a graph is a subgraph in which any two nodes are connected to each other by sequences of edges, and which is connected to no additional nodes in the original graph.

To illustrate how p_c^q can be determined, consider the following two examples:

- Imagine n firms are arranged in a straight line and each relationship may transmit shocks with probability p_0. The probability that every relationship in the line transmits shocks is p_0^{n-1}. Given how shocks spread from

[31] An independent sequence is 0-dependent using this terminology.

one firm to another, the probability that at least one negative shock spreads over $n-1$ different firms equals

$$\begin{aligned} & (1-(1-q)^n)\,\mathbb{P}\,[\text{every relationship in the line transmits shocks}] \\ \approx\ & (1-e^{-nq})p_0^{n-1} \quad (\text{for large } n) \end{aligned} \tag{6}$$

which tends toward zero as $n \to \infty$. Thus, $p_c^q = 1$.

- Suppose n firms are arranged in a circle. The probability that every relationships in the circle transmits shocks is p_0^n, which tends toward zero as $n \to \infty$. Following the previous argument, it is easy to see that $p_c^q = 1$.

Taking results from bond percolation, Table I reports critical probabilities for several symmetric network topologies assuming that every relationship in the graph transmits shocks with the same probability. As Table I shows, p_c^q varies across network topologies. For instance, if the limiting topology of the sequence $\{\mathcal{G}_n\}_{n\geq 1}$, $\mathcal{G}_\infty$, is the two-dimensional honeycomb lattice, then $p_c^q = 1 - 2\sin\left(\frac{\pi}{18}\right) \approx 0.65$, whereas if $\mathcal{G}_\infty$ is the two-dimensional square lattice then $p_c^q = \frac{1}{2}$.

The previous analysis determines conditions under which $\sqrt{n}\widetilde{W}_n$ is normally distributed for some symmetric network topologies. But what happens in other networks? In particular, under what conditions is $\sqrt{n}\widetilde{W}_n$ asymptotically normally distributed in large asymmetric networks? Using random walks on trees, Lyons (1990) shows that if $\mathcal{G}_\infty$ is a tree then

$$p_c = \frac{1}{\text{branching number of } \mathcal{G}_\infty}, \tag{7}$$

where the branching number of a tree is the average number of branches per node in a tree.[32] A tree is a connected graph in which two given nodes are connected by exactly one sequence of edges. A tree is said to be z-regular if each node has degree z. If $\mathcal{G}_\infty$ is a z-regular tree, the average number of branches per node is $z-1$ so $p_c = \frac{1}{z-1}$; which is consistent with Table I.[33]

One can generalize the previous result for topologies where $\mathcal{G}_\infty$ is sparse and locally treelike. G_n is said to be sparse if G_n has m edges and $m = O(n)$. Notation $m = O(n)$ indicates that m grows, at most, linearly with n so there exists a positive number c such that $\left|\frac{m}{n}\right| < c$ for all n. Namely, G_n is sparse if only a small fraction of the possible $\frac{n(n-1)}{2}$ edges are present. G_∞ is said to be locally treelike if in the limit an arbitrarily large neighborhood around any node takes the form of a tree. Reformulating percolation in trees as a message passing process, Karrer,

[32] For a concrete definition of the branching number see Lyons (1990, page 935).

[33] To motivate the previous result, it is informative to compute the percolation threshold in the Bethe lattice with z neighbors per every node. Start at the root and check whether there is a chance of finding an infinite open path from the root. Starting from the root, one has $(z-1)$ new edges emanating from each new node in each layer of the lattice. Each of these $(z-1)$ new edges leads to one new node, which is affected with probability p. On average, $(z-1)p$ nodes are affected at each layer of the lattice. If $(z-1)p < 1$ then the average number of affected nodes decreases in each layer by a factor of $(z-1)p$. As a consequence, if $(z-1)p < 1$ the probability of finding an infinite open path goes to zero exponentially in the path length. Thus, $p_c = \frac{1}{z-1}$ for the Bethe lattice with z neighbors for every node.

Newman, and Zdeborová (2014) show that if $\mathcal{G}_\infty$ is sparse and locally treelike then

$$p_c \quad = \quad \frac{1}{\epsilon_H} \tag{8}$$

where ϵ_H is the leading eigenvalue of the $2n \times 2n$ matrix

$$M \quad = \quad \begin{pmatrix} A & \mathbb{I} - D \\ \mathbb{I} & 0 \end{pmatrix} \tag{9}$$

where A is the adjacency matrix that represents $\mathcal{G}_n$, $\mathbb{I}$ is the $n \times n$ identity matrix, and D is the diagonal matrix with the number of relationships per firm along the diagonal.

Now to allow for propensities to vary across relationships, one can extend the previous argument in the following way. Let $\mathcal{C}_n$ be the set of graphs composed by those connected components of $\mathcal{G}_n$ whose number of nodes grows, at most, linearly with n. Mathematically,

$$\mathcal{C}_n \quad \equiv \quad \{G \text{ is a connected component of } \mathcal{G}_n : \text{the number of nodes in } G = O(n)\} \tag{10}$$

Because in the limit one only needs to pay attention to components of G_n that potentially grow linearly with n, the following inequality

$$\lim_{n \to \infty} \left\{ \max_{(i,j) \,\in\, \mathcal{C}_n} \widetilde{p}_{ij} \right\} \quad < \quad p_c^q, \tag{11}$$

ensures that shocks to individual firms would remain locally confined provided that sets of firms of finite size become negligible as the economy grows large. As a consequence, if inequality (11) holds then $\sqrt{n}\widetilde{W}_n$ and $\Delta\widetilde{c}$ are normally distributed as n grows large.

□

Proof of Proposition 1. I look for an equilibrium such that the price dividend ratio is stationary. I conjecture that if c is the current aggregate consumption and s the current state of ζ_t, then $P_a(c,s) = w_s^a c$, in which P_a is the price of aggregate wealth and w_s^a a number that depends on state s. If $s_t = s$ and $s_{t+1} = s'$, the realized gross return at period $t+1$ of the asset that delivers aggregate consumption as its dividend each period, $\widetilde{R}_{a,t+1}$, equals

$$\widetilde{R}_{a,t+1} \quad = \quad \frac{\widetilde{P}_{a,t+1} + \widetilde{C}_{t+1}}{P_{a,t}} \quad = \quad \frac{w_{s'}^a + 1}{w_s^a} \frac{\widetilde{C}_{t+1}}{C_t} \tag{12}$$

Setting $\widetilde{R}_{i,t+1} = \widetilde{R}_{a,t+1}$ in equation (4) yields,

$$\begin{aligned} \mathbb{E}_t \left(\left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho}} \right) \quad &= \quad 1 \\ \Rightarrow \mathbb{E} \left(\left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\frac{w_{s'}^a + 1}{w_s^a} \frac{\widetilde{C}_{t+1}}{C_t} \right]^{\frac{1-\gamma}{1-\rho}} \middle| s \right) \quad &= \quad 1 \end{aligned} \tag{13}$$

Provided that s_t follows a Markov process, equation (13) can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{1-\gamma} \Big| s' \right) \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{1-\gamma}{1-\rho}} \right) = 1 \tag{14}$$

Reordering equation (14) yields,

$$w^a_s = \beta \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}} \Big| s' \right) (w^a_{s'}+1)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma}} \tag{15}$$

which completes the proof.

REMARK 1: *If $\sqrt{n}\widetilde{W}_{n,t+1}$ is normally distributed, then*

$$\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}} \Big| s \right) = \exp\left(\frac{(1-\gamma)(\alpha_0+\alpha_1\bar{d}-\alpha_2\mu_s-\bar{\mu})}{\tau} + \frac{(1-\gamma)^2}{2}\left(\frac{\alpha_2^2\sigma_s^2-\sigma_x^2}{\tau^2} \right) \right) \tag{16}$$

where

$$\mu_s \equiv \lim_{n\to\infty} \mathbb{E}\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}} \Big| s \right) \quad and \quad \sigma_s^2 \equiv \lim_{n\to\infty} Var\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}} \Big| s \right)$$

and the above constants are assumed to be finite so that equation (16) is well-defined.

REMARK 2 (Price of Market Return): *If consumption and output growth differ, I compute the price of the market return as follows. I conjecture that if y is the current aggregate output and s the current state of ζ_t, then $P_m(c,s) = w^m_s y$, where P_m is the price of the market portfolio and w^m_s a number that depends on state s. If $s_t = s$ and $s_{t+1} = s'$, then the realized gross return at period $t+1$ of the asset that delivers aggregate output as its dividend each period, $\widetilde{R}_{m,t+1}$, equals*

$$\widetilde{R}_{m,t+1} = \frac{\widetilde{P}_{m,t+1}+Y_{t+1}}{P_{m,t}} = \frac{w^m_{s'}+1}{w^m_s}\frac{Y_{t+1}}{Y_t} \tag{17}$$

Setting $\widetilde{R}_{i,t+1} = \widetilde{R}_{m,t+1}$ in equation (4) yields,

$$\begin{aligned} \mathbb{E}_t\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho}-1} \widetilde{R}_{m,t+1} \right) &= 1 \\ \Rightarrow \mathbb{E}\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\frac{w^a_{s'}+1}{w^a_s}\left(\frac{\widetilde{C}_{t+1}}{C_t} \right) \right]^{\frac{1-\gamma}{1-\rho}-1} \left(\frac{w^m_{s'}+1}{w^m_s}\widetilde{X}_{t+1} \right) \Big| s \right) &= 1 \end{aligned} \tag{18}$$

where $\widetilde{X}_{t+1} = \frac{Y_{t+1}}{Y_t}$. Provided that s_t follows a Markov process, equation (18) can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'=\{LL,LH,HL,HH\}} \omega_{s,s'} \mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\gamma} \widetilde{X}_{t+1} \Big| s' \right) \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{1-\gamma}{1-\rho}-1} \left(\frac{w^m_{s'}+1}{w^m_s} \right) \right) = 1 \tag{19}$$

Reordering equation (19) yields,

$$w_s^m \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \{LL,LH,HL,HH\}} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1} + \widetilde{x}_{t+1}} \middle| s' \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{1-\gamma}{1-\rho} - 1} (w_{s'}^m + 1) \right) \tag{20}$$

It follows from (3) that $-\gamma \Delta \widetilde{c}_{t+1} + \widetilde{x}_{t+1} = \bar{\mu} + (\tau - \gamma)\Delta \widetilde{c}_{t+1} + \sigma_x \widetilde{\xi}_{t+1}$*. Therefore, (20) equals to*

$$w_s^m \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{\mu} + \frac{\sigma_x^2}{2}} \left(\sum_{s' \in \{LL,LH,HL,HH\}} \omega_{s,s'} \mathbb{E}\left(e^{(\tau-\gamma)\Delta \widetilde{c}_{t+1}} \middle| s' \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{1-\gamma}{1-\rho} - 1} (w_{s'}^m + 1) \right) \tag{21}$$

□

Proof of Proposition 2. Setting $\widetilde{R}_{i,t+1} = R_f$ in equation (4) yields,

$$\mathbb{E}\left(\left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho} - 1} \middle| s \right) \quad = \quad \frac{1}{R_f(s)}. \tag{22}$$

Provided that s_t follows a Markov process and $P_a(c, s) = w_s^a c$, the left hand side of equation (22) can be rewritten as the following sum

$$\beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\gamma} \middle| s' \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right)$$

Therefore,

$$\frac{1}{R_f(s)} \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1}} \middle| s' \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right),$$

which completes the proof □

Proof of Proposition 3. Consider $s_t = s$ and $s_{t+1} = s'$. Equation (4) can be rewritten as,

$$P_{i,t} \quad = \quad \mathbb{E}_t \left(\widetilde{M}_{t+1} \left(\widetilde{P}_{i,t+1} + y_{i,t+1} \right) \right) \qquad i = 1, \cdots, n \tag{23}$$

where

$$\widetilde{M}_{t+1} \quad \equiv \quad \left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho} - 1}$$

represents the pricing kernel. Dividing equation (23) by Y_t yields

$$\frac{P_{i,t}}{Y_t} \quad = \quad \mathbb{E}_t \left(\widetilde{M}_{t+1} \widetilde{X}_{t+1} \frac{\widetilde{P}_{i,t+1}}{Y_{t+1}} \right) + \mathbb{E}_t \left(\widetilde{M}_{t+1} \frac{y_{i,t+1}}{Y_t} \right) \qquad i = 1, \cdots, n \tag{24}$$

which can be rewritten as

$$v_{i,t} \quad = \quad \mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{X}_{t+1}v_{i,t+1}\right) + \mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \qquad i = 1, \cdots, n \tag{25}$$

with $v_{i,t} \equiv v_i(s) \equiv \frac{P_{i,t}}{Y_t}$. Provided that s_t follows a Markov process and $P_a(c,s) = w_s^a c$, the first term in the right hand side of equation (25) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{X}_{t+1}v_{i,t+1}\right) \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{\mu}+\frac{\sigma_x^2}{2}} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}}\middle|s'\right) v_i(s')\right) \tag{26}$$

whereas the second term in the right hand side of equation (25) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \quad = \quad e^{\alpha_0+\alpha_1 d_i}\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\right) \tag{27}$$

The expectation term in the right hand side of equation (27) can be written as

$$\begin{aligned}
\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\right) &= \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=LL,LH,HL,HH}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}e^{-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\middle|s'\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right)\\
&= \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=LL,LH,HL,HH}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\middle|s'\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right)
\end{aligned}$$

As a consequence,

$$\begin{aligned}
v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{\mu}+\frac{\sigma_x^2}{2}} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}}\middle|s'\right) v_i(s')\right)\\
&+ \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0+\alpha_1 d_i}\left(\sum_{s'=LL,LH,HL,HH}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\middle|s'\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right) \qquad i = 1, \cdots, n
\end{aligned}$$

Define $\pi_i(s') \equiv \mathbb{E}\left[\widetilde{\varepsilon}_{i,t+1}|s_{t+1} = s'\right]$. It is worth noting that

$$\begin{aligned}
-\gamma\Delta\widetilde{c}_{t+1} - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1} &= -\gamma\underbrace{\left(\frac{1}{\tau}\left\{\alpha_0+\alpha_1\bar{d}-\alpha_2\left(\sum_{j\neq i}\frac{\widetilde{\varepsilon}_{j,t+1}}{\sqrt{n}}\right)-\sigma_x\widetilde{\xi}_{t+1}-\bar{\mu}\right\}\right)}_{\Delta\widetilde{c}_{-i,t+1}} - \alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1}\\
&= -\gamma \quad \Delta\widetilde{c}_{-i,t+1} - \alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1}
\end{aligned} \tag{28}$$

Because $\Delta\widetilde{c}_{-i,t+1}$ and $\widetilde{\varepsilon}_{i,t+1}$ are independent

$$\begin{aligned}
\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{-i,t+1}-\alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1}}\middle|s\right) &= \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{-i,t+1}}\middle|s\right)\left\{\pi_i(s)e^{-\alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)}+(1-\pi_i(s))\right\}\\
&\approx \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}}\middle|s\right)(1-\pi_i(s))
\end{aligned} \tag{29}$$

where the last approximation is accurate for large n. If the distribution of $\Delta\widetilde{c}_{t+1}$ is known, the expectation of

$-\gamma\Delta\widetilde{c}_{t+1} - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}$ can be approximated using equation (29). Therefore,

$$\begin{aligned} v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{\mu}+\frac{\sigma_x^2}{2}} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}} \big| p_{s'} \right) v_i(s') \right) \\ &+ \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0+\alpha_1 d_i} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}} \big| s \right) (1-\pi_i(s)) \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right) \qquad i = 1,\cdots,n \end{aligned}$$

which completes the proof ☐

Proof of Proposition 4. Recall that

$$\operatorname{Var}\left(\widetilde{M}_{t+1} \big| s \right) = \mathbb{E}\left(\widetilde{M}_{t+1}^2 \big| s \right) - \mathbb{E}^2\left(\widetilde{M}_{t+1} \big| s \right) \qquad (30)$$

The first term in the right hand side of equation (30) can be rewritten as

$$\mathbb{E}\left(\widetilde{M}_{t+1}^2 \big| s \right) = \beta^{2\left(\frac{1-\gamma}{1-\rho}\right)} \left(\sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-2\gamma} \bigg| s' \right) \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)} \right) \qquad (31)$$

Provided that $\lambda_m(s) \equiv -\frac{\operatorname{Var}\left(\widetilde{M}_{t+1}|s\right)}{\mathbb{E}\left(\widetilde{M}_{t+1}|s\right)}$ and $\mathbb{E}\left(\widetilde{M}_{t+1}|s\right) = \frac{1}{R_f(s)}$, it then follows from equation (30) that

$$\lambda_m(s) = \frac{1}{R_f(s)} - R_f(s) \left(\beta^{2\left(\frac{1-\gamma}{1-\rho}\right)} \sum_{s'=LL,LH,HL,HH} \omega_{s,s'} \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)} \mathbb{E}\left(e^{-2\gamma\Delta\widetilde{c}_{t+1}} \big| s' \right) \right)$$

which completes the proof ☐

Appendix C: Simulation of the Model

This section describes the algorithm I use to compute firms' probabilities of facing negative cash-flow shocks in each state of nature so one can compute asset prices and returns at the firm level using proposition 4. Let s_t denote the state of the parameter vector ζ_t at period t. To simplify the computation of probabilities $\{\pi_i(s_t)\}_{i=1}^n$, I restrict the topology of $\mathcal{G}_n$. In general topologies, computing $\{\pi_i(s_t)\}_{i=1}^n$ is difficult, because the number of states that need to be considered increases exponentially with n. In economies with no cycles, however, computing $\{\pi_i(s_t)\}_{i=1}^n$ is easier. In those economies, computing $\{\pi_i(s_t)\}_{i=1}^n$ can be framed as a recursive problem, as the following algorithm describes.

Algorithm *Firms' Probabilities (G_n, s_t, q)*

(∗ Description: Algorithm that computes firms' probabilities of facing negative shocks if G_n is a forest ∗)

Input: G_n (a forest), s_t, q.

Output: The set of probabilities of firms facing a negative cash-flow shock at time t, $\{\pi_i(s_t)\}_{i=1}^n$

1. **for** each firm $i \in G_n$

2. Determine the subgraph of G_n wherein firm i participates. Denote such a graph as T_i and label firm i as its root.[34]

3. **if** firm i has a no connections

4. **return** $\pi_i(s_t) = q$

5. **else** **return** Prob(i,T_i,s_t,q)

where Prob(i,T_i,s_t,q) corresponds to the following recursive program:

Algorithm *Prob(i,T_i,s_t,q)*

($*$ Description: Recursive algorithm that computes firm i's probability of facing a negative cash-flow shock $*$)

Input: A node i in $\mathcal{G}_n$, the tree T_i wherein node i is the root, s_t, and q.

Output: $\pi_i(s_t)$

1. Determine the set of children of node i in T_i, say $\mathcal{C}_i$.[35]

2. **if** $\mathcal{C}_i = \emptyset$

3. **return** $\pi_i(s_t) = q$

4. **else** **if** every node in $\mathcal{C}_i$ has no children

5. **return** $\pi_i(s_t) = q + (1-q)\left(1 - \mathbb{E}\left[\prod_{k\in\mathcal{C}_i}(1 - q\widetilde{p}_{ikt})\big|s_t\right]\right)$

6. **else** **return** $\pi_i(s_t) = q + (1-q)\left(1 - \mathbb{E}\left[\prod_{k\in\mathcal{C}_i}(1 - \widetilde{p}_{ikt}\text{Prob}(k, T_{i,k}, s_t, q))\right)\big|s_t\right]$ [36]

In economies with no cycles, it is also simple to compute the first two moments of the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ at $t+1$. Let μ_s, σ_s^2 denote the mean and variance of $\sqrt{n}\widetilde{W}_{n,t+1}$ if $s_{t+1} = s$, respectively. In other words,

$$\mu_s = \lim_{n\to\infty} \mathbb{E}\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\bigg| s\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi_i(s)}{\sqrt{n}} \qquad s = LL, LH, HL, HH. \tag{1}$$

and

$$\begin{aligned} \sigma_s^2 &= \lim_{n\to\infty} \text{Var}\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\bigg| s\right) \qquad (2) \\ &= \lim_{n\to\infty} \left\{\frac{1}{n}\sum_{i=1}^{n} \pi_i(s)\left(1-\pi_i(s)\right) + \frac{1}{n}\sum_{(i,j)\in\mathcal{G}_n} \text{Cov}\left(\widetilde{\varepsilon}_{i,t+1}, \widetilde{\varepsilon}_{j,t+1}\big| s_t = s\right)\right\} \qquad s = LL, LH, HL, HH. \end{aligned}$$

The second term in the equation above can be simplified further. If there exists a path between firms i and j after edges are removed at period $t+1$, then $\widetilde{\varepsilon}_{i,t+1} = \widetilde{\varepsilon}_{j,t+1}$. If there is no path between firms i and j in $\mathcal{G}_n$, variables $\widetilde{\varepsilon}_{i,t+1}$ and $\widetilde{\varepsilon}_{j,t+1}$ are independent. It then follows that

$$\begin{aligned} \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\bigg| \text{ there is a path between } i \text{ and } j\right] &= \text{Var}_t\left[\widetilde{\varepsilon}_{i,t}\right] + \mathbb{E}_t^2\left[\widetilde{\varepsilon}_{i,t}\right] = \pi_i(s)(1-\pi_i(s)) + \pi_i^2(s) \\ \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\bigg| \text{ there is no path between } i \text{ and } j\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\right]\mathbb{E}_t\left[\widetilde{\varepsilon}_{j,t}\right] = \pi_i(s)\pi_j(s). \end{aligned}$$

[34] Note that such a graph is a tree provided that G_n is a forest.

[35] In a rooted tree, the parent of a node is the node connected to it on the path to the root. Every node except the root has a unique parent. A child of a node v is a node of which v is the parent.

[36] Tree $T_{i,k}$ denotes the branch of tree T_i that starts at node k.

Hence,

$$\begin{aligned}\mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle|\text{ there is a path between } i \text{ and } j\right]\mathbb{P}\left[\text{there is a path between } i \text{ and } j \text{ at } t\right]\\ &+ \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle|\text{ there is no path between } i \text{ and } j\right]\mathbb{P}\left[\text{there is no path between } i \text{ and } j \text{ at } t\right]\\ &= \left(\pi_i(s)(1-\pi_i(s))+\pi_i^2(s)\right)\mathbf{P}_{ij}(s)+\pi_i(s)\pi_j(s)\left(1-\mathbf{P}_{ij}(s)\right),\end{aligned}$$

where $\mathbf{P}_{ij}(s) \equiv \mathbb{P}\left[\text{there is a path between } i \text{ and } j \text{ if } s_t = s\right]$. Thus,

$$\begin{aligned}\mathrm{Cov}_t\left[\widetilde{\varepsilon}_{i,t},\widetilde{\varepsilon}_{j,t}\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\right]-\mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\right]\mathbb{E}_t\left[\widetilde{\varepsilon}_{j,t}\right]\\ &= \left(\pi_i(s)(1-\pi_i(s))+\pi_i^2(s)\right)\mathbf{P}_{ij}(s)+\pi_i(s)\pi_j(s)\left(1-\mathbf{P}_{ij}(s)\right)-\pi_i(s)\pi_j(s)\\ &= \pi_i(s)\left(1-\pi_j(s)\right)\mathbf{P}_{ij}(s).\end{aligned}$$

Therefore,

$$\sigma_s^2 = \lim_{n\to\infty}\left\{\frac{1}{n}\sum_{i=1}^{n}\pi_i(s)\left(1-\pi_i(s)\right)+\frac{1}{n}\sum_{(i,j)\in\mathcal{G}_n}\pi_i(s)\left(1-\pi_j(s)\right)\mathbf{P}_{ij}(s)\right\} \qquad s = LL, LH, HL, HH.$$

To compute $\mathbf{P}_{ij}(s)$ I need to determine the set of paths that connect firms i and j on $\mathcal{G}_n$. If there is more than one path connecting firms i and j, computing $\mathbf{P}_{ij}(s)$ is difficult because shocks can be transmitted by any of those paths. On the other hand, if there is only one path connecting any two given firms, say firms i and j, computing $\mathbf{P}_{ij}(s)$ becomes easier because there is a unique path connecting firms i and j. It then becomes handy to restrict the topology of $\mathcal{G}_n$ so that it does not have cycles. The following remark describes $\mathbf{P}_{ij}(s)$ when $\mathcal{G}_n$ is a forest.

REMARK 3: *Suppose $\mathcal{G}_n$ is a forest; namely, there are no cycles. Then, every component of $\mathcal{G}_n$ is a tree. Provided that any two given firms are jointed by a unique path (in case such a path exists),*

$$\boldsymbol{P}_{ij}(s) = \begin{cases}\mathbb{E}\left[\prod_{(k,l)\in\mathcal{P}_{i,j}}\widetilde{p}_{kl}\middle|s\right] & \textit{where } \mathcal{P}_{i,j} \textit{ is the (unique) path between } i \textit{ and } j \textit{ in } \mathcal{G}_n\\ 0 & \textit{there is no path between } i \textit{ and } j.\end{cases} \tag{3}$$

Appendix D: Tables and Figures

This section contains the tables and figures mentioned in the paper and in the appendix.

Table I
Critical probability for different symmetric network topologies

The table reports critical probabilities for different symmetric network topologies. Besides reporting the two examples described in Appendix A, the table reproduces a subset of the values reported in Stauffer and Aharony (1994, Table 1). The first column reports the topology of $\mathcal{G}_\infty$. The second column reports the number of neighbors of any given node in $\mathcal{G}_\infty$. The third column reports the critical probability, $p_c(\mathcal{G}_\infty)$. Despite the fact that $\mathcal{G}_\infty$ may be highly connected, if $\max \widetilde{p}_t < p_c(\mathcal{G}_\infty)$, then no infinite component emerges as $n \to \infty$, and thus $\sqrt{n}\widetilde{W}_n$ is asymptotically normally distributed. For illustrative purposes, figure 3(a) depicts a 2D honeycomb lattice, figure 3(b) depicts a 2D squared lattice, figure 3(c) depicts a 2D triangular lattice, and figure 3(d) depitcs a Bethe lattice with $z = 3$. The Bethe lattice of degree z is defined as an infinite tree in which any node has degree z. For finite n such topologies are called Cayley Trees.

Topology of $\mathcal{G}_\infty$	**Number of neighbors**	$p_c(\mathcal{G}_\infty)$
Infinite line (1D lattice)	2	1
Infinite circle	2	1
2D honeycomb lattice	3	$1 - 2\sin\left(\frac{\pi}{18}\right)$
2D squared lattice	4	$\frac{1}{2}$
2D triangular lattice	6	$2\sin\left(\frac{\pi}{18}\right)$
Bethe lattice	z	$\frac{1}{z-1}$

Table II
Major Industry Groups in Customer–Supplier Database

The table reports the distribution of firms across major industry groups in the Cohen and Frazzini (2008) customer–supplier database for which monthly return data were available from 1980 to 2004. Major industry groups are defined by the first two numbers of SIC codes.

Industry	**Number of firms**
Agriculture, forestry, and fishing	14
Construction	32
Finance, insurance, and real estate	308
Manufacturing	2,639
Mining	337
Public Administration	7
Retail	240
Service	1,099
Transportation, communications, electric, gas, and sanitary	432
Wholesale	234
Total	5,342

Table III
Characteristics of Customer–Supplier Networks

The table reports characteristics of customer-supplier networks generated at an annual frequency using the Cohen and Frazzini (2008) dataset from 1980 to 2004. Two firms are connected in the network of year t if one of them represents at least 10% of the other firm's sales during that year. The number of components (clusters) in each network is computed via two consecutive depth-first searches. Provided that degree distributions exhibit fat tails, one can approximate them via power law distributions in at least the upper tail. Namely, the probability of a given degree d in the network of year t, $\mathbb{P}^t(d)$, can be expressed as $\mathbb{P}^t(d) = a_t d^{-\xi_t}$, where $a_t > 0$ and $\xi_t > 1$ are the parameters to be estimated. The last row shows the average and standard deviation of the maximum likelihood estimators for ξ_t over the sample period. Finally, the column benchmark reports the characteristics of the network in the benchmark economy.

Characteristic	**Mean**	**Standard Deviation**	**Benchmark**
Number of firms per customer–supplier network	1,038	415	1,038
Number of relationships per customer–supplier network	1,066	635	918
Number of components per network	174	25	120
Size of the largest component	482	382	482
Size of the second largest component	54	31	54
Size of the third largest component	19	17	19
Size of the fourth largest component	11	7	11
Size of the fifth largest component	9	5	9
Exponent of fitted power law to the degree distribution	2.36	0.18	2.36

Table IV
Benchmark Parameterization

The table reports the list of parameter values in the benchmark parameterization. I set $\bar{\mu} = -0.019/12$ so the difference between unconditional means of consumption and dividend growth generated by the benchmark economy are similar to the ones found in data. I follow Bansal and Yaron (2004), and I set $\tau = 3$. I set $\sigma_x = 0.05/\sqrt{12}$ so that the difference between unconditional means of consumption and dividend growth generated by the benchmark economy is similar to the ones found in data. I divide the rest of the parameter values into three groups. Parameters in the first group define the preferences of the representative investor: β represents the time discount factor, γ represents the coefficient of relative risk aversion for static gambles, and ρ represents the inverse of the inter-temporal elasticity of substitution. Parameters in the second group describe firms' cash-flows: α_0 measures the part of firms' cash-flows unrelated to inter-firm relationships, α_1 measures the marginal benefit a firm receives from each relationship, and α_2 measures the decrease in a firm's cash flow if a firm faces a negative shock. Given a network topology, parameters in the third group define the stochastic process that determines the propagation of shocks across firms. Parameter q measures how frequently firms face negative idiosyncratic shocks. The rest of parameters define the cross-sectional distribution from which propensities of relationships to transmit shocks are determined: ζ_{1L}, ζ_{1H}, ζ_{2L}, and ζ_{2H}.

Preferences			**Firms' Cash-flows**			**Propagation of shocks**				
β	γ	ρ	α_0	α_1	α_2	ζ_{1L}	ζ_{1H}	q	ζ_{2L}	ζ_{2H}
0.997	10	0.65	0.27	0.05	0.0626	0.90	1.19	0.129	52.4	72.7

Table V
Moments under the Benchmark Parameterization

The table reports the first two moments of consumption and dividend growth as well as a set of key asset pricing moments. Column **Data** reports moments found in data. Column **Model** reports moments generated under the benchmark parameterization described in Table IV. Column **BY2004** reports moments generated under the long-run risks model of Bansal and Yaron (2004). Data on consumption and dividends is obtained from Robert Shiller's website http://www.econ.yale.edu/ shiller/data.htm. Moments on the return on aggregate wealth, risk-free rate, equity premium, and Sharpe ratio are based on data from 1928 to 2014 and obtained from Aswath Damodaran's website: http://pages.stern.nyu.edu/∼adamodar/. The annual return on aggregate wealth is approximated by the annual return of the S&P 500. The return on the risk-free asset is approximated by the yield on three-month T-bills. All values are in percentage with the exception of average Sharpe ratios.

Moments	**Data**	**Model**	**BY2004**
Average annual log of consumption growth rate	1.9	1.9	1.8
Annual volatility of log consumption rate	3.5	3.5	2.8
Average annual log dividend growth rate	3.8	3.8	1.8
Annual volatility of the log dividend growth rate	11.63	11.7	12.3
Average annual market return (S&P 500)	11.53	12	7.2
Annual volatility of the market return	19	18.92	19.42
Average annual risk-free rate (3-month T-bill)	3.53	2.16	0.86
Annual volatility of risk-free rate	3	0.7	0.97
Average annual equity risk premium	8	10	6.33
Average annual Sharpe ratio	0.4	0.52	0.33

Table VI
Similarities between the calibrated model and the LRR model

The table reports averages and standard deviations of similarity measures between time series generated with either the calibrated model or the benchmark parameterization in the LRR model of Bansal and Yaron (2004). To compute averages and standard deviations, I sample from the calibrated model and the LRR model to construct two finite-sample empirical distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$, and one for the conditional volatility of consumption growth, $\text{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$. Reported values are based on 300 simulated samples over 620 periods. The first 100 periods in each sample are disregarded to eliminate bias coming from the initial condition. All similarity measures report scores computed as $\frac{1}{1+distance}$, where *distance* is defined according to each similarity measure. Let $\mathbf{X}_T = (X_1, \cdots, X_T)$ and $\mathbf{Y}_T = (Y_1, \cdots, Y_T)$ denote realizations from two time series, $X = \{X_t\}$ and $Y = \{Y_t\}$. The first and second similarity measures focus on the proximity between X and Y at specific points of time. The euclidean distance (ED) is defined as $\sqrt{\sum_{t=1}^{T}(X_t - Y_t)^2}$, whereas the dynamic time warping (DTW) distance is defined as $\min_r\left(\sum_{i=1}^{m}|X_{a_i} - Y_{b_i}|\right)$, where $r = ((X_{a_1}, Y_{b_1}), \cdots, (X_{a_m}, Y_{b_m}))$ is a sequence of m pairs that preserves the order of observations, i.e., $a_i < a_j$ and $b_i < b_j$ if $j > i$. DTW seeks to find a mapping such that the distance between X and Y is minimized. This way of computing distance allows two time series that are similar but locally out of phase to align in a nonlinear manner. The third measure focuses on correlation-based distances. It uses the partial autocorrelation function (PACF) to define the distance between time series. In particular, distance is defined as $\sqrt{(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})'\Omega(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})}$, where Ω is a matrix of weights, whereas $\hat{\rho}_{X_t}$ and $\hat{\rho}_{Y_t}$ are the estimated partial autocorrelations of X and Y, respectively. The fourth and fifth measures assume that a specific model generates both time series. The idea is to fit the specific model to each time series and then measure the dissimilarity between the fitted models. The fourth measure computes the distance between two time series as the ED between the truncated AR operators. In this case, distance is defined as $\sqrt{\sum_{j=1}^{k}(e_{j,X_t} - e_{j,Y_t})^2}$, where $e_{X_t} = (e_{1,X_t}, \cdots, e_{k,X_t})$ and $e_{Y_t} = (e_{1,Y_t}, \cdots, e_{k,Y_t})$ denote the vectors of $AR(k)$ parameter estimators for X and Y, respectively. The fifth measure computes dissimilarity between two time series in terms of their linear predictive coding in ARIMA processes, as in Kalpakis, Gada, and Puttagunta (2001). The last measure defines distance based on nonparametric spectral estimators. Let f_{X_T} and f_{Y_T} denote the spectral densities of X_T and Y_T, respectively. In this case, the dissimilarity measure is given by a nonparametric statistic that checks the equality of the log-spectra of the two time series. It defines distance as $\sum_{k=1}^{n}\left[Z_k - \hat{\mu}(\lambda_k) - 2\log(1 + e^{Z_k - \hat{\mu}(\lambda_k)})\right] - \sum_{k=1}^{n}\left[Z_k - 2\log(1 + e^{Z_k})\right]$, where $Z_k = \log(I_{X_T}(\lambda_k)) - \log(I_{Y_T}(\lambda_k))$, and $\hat{\mu}(\lambda_k)$ is the local maximum log-likelihood estimator of $\mu(\lambda_k) = \log(f_{X_T}(\lambda_k)) - \log(f_{Y_T}(\lambda_k))$ computed with local lineal smoothers of the periodograms. All similarity measures are computed using the **R** package TSclust (see Montero and Vilar (2014)).

	$\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$		$\text{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$	
Similarity measure	**Mean**	**Standard Deviation**	**Mean**	**Standard Deviation**
ED	0.958	0.012	0.974	0.008
DTW	0.758	0.091	0.723	0.105
PACF	0.736	0.043	0.743	0.043
ED in AR	0.908	0.100	0.910	0.097
Linear predictive in ARIMA	0.726	0.325	0.729	0.313
Spectral distance	1.0	0.000	1.0	0.000

Table VII
Eigenvector Centrality Summary Statistics

The table reports averages of summary statistics for log(eigenvector centrality). To compute averages in the third and fourth columns, I use customer–supplier data on years 1982, 1987, 1992, 1997, and 2002 to be consistent with the years used by Ahern (2013). Using data reported in Ahern (2013, Internet Appendix Table II), the second column presents averages of the statistics for log(eigenvector centrality) in inter-sectoral trade networks. The third column presents averages in annual customer–supplier networks in which two firms are connected if one firm represents at least 10% of the other firm's annual sales. The fourth column presents averages in annual customer–supplier networks in which two firms are connected if one firm represents at least 20% of the other firm's annual sales. The fifth column reports the statistics for log(eigenvector centrality) in the network of the calibrated economy.

Statistic	**Inter-sectoral Networks**	**Customer–Supplier Networks** (10%)	**Customer–Supplier Networks** (20%)	**Calibrated Network**
Number of sectors/firms	474	750	382	400
Mean	−6.68	−6.74	−6.62	−6.09
Standard deviation	1.48	1.07	1.31	1.71
Skewness	0.87	4.04	3.28	1.54
Kurtosis	4.45	18.50	12.38	3.70
Minimum	−10.21	−7.01	−7.01	−7.01
1st percentile	−9.39	−7.01	−7.01	−7.01
25th percentile	−7.71	−7.01	−7.01	−7.01
Median	−6.85	−7.01	−7.01	−6.09
75th percentile	−5.90	−7.01	−7.01	−6.42
99th	−2.27	−1.83	−1.67	−2.30
Maximum	−0.17	−0.46	−0.34	−0.74

Table VIII
Performance of Centrality Portfolios

The table reports monthly average raw returns, alphas and loadings from the five-factor model for three portfolios of stocks sorted by annual centrality and the portfolio that is long on the low tercile and short on the high tercile of centrality. The bottom row provides the t-statistics for the low minus high portfolio. Firms are assigned into terciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample is from June 1981 to December 2004. Raw returns and alphas are in percent.

	Raw	Alphas				5-Factor Loadings				
Tercile	Return	CAPM	3-Factor	4-Factor	5-Factor	MKT	SMB	HML	RMW	CMA
Low	2.06	0.89	1.12	1.01	1.26	0.94	0.12	-0.34	-0.36	0.11
2	1.96	0.86	0.87	0.83	0.77	1.00	0.06	-0.12	0.07	0.22
High	1.71	0.64	0.72	0.75	0.72	0.94	-0.23	-0.11	-0.05	0.05
Low - High	0.34	0.25	0.40	0.25	0.54	-0.006	0.36	-0.23	-0.30	0.05
t-statistic	[1.72]	[1.26]	[2.3]	[1.44]	[3.05]	[-0.13]	[6.10]	[-2.67]	[-3.94]	[0.44]

Table IX
Performance of Centrality Portfolios in Manufacturing Stocks

The table reports monthly average raw returns, alphas and loadings from the five-factor model for three portfolios of manufacturing stocks sorted by annual centrality and the portfolio that is long on the low tercile and short on the high tercile of centrality. The bottom row provides the t-statistics for the low minus high portfolio. Manufacturing firms are assigned into terciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample considers from June 1981 to December 2004. Raw returns and alphas are in percent. Manufacturing stocks represent about 50% of the stocks in the database.

	Raw	Alphas				5-Factor Loadings				
Tercile	Return	CAPM	3-Factor	4-Factor	5-Factor	MKT	SMB	HML	RMW	CMA
Low	2.02	0.85	1.11	1.01	1.14	0.93	0.29	-0.54	-0.20	0.29
2	1.85	0.74	0.84	0.87	0.75	0.97	0.13	-0.30	-0.005	0.33
High	1.79	0.71	0.82	0.85	0.81	0.93	-0.22	-0.19	-0.09	0.15
Low - High	0.23	0.13	0.28	0.15	0.33	0.002	0.52	-0.34	-0.11	0.14
t-statistic	[1.02]	[0.58]	[1.49]	[0.80]	[1.64]	[0.04]	[7.76]	[-3.50]	[-1.27]	[1.08]

Table X
Performance of Centrality Portfolios in Service Stocks

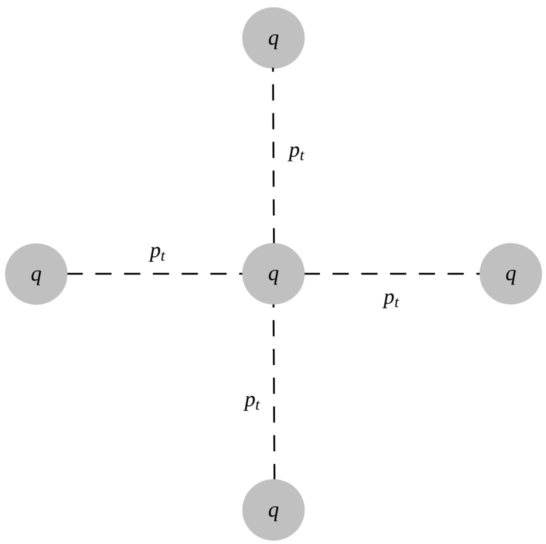
The table reports monthly average raw returns, alphas and loadings from the five-factor model for three portfolios of service stocks sorted by annual centrality and the portfolio that is long on the low tercile and short on the high tercile of centrality. The bottom row provides the t-statistics for the low minus high portfolio. Service firms are assigned into terciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample considers from June 1981 to December 2004. Raw returns and alphas are in percent. Service stocks represent about 20% of the stocks in the database.

	Raw	Alphas				5-Factor Loadings				
Tercile	Return	CAPM	3-Factor	4-Factor	5-Factor	MKT	SMB	HML	RMW	CMA
Low	2.86	1.38	1.87	1.69	2.34	1.14	0.58	-0.42	-0.71	-0.46
2	2.32	1.03	1.34	1.27	1.39	1.07	0.40	-0.43	-0.009	-0.09
High	1.54	0.19	0.87	1.13	0.99	1.00	0.06	-0.67	0.15	-0.66
Low - High	1.31	1.19	1.00	0.55	1.35	0.14	0.51	0.24	-0.87	0.20
t-statistic	[2.38]	[2.14]	[1.79]	[0.98]	[2.36]	[1.03]	[2.73]	[0.88]	[-3.58]	[0.54]

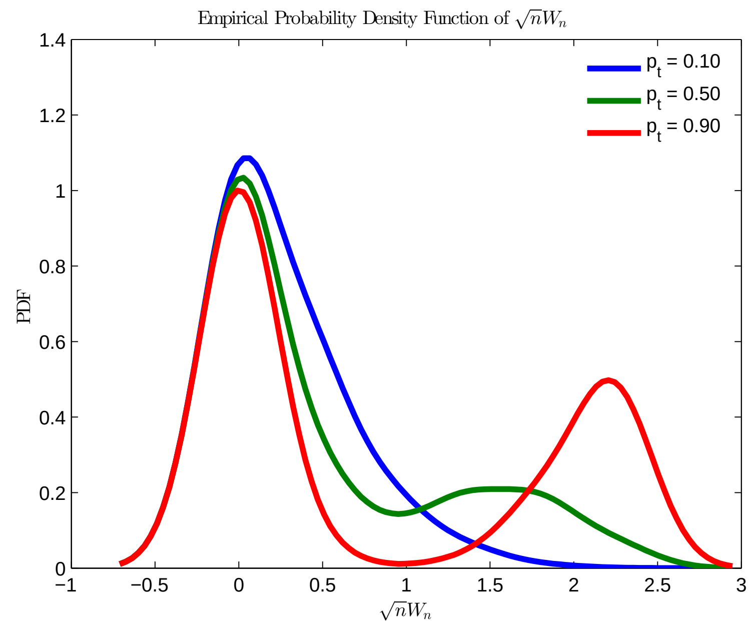
Table XI
Performance of Centrality Portfolios in Manufacturing and Service Stocks

The table reports monthly average raw returns, alphas and loadings from the five-factor model for three portfolios of manufacturing and service stocks sorted by annual centrality and the portfolio that is long on the low tercile and short on the high tercile of centrality. The bottom row provides the t-statistics for the low-high portfolio. Manufacturing and service firms are assigned into terciles at the end of October every year and the value-weighted portfolios are not rebalanced for the next 12 months. The sample considers from June 1981 to December 2004. Raw returns and alphas are in percent. Manufacturing and service stocks represent about 70% of the stocks in the database.

	Raw	Alphas				5-Factor Loadings				
Tercile	Return	CAPM	3-Factor	4-Factor	5-Factor	MKT	SMB	HML	RMW	CMA
Low	2.17	0.93	1.26	1.13	1.35	0.98	0.28	-0.59	-0.30	0.24
2	1.98	0.85	0.99	0.99	0.96	0.96	0.15	-0.33	-0.06	0.25
High	1.80	0.71	0.87	0.91	0.92	0.92	-0.24	-0.18	-0.13	-0.01
Low - High	0.36	0.22	0.39	0.21	0.42	0.05	0.52	-0.41	-0.17	0.26
t-statistic	[1.48]	[0.92]	[1.91]	[1.04]	[2.00]	[1.11]	[7.42]	[-3.95]	[-1.87]	[1.89]

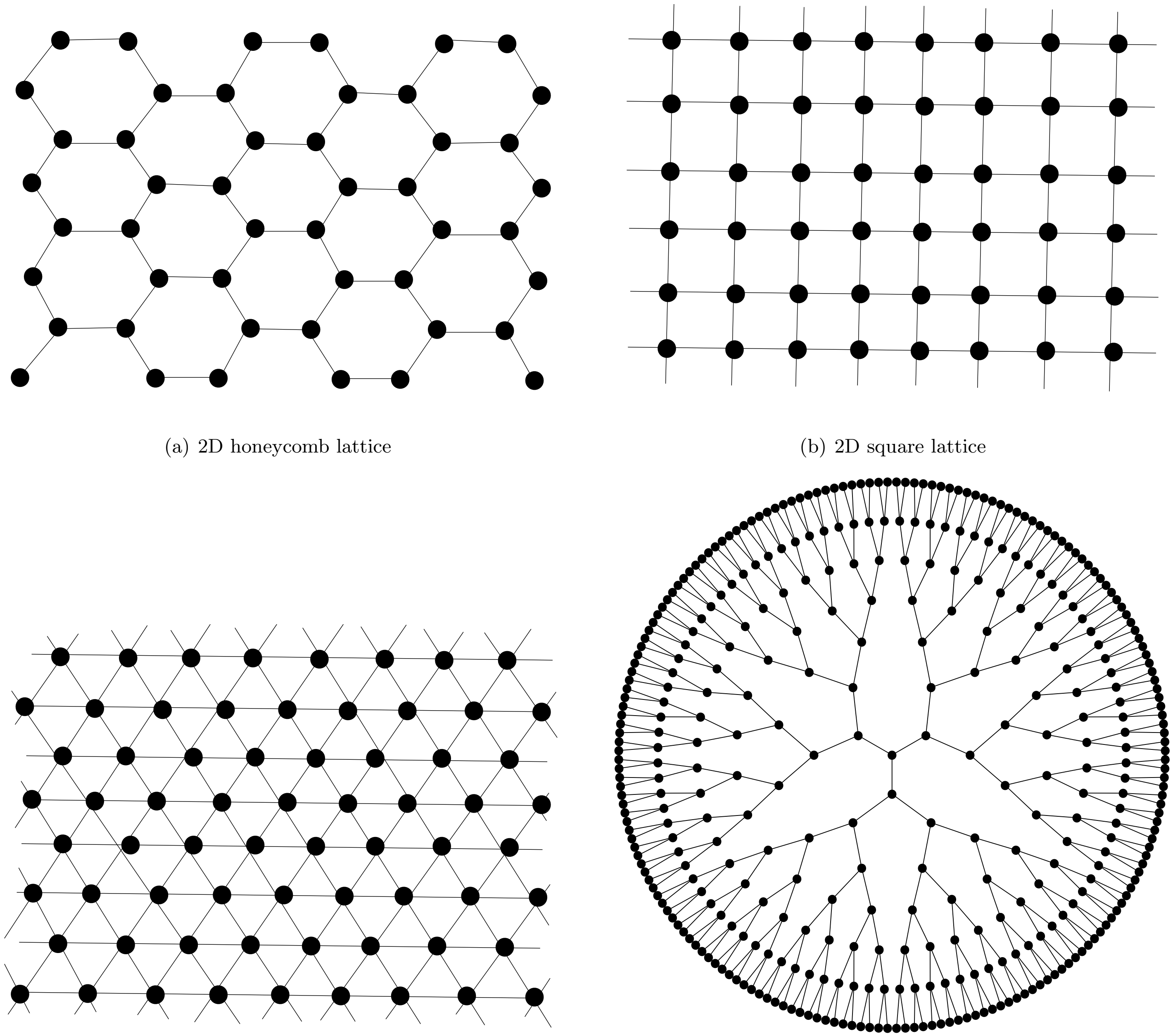


(a) Network Topology



(b) Empirical density function of $\sqrt{n}\widetilde{W}_{n,t}$

Figure 2. The figure illustrates how changes in the propensity of inter-firm relationships to transmit shocks at t, $\widetilde{p}_t$, affect the distribution of $\sqrt{n}\widetilde{W}_{n,t}$. Figure 2(a) depicts an economy with $n = 5$ firms, whereas figure 2(b) depicts estimates of the density function of $\sqrt{n}\widetilde{W}_{n,t}$ for different values of $\widetilde{p}_t$. These estimates are computed via normal kernel smoothing estimators using function ksdensity($\cdot$) in MATLAB.



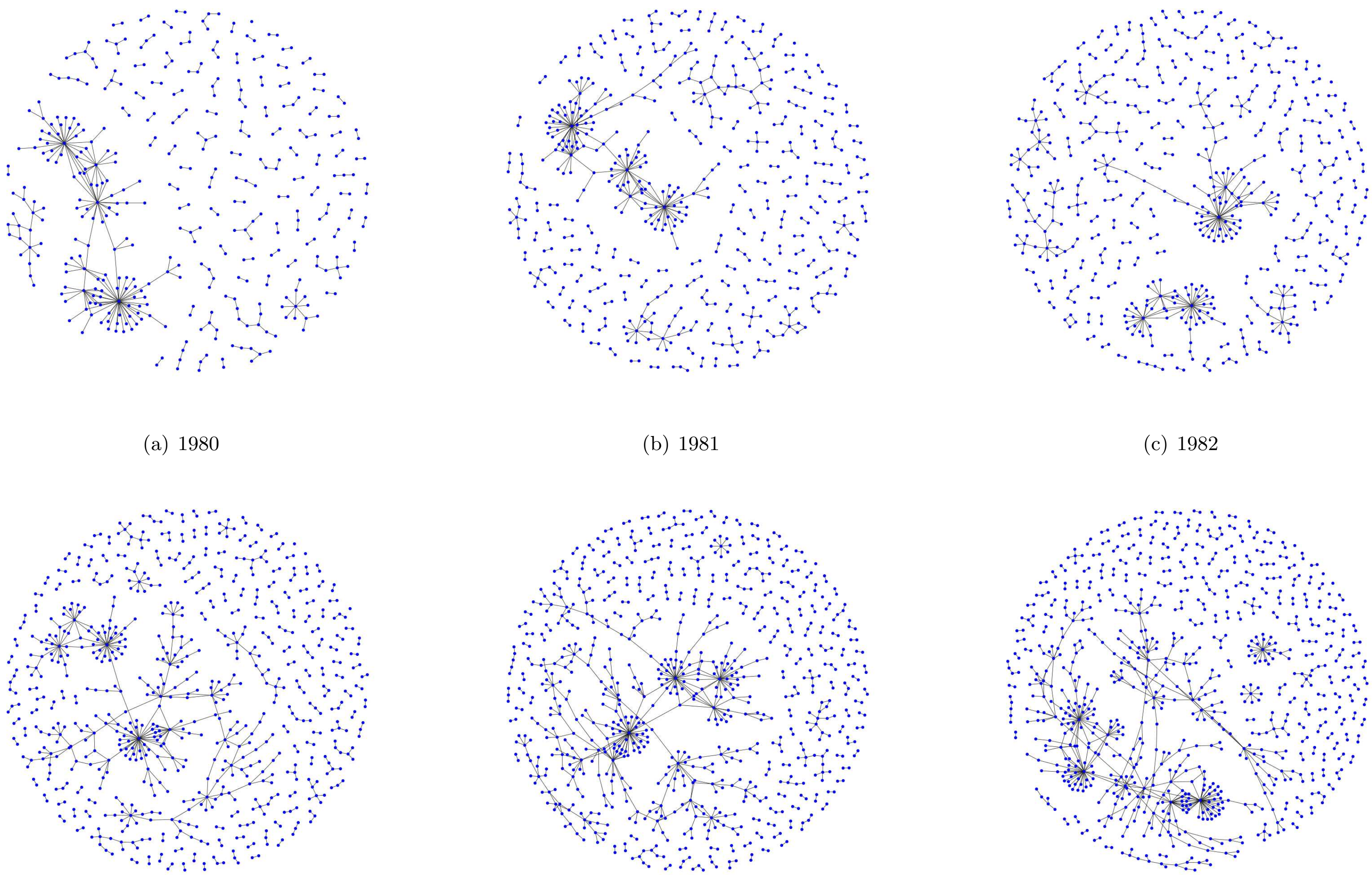
(a) 2D honeycomb lattice

(b) 2D square lattice

(c) 2D triangular lattice

(d) Bethe lattice with degree 3

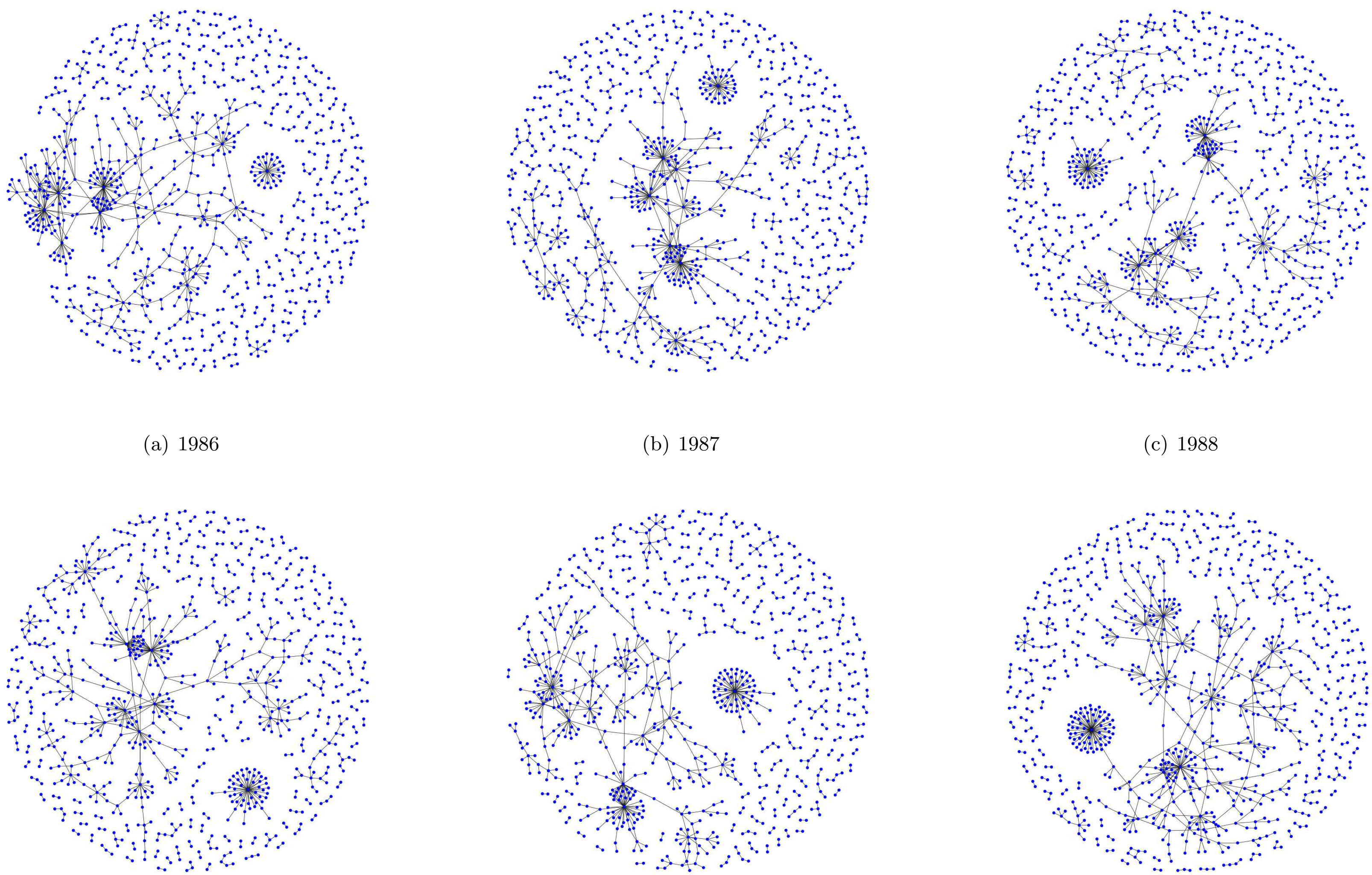
Figure 3. The figure shows the topologies of the symmetric networks considered in corollary 1 and Table I.



(a) 1980 (b) 1981 (c) 1982

(d) 1983 (e) 1984 (f) 1985

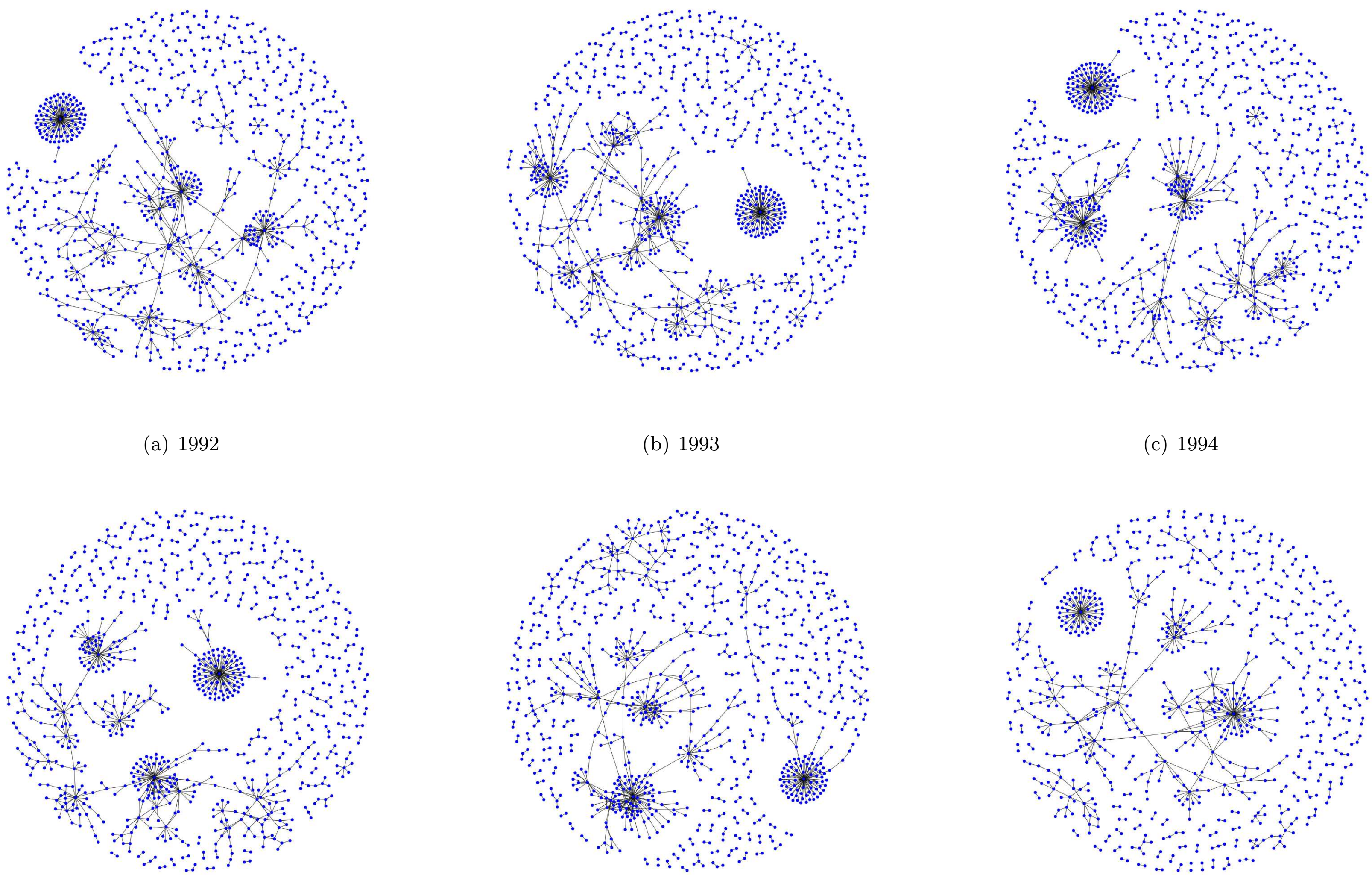
Figure 4. The figure shows customer–supplier networks from 1980 to 1985.



(a) 1986 (b) 1987 (c) 1988

(d) 1989 (e) 1990 (f) 1991

Figure 5. The figure shows customer–supplier networks from 1986 to 1991.



(a) 1992 (b) 1993 (c) 1994

(d) 1995 (e) 1996 (f) 1997

Figure 6. The figure shows customer–supplier networks from 1992 to 1997.

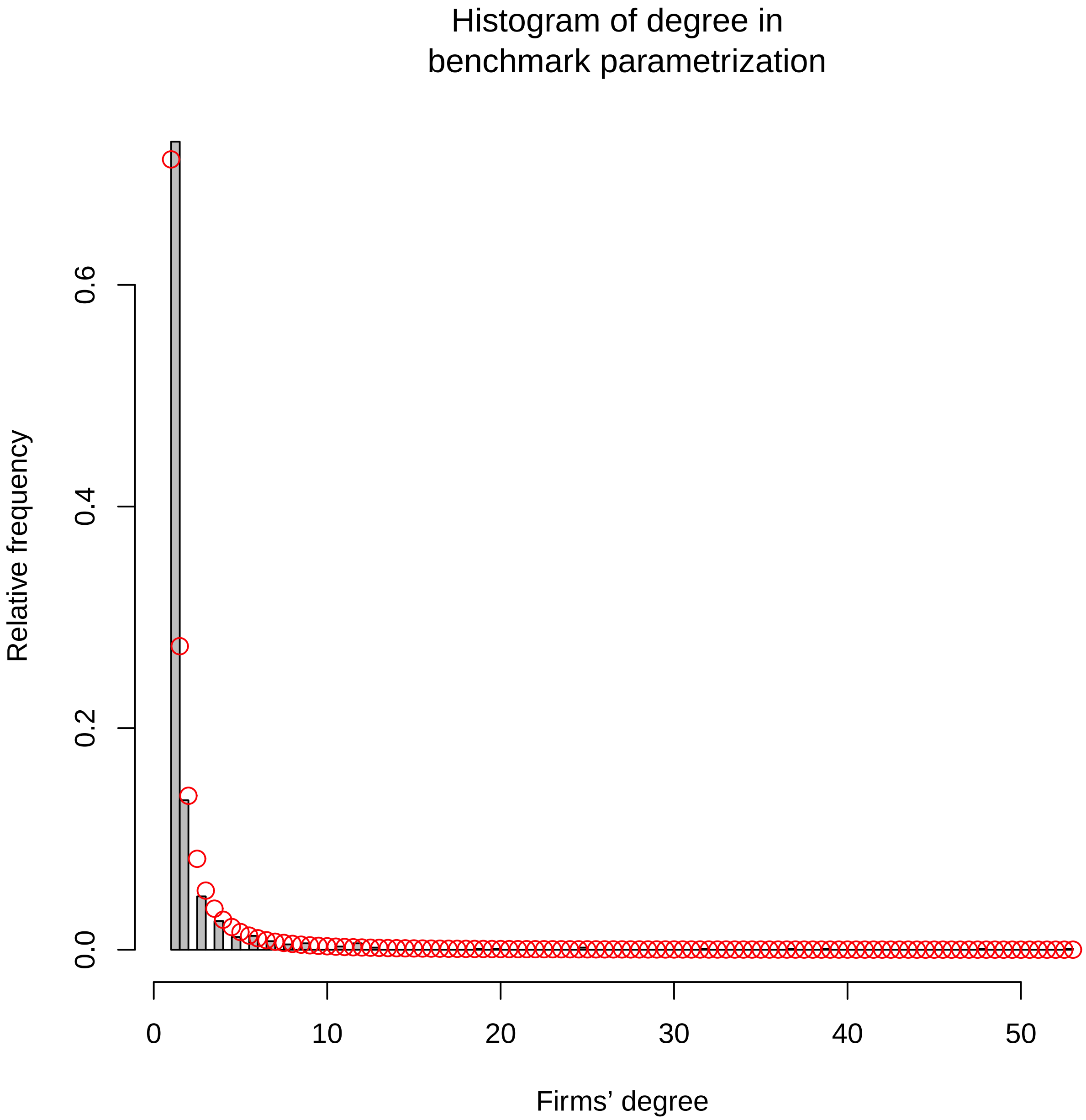
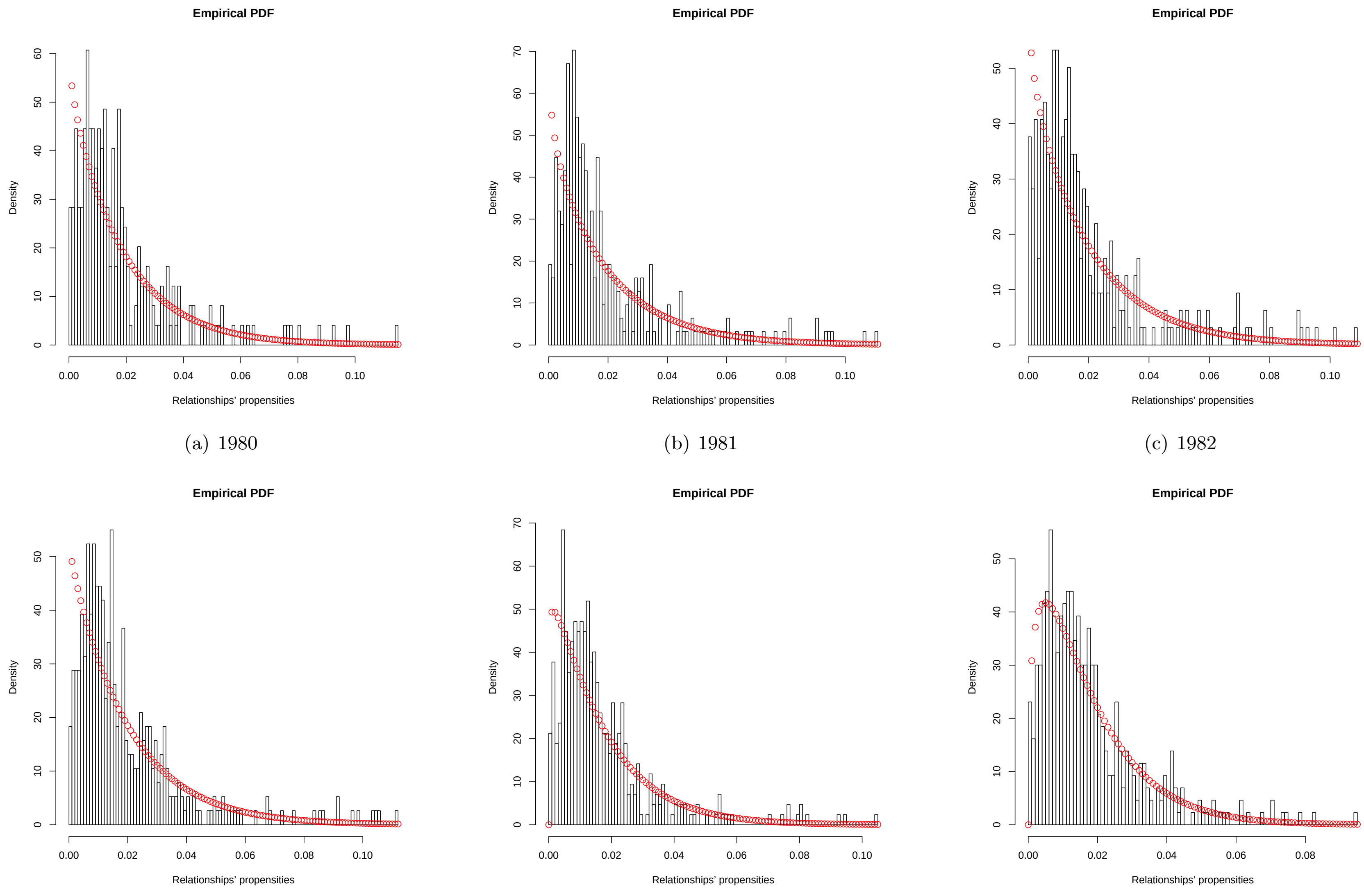


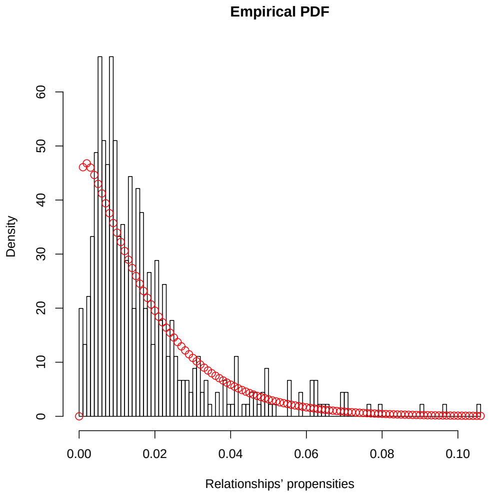
Figure 7. The figure shows the degree distribution under benchmark parameterization (shown with bars). Dots represent a power law distribution with exponent 2.36.



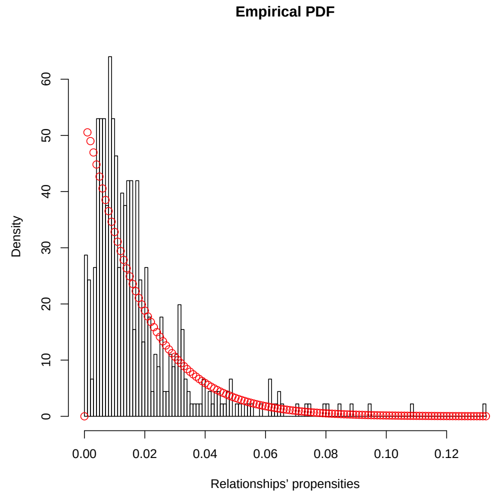
(a) 1980 (b) 1981 (c) 1982

(d) 1983 (e) 1984 (f) 1985

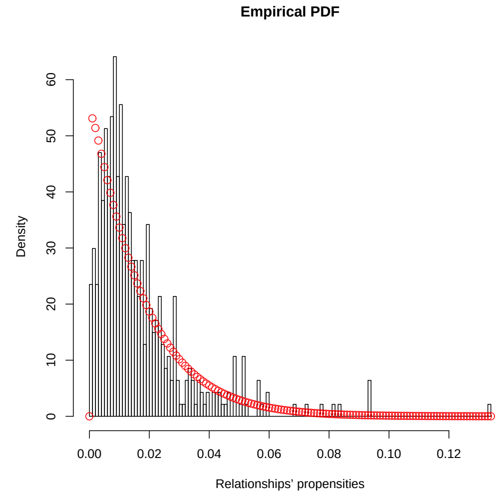
Figure 8. The figure shows annual density functions of relationship propensities in customer–supplier networks from 1980 to 1985. In the above figures, bars depict annual empirical probability density functions whereas red dots comes from fitting a beta distribution to each empirical probability density function.



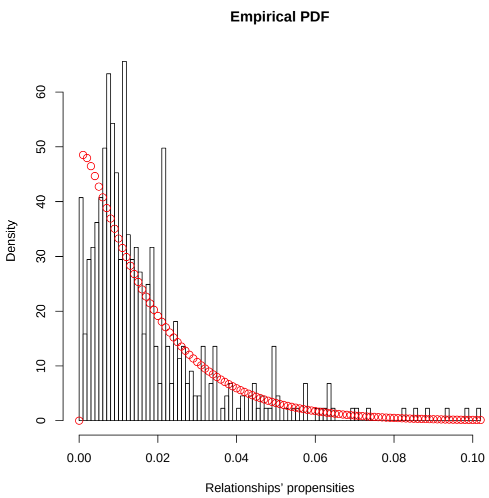
(a) 1986



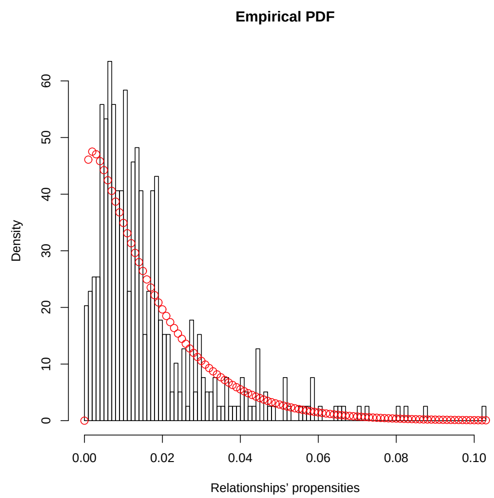
(b) 1987



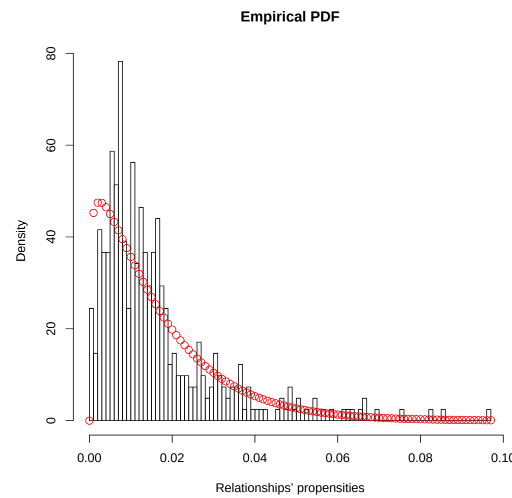
(c) 1988



(d) 1989



(e) 1990



(f) 1991

Figure 9. The figure shows annual density functions of relationship propensities in customer–supplier networks from 1986 to 1991. In the above figures, bars depict annual empirical probability density functions whereas red dots comes from fitting a beta distribution to each empirical probability density function.

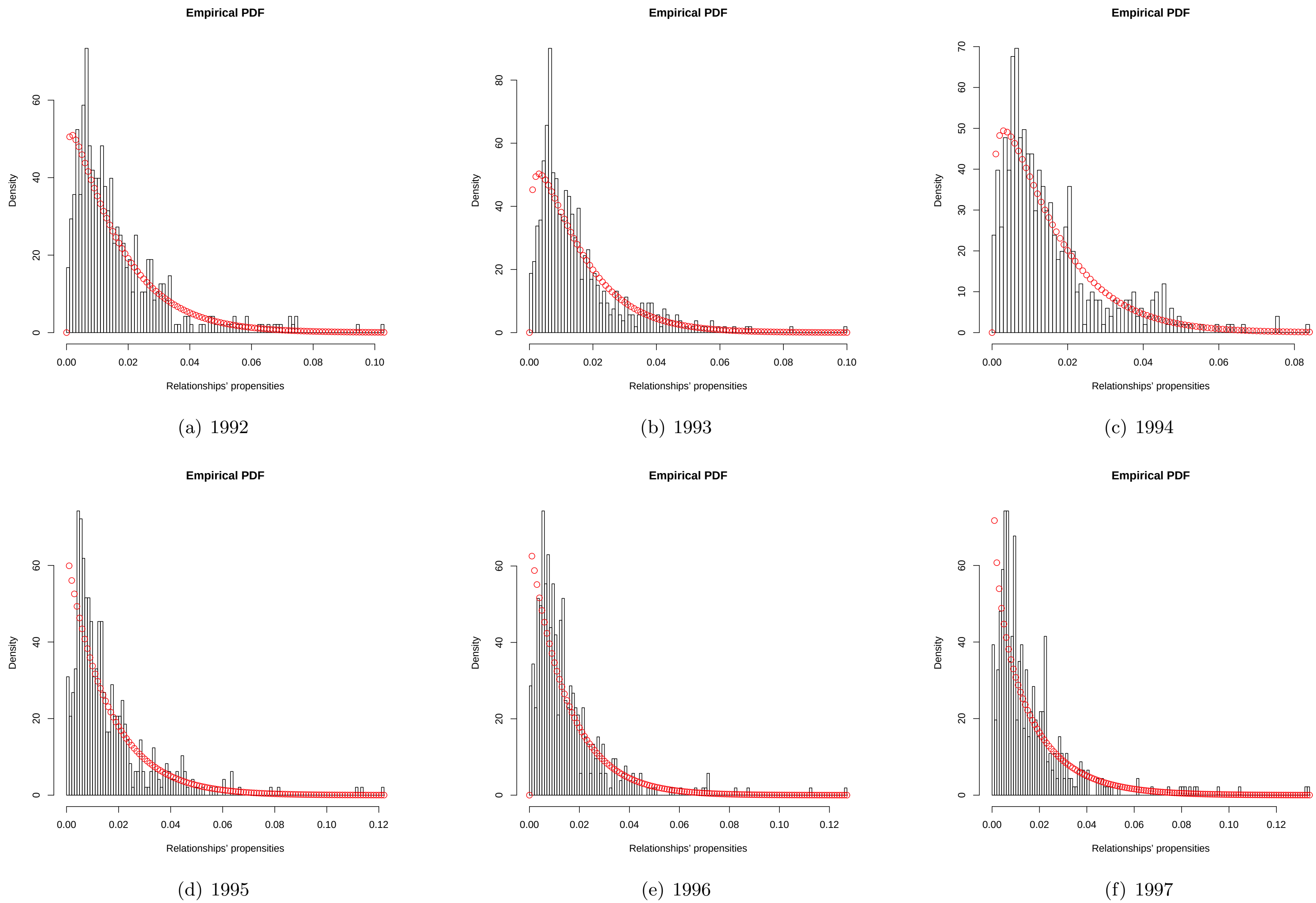


Figure 10. The figure shows annual density functions of relationship propensities in customer–supplier networks from 1992 to 1997. In the above figures, bars depict annual empirical probability density functions whereas red dots comes from fitting a beta distribution to each empirical probability density function.

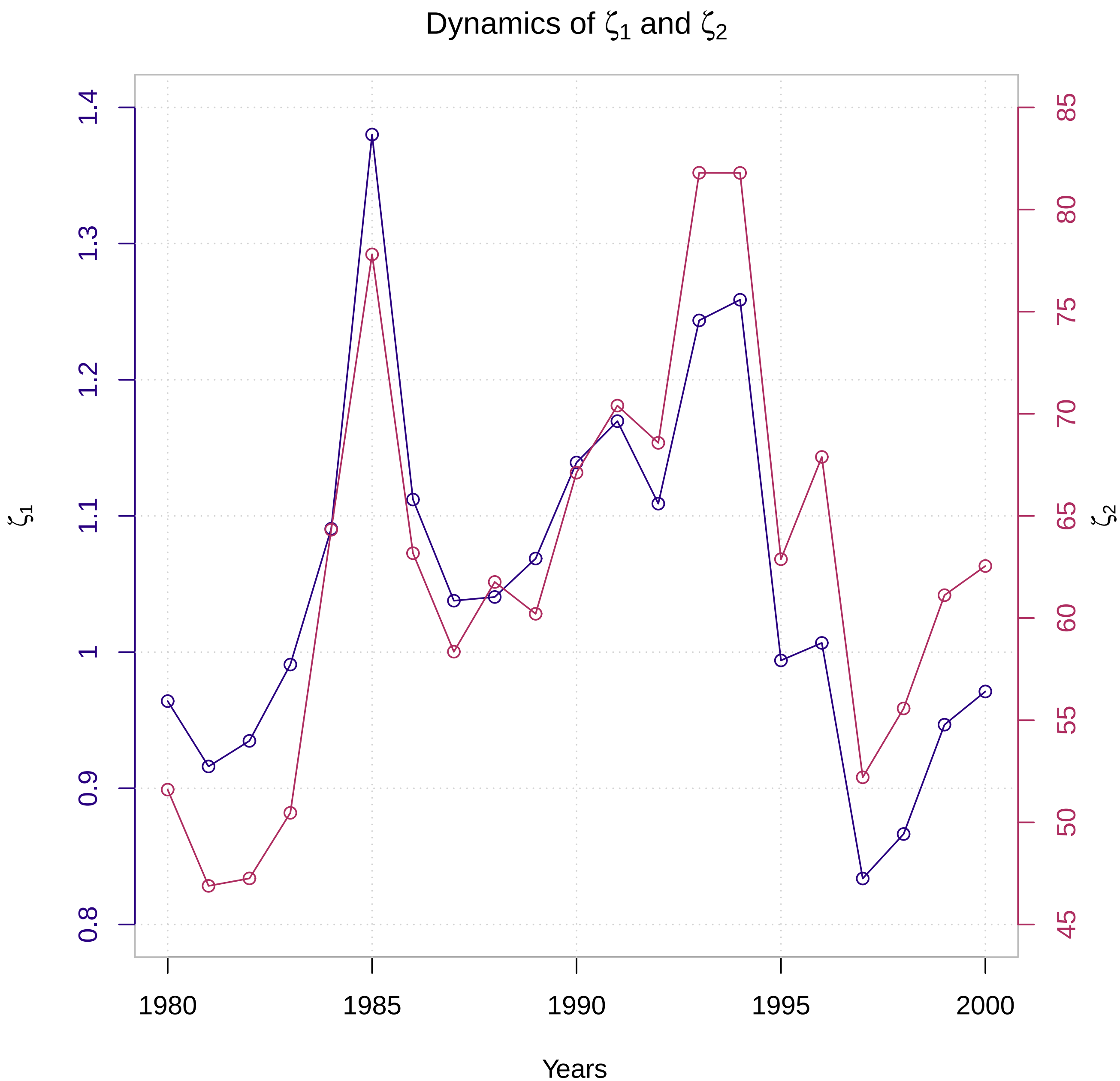
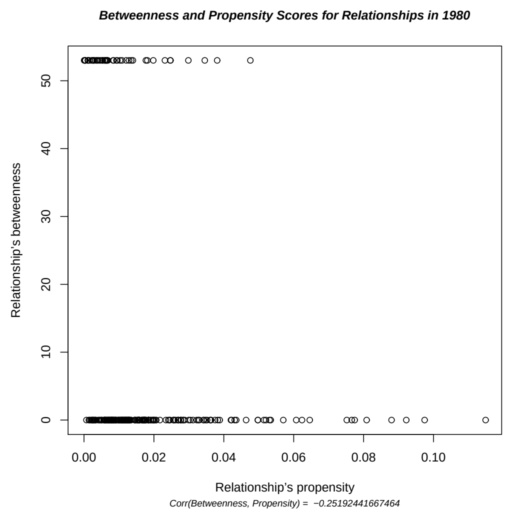
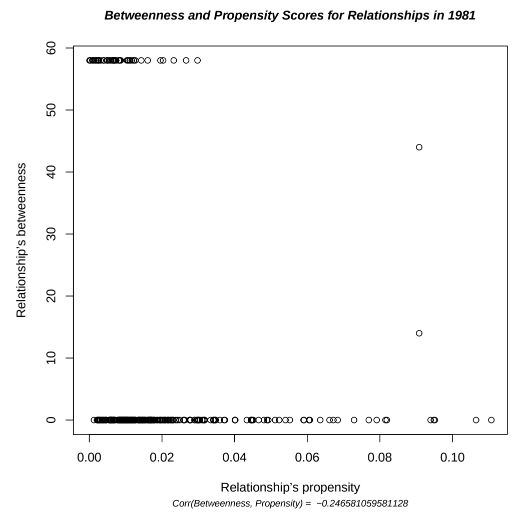


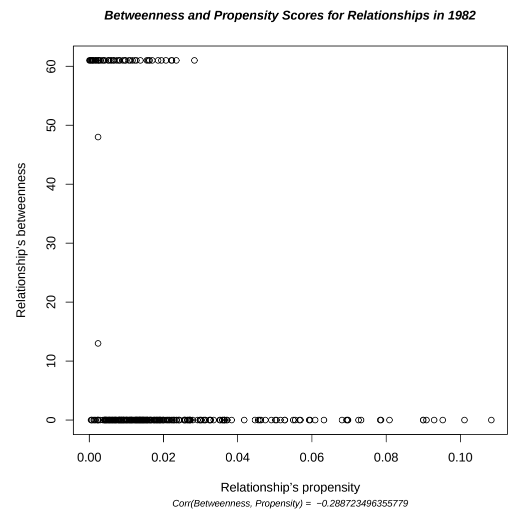
Figure 11. The figure shows annual estimates of parameters ζ_1 and ζ_2. I obtain these estimates by fitting Beta distributions to the annual link weight distributions using maximum likelihood.



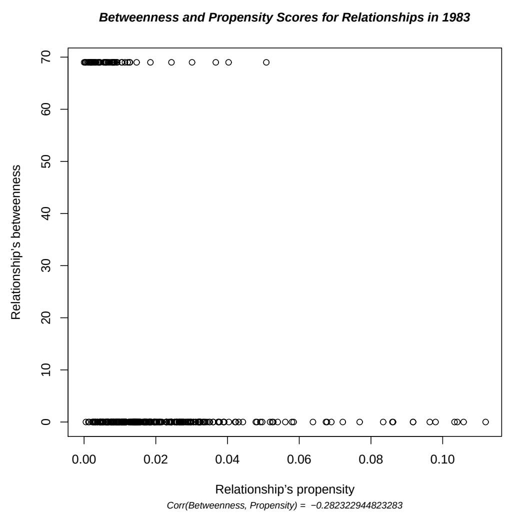
(a) 1980



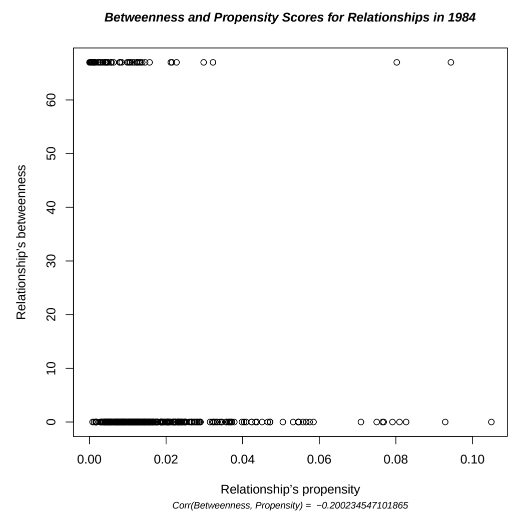
(b) 1981



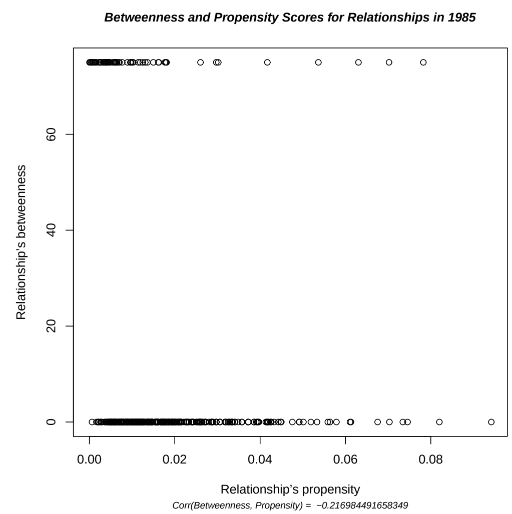
(c) 1982



(d) 1983

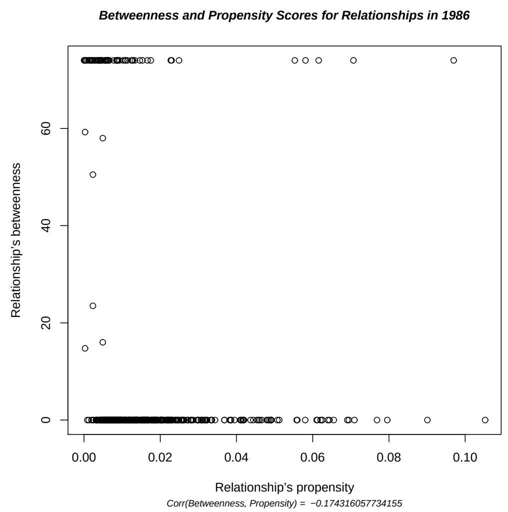


(e) 1984

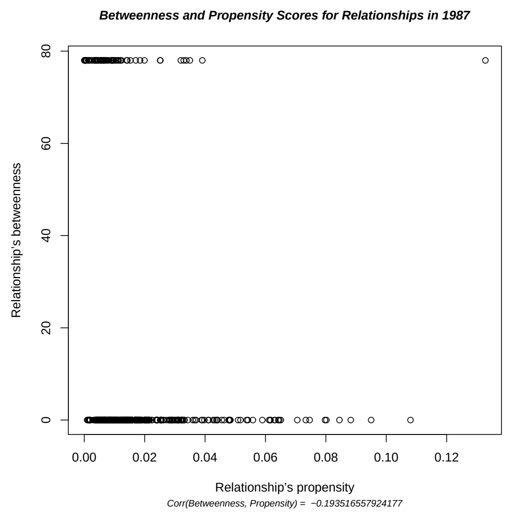


(f) 1985

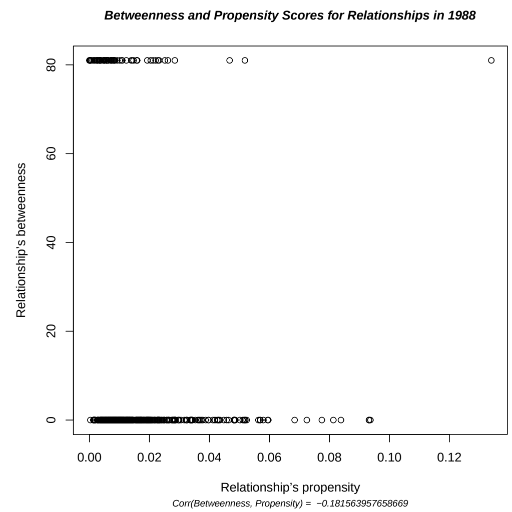
Figure 12. The figure shows the relationship between link betweenness centrality and link weights in customer–supplier networks from 1980 to 1985.



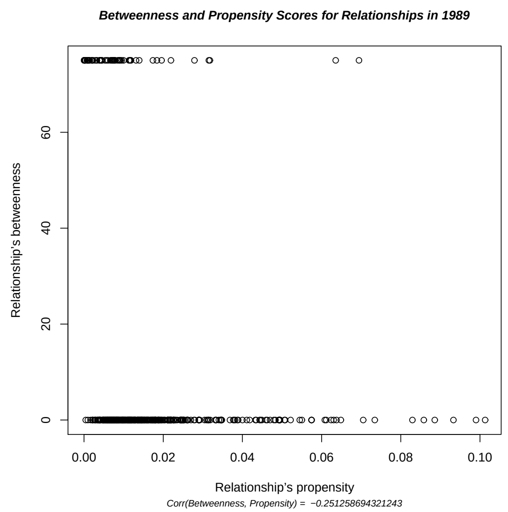
(a) 1986



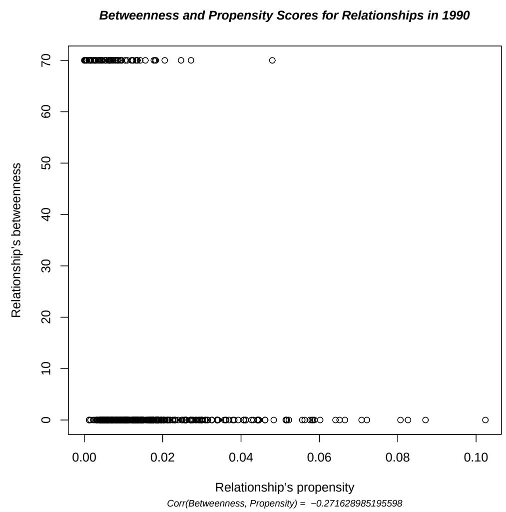
(b) 1987



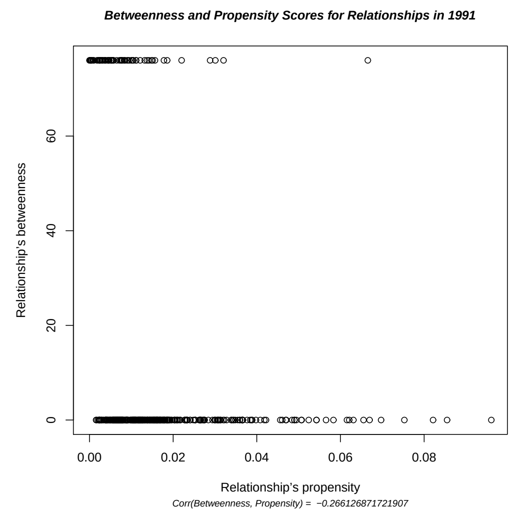
(c) 1988



(d) 1989

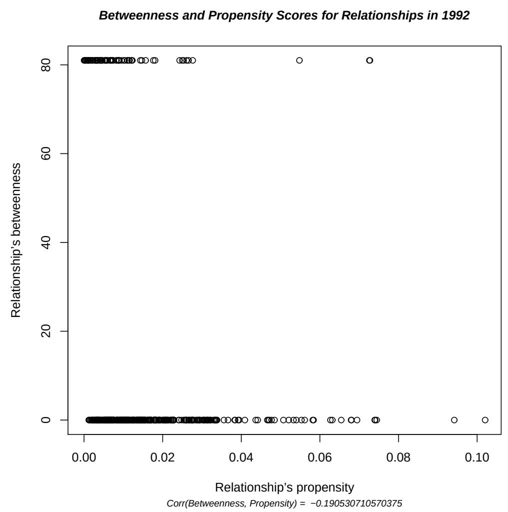


(e) 1990

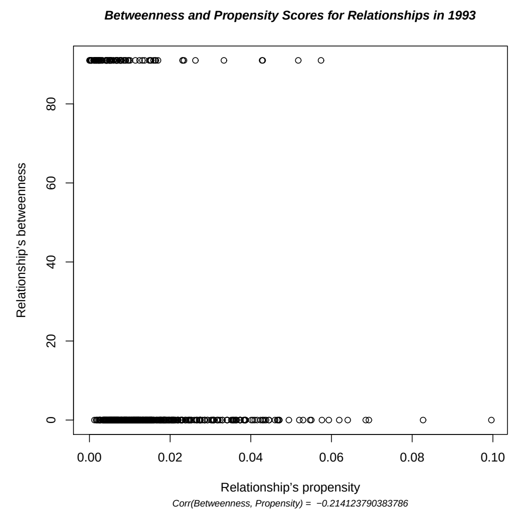


(f) 1991

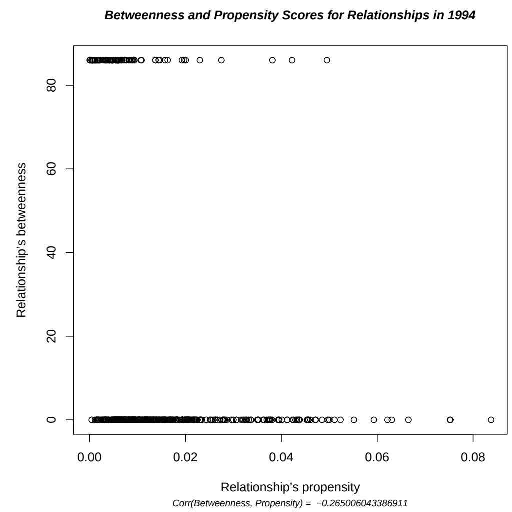
Figure 13. The figure shows the relationship between link betweenness centrality and link weights in customer–supplier networks from 1986 to 1991.



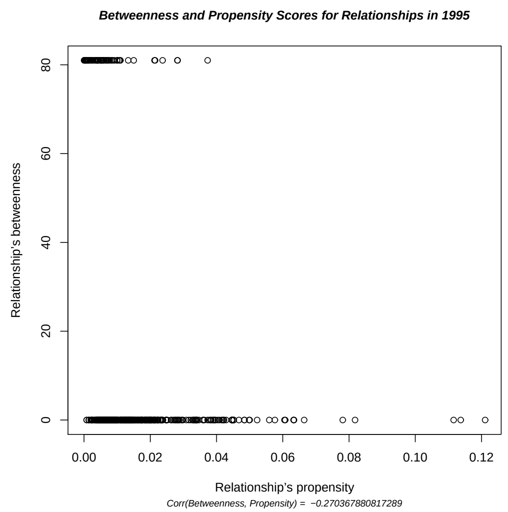
(a) 1992



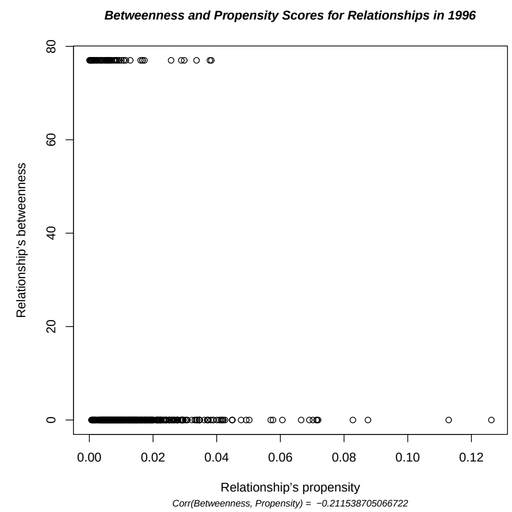
(b) 1993



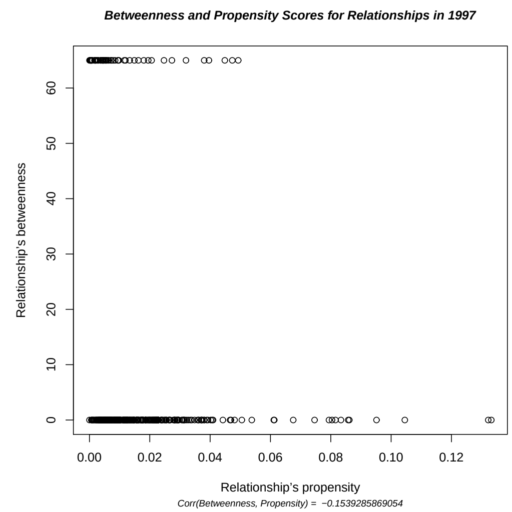
(c) 1994



(d) 1995



(e) 1996



(f) 1997

Figure 14. The figure shows the relationship between link betweenness centrality and link weights in customer–supplier networks from 1992 to 1997.

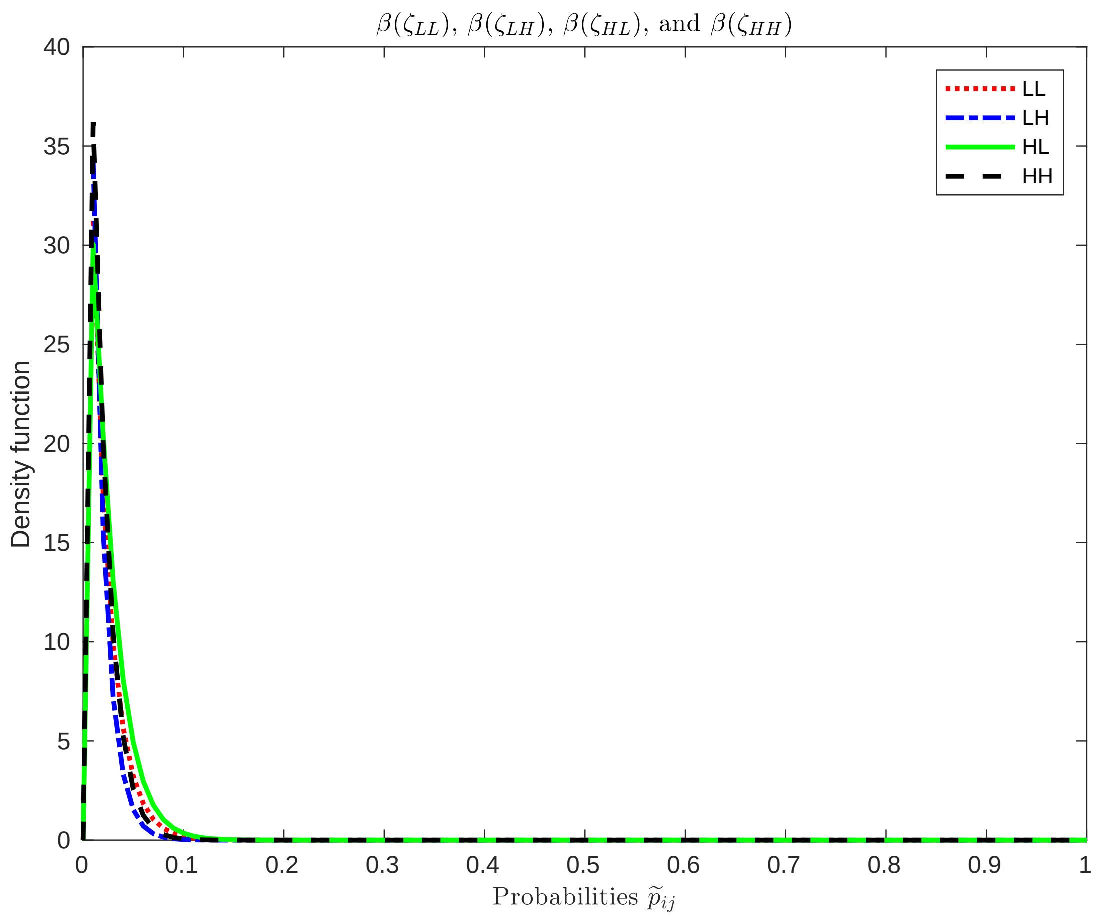


Figure 15. The figure shows probability density functions for Beta distributions with shape parameter vectors ζ_{LL}, ζ_{LH}, ζ_{HL}, and ζ_{HH}.

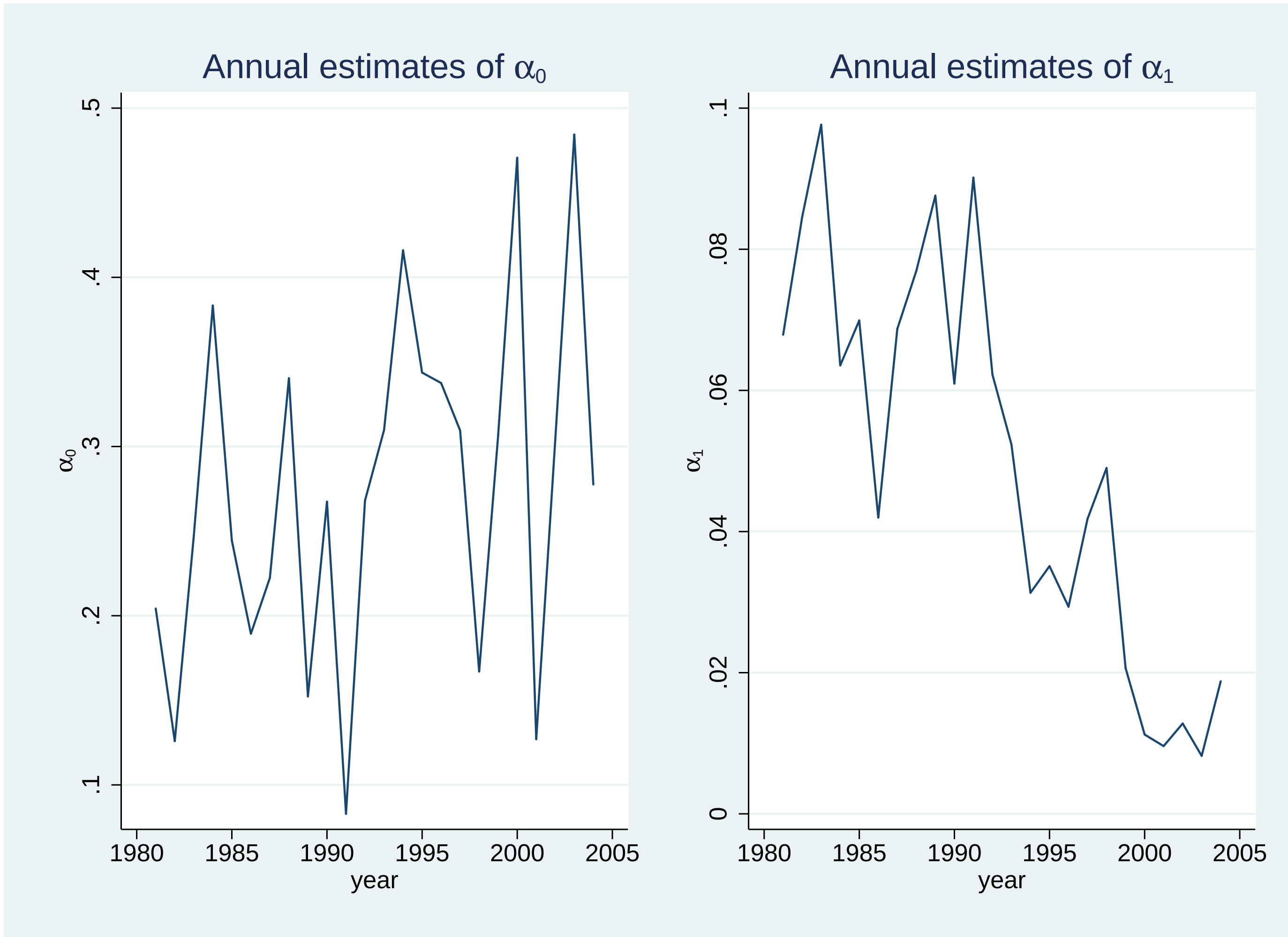
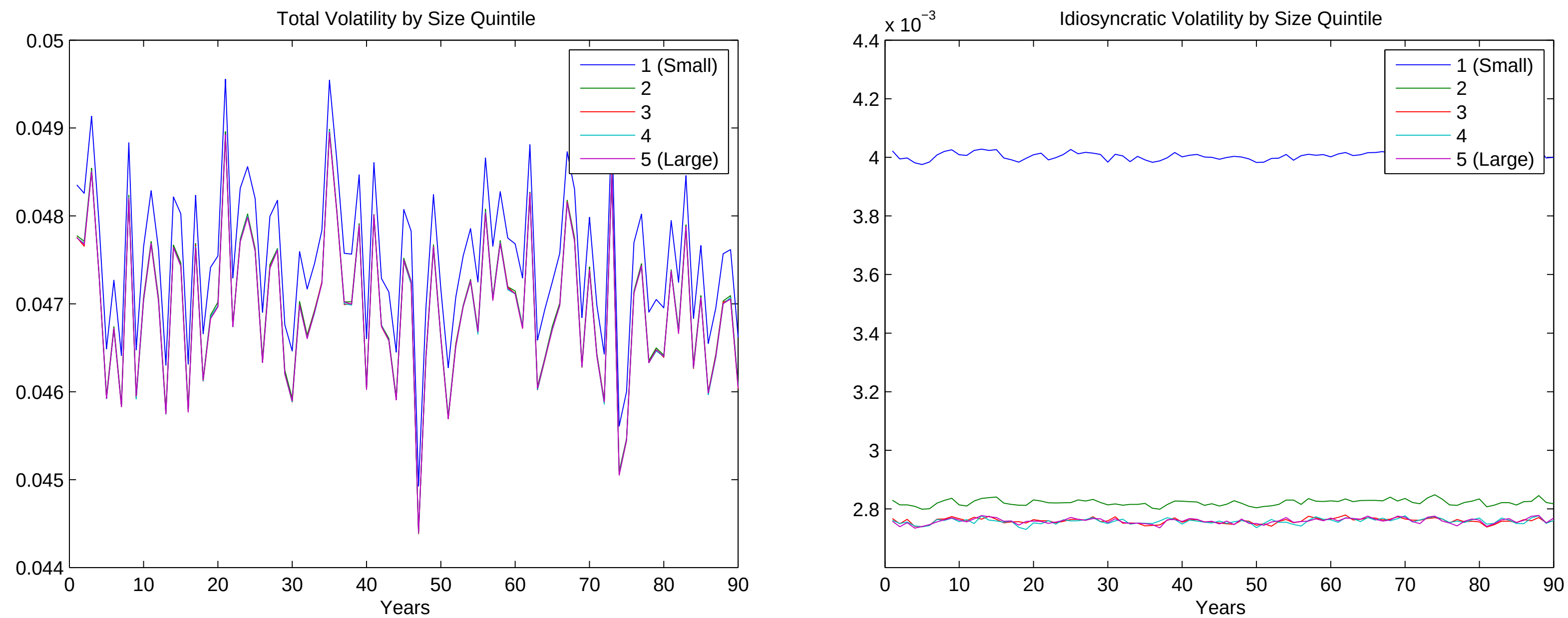


Figure 16. The figure shows annual estimates of parameters α_0 and α_1 in equation (1).



(a) Total Volatility by Size Quintile

(b) Idiosyncratic Volatility by Size Quintile

Figure 17. Annualized firm-level volatilities averaged within size quintiles. I simulate 200 panels with 400 firms over 1,500 periods and disregard the first 500 periods to eliminate biases coming from the initial condition. Within each panel, I compute firm-level total volatility as the annualized standard deviation of monthly firm level realized returns. I construct firm-level idiosyncratic volatility as the annualized standard deviation of residuals computed from monthly CAPM regressions of firm-level excess realized returns on the excess realized return of the market portfolio. This procedure yields panels of firm-year total and idiosyncratic volatilities estimates. Then, at the beginning of each year I sort firms based on size and average annual volatilities within size quintiles. This procedure yields five time series of total and idiosyncratic volatilities per panel. Figure 17(a) plots total volatilities per quintile, whereas figure 17(b) plots idiosyncratic volatilities per quintile averaged over the 200 panels.