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Credit Default Swaps and Debt Contracts: Spillovers and Extensive Default Premium Choice*

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Abstract

This paper highlights two new effects of credit default swap (CDS) markets on credit markets. First, when firms' cash flows are correlated, CDS trading impacts the cost of capital and investment for all firms, even those that are not CDS obligors. Second, CDSs generate a tradeoff between default premiums and default risk. CDSs alter firm incentives to invest along the extensive default premium margin, even absent maturity mis-match. Firms are more likely to issue safe debt when default premiums are high and vise versa. The direction of the tradeoff depends on whether investors use CDSs for speculation or hedging.

Keywords: credit derivatives, spillovers, investment, default risk.

JEL Classification: D52, D53, E44, G10, G12

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1 Introduction

The global financial crisis of 2007-2008 underscored the need to better understand how financial market participants price and take risk. Credit default swaps (CDSs) are a particular type of financial instrument that market participants used with increasing regularity in the build-up to the crisis. According to the Bank for International Settlements (BIS) the notional size of the CDS market (value of all outstanding contracts) at its peak before the market crash in 2007 was $57 trillion. While that number has abated mainly as a result of the multilateral netting of contracts, its size as of the first half of 2015 was $15 trillion. Clearly the CDS market remains large and active, and continues to engender a variety of research efforts aiming to better understand the effects of CDS markets on capital markets.

We investigate the role of CDS trading on the cost of capital and *endogenous* investment choice of multiple firms in an economy where some firms are obligors for CDS contracts and other are not.[1] We first show that the general equilibrium effects on pricing and investment of CDS trading impact the debt contracts of all firms, even the debt contracts for firms for whom CDS contracts do *not* trade. We then show that CDS trading impacts default premiums and alters the incentives to issue risky rather than default-risk free debt.

Building on previous work by Fostel and Geanakoplos (2012, 2016) and Che and Sethi (2015), CDS trading affects real outcomes through its influence on investors' marginal valuation of holding cash to as collateral to sell CDSs rather than holding cash to buy bonds. As originally shown by Fostel and Geanakoplos (2012) and subsequently by Che and Sethi (2015), the collateral used to back CDS contracts re-allocates capital from the bond market to the CDS market, which changes the demand for bonds. We use the re-allocation channel to show that in equilibrium, when assets (firm production in our model) have correlated cash flows, an optimistic marginal buyer prices the two assets so that the relative returns to investing in either asset will be equal. The equivalent asset returns imply that when CDS trading alters the demand for one type of bond, it alters the demand for the other as well. Firms respond to changes in the cost of capital by changing their demand for investment.

[1]Using data from either Markit or the Depository Trust and Clearing Corporation (DTCC), there are only around 3000 firms for which a CDS contract ever exists. We do not attempt to endogenize CDS issuance in this paper; we simply take as given that the market is active only for a portion of the firms in the economy.

We also show that endogenous default premiums coupled with endogenous investment demand lead to interesting firm financing decisions regarding the choice to issue debt that is default risk free rather than risky. Specifically, as the default premium falls, the benefit of raising elevated levels of capital that cannot be repaid when cash flow is low is reduced. As fundamentals improve firms switch to issuing fewer bonds that can be fully repaid in all states, hence the extensive default premium choice. We then show that for a given set of economy fundamentals, the extensive default risk margin changes when bond buyers are permitted to trade CDSs because CDS trading alters default premiums.

The framework in our paper is a static, general equilibrium model with two states characterized by a commonly known aggregate productivity shock with high values in the up state and low values in the down state. There are two firm types endowed with different production technologies that are symmetrically impacted by the technology shock *i.e.* the production technologies generate high cash flows in the up state and low cash flows in the down state. To produce, each firm endogenously issues non-contingent, collateralized debt. The firms know the true probability over the high and low cash-flow states, but this is unobservable.[2] We assume a portion of the cash flows from production are not pledgeable so that production generates positive profits and control rents. Debt financing comes from investors with heterogeneous beliefs about the probability of the future states. We first solve the baseline model without CDS for equilibrium bond prices, firm investment, and the states in which firms choose to issue debt contracts characterized by positive rather than zero default premiums. Similarly to Che and Sethi (2012), we then extend the model to incorporate CDS contracts, where CDS purchasers must also own the underlying bond, which we call Covered CDS Economies. This restriction on CDS ownership is then removed so investors are free to purchase CDS without owning the underlying bond, in what we call *naked CDS economies*.[3]

We begin by characterizing the equilibrium based on whether the debt contracts

[2]Alternatively, one could assume that the state probabilities reveal a non-verifiable or non-contractible signal about cash flows.

[3]Norden and Radoeva (2013) document that there is clear firm heterogeneity in the size of the CDS market relative to the size of the underlying bond market supporting the CDS contracts. One natural way to interpret the covered CDS economy is as a CDS market that is "small" relative to the size of the underlying bond market whereby a sufficient level of investor capital remains available to purchase bonds. Naked CDS economies in our framework would correspond to very large outstanding CDS markets relative to the underlying bond market supporting those trades.

issued have a positive or zero default premium. Low values of the technology shock always result in debt contracts that contain positive risk premiums. Such debt contracts allow the firm to invest as much as possible in the up state where the firm is always the residual equity claimant. As a result of limited liability, creditors simply take possession of firm assets in the down state. Alternatively, the capital raised for production is limited when financed via debt contracts that are repaid even when cash flows are low. The lower investment level also limits the residual equity claims in the up state, when debts can be fully repaid using either debt contract. For given intermediate values of the technology shock, debt contracts that are default-risk-free are only possible when the likelihood of the up state is sufficiently low, because firms earn equity claims in the up state only when debt issues have a positive default premium (creditors become the residual claimants in the down state). The higher the likelihood than an up state will occur, the more firms invest. The only way debt contracts with a zero default premium can be repaid in the down state as investment levels increase is if the value of the technology shock also increases. The uninteresting case is when technology shocks are sufficiently high that debt contracts never contain a positive default premium because the cash flows in the down state are always sufficient to repay debt.

We then introduce covered CDSs into this characterization of equilibrium default premiums. Consistent with Che and Sethi (2015), covered CDSs raise bond prices and lower borrowing costs, because optimistic investors can hold more credit risk when using their cash as collateral to sell CDSs as a result of the implicit leverage embedded in CDSs rather than using their cash to buy bonds outright. The concentration of credit risk held by optimists increases the remaining supply of capital that can then finance other firm investment needs. Our analysis thus extends this mechanism to show that even when CDSs trade only on one bond, both firms' debt contracts are affected.[4] More interestingly, because investment is *endogenous*, firms respond to lower default premiums by issuing more debt. The consequence for the extensive default premium margin is that firms default in *more* states. With *fixed* investment, firms default in *fewer* states as in Che and Sethi (2015). The difference is that the lower default premiums on risky debt contracts incentivizes firms to issue more debt

[4]We do not endogenize which firm is the CDS obligor. Oehmke and Zawadowski (2016b) provide one rationale for why CDS emerge on certain firms and no others, such as debt covenants and fragmentation. We take this fact as given, and explore the general equilibrium financing implications. The qualitative results of our model hold regardless of which firm serves as the CDS obligor because the mechanism is symmetric.

at a lower cost. CDSs are redundant if the firm issues risk free debt, which means the terms of such debt contracts are unaffected by CDS trading. Thus, for a given technology fundamental in which a firm is indifferent between the two contracts in an economy with out CDSs, the firm strictly prefers to issue the risky debt contract in the economy with covered CDSs.

The restriction that CDS buyers must own the underlying bonds is then removed and naked CDS positions are the permitted. Naked CDSs also generate spillovers even when only one firm type serves as the CDS obligor. Pessimists increase the demand for CDSs, which raises CDS prices. Higher CDS prices reduce the amount of capital CDS sellers have to post to sell CDSs. The lower collateral requirement increases the embedded leverage that optimist receive and attracts more capital into derivative markets from natural bond buyers. The result is lower investor capital to fund debt for all firms and reduced investment levels. Consequently, default premiums also rise, which disincentives issuing risky debt. The tradeoff between default premiums and risky debt choice again stands in contrast to Che and Sethi (2015). Thus, both types of CDS contracts lead to a tradeoff between borrowing costs and default risk when investment is endogenous. Taken together, the model predicts that CDS markets have "unintended" consequences on corporate debt markets that are novel to the theoretical CDS literature.

The growing body of theoretical CDS literature examines how CDSs affect, bond, equity, and sovereign debt markets (see Augustin et. al (2014) for a complete and thorough survey on the broad literature.) Our work is most closely related to a class of heterogeneous agent models developed by Fostel and Geanokoplos (2012, 2016). Fostel and Geanokoplos (2012) show that in an endowment economy, financial innovation in credit derivative markets alters asset collateral capacities, and asset prices. Fostel and Geanakoplos (2016) study the effect that credit derivatives have on whether agents engage in production, showing that credit derivatives can lead to investment beneath the first best level obtained in an Arrow-Debreu economy and can robustly destroy equilibrium. Our model is distinguishable from their models because we have multiple firm types and consider the extensive margin between default-able and risk-less debt levels, which allows us to characterize spillovers and the tradeoff between borrowing costs and default risk. They are also more interested in the manner in which financial contracts are collateralized affects investment efficiency. Che and Sethi (2015) study how CDS affect borrowing costs for a representative firm with a random output draw that raises an exogenous amount of capital. Our model adds several relevant features by explicitly modeling an endogenous production environment with different firm types. Our model also gives rise to a more in-depth discussion of the investment and default decisions because of endogenous investment. Oehmke and Zawadowski (2015a) study the effects CDSs have on bond market pricing when investors have not only heterogeneous beliefs, but also heterogeneous trading frequencies. Their model is more suited to studying the effect of CDSs on secondary bond market activity. The authors do not consider investment in production or default. Common to all of these models, however, is the underlying mechanism through which CDS generate effects on the real economy. Mainly, the embedded leverage in the derivative contracts alters investor use of collateral that would otherwise be used to purchase bonds.

In a different stand of research Bolton and Oehmke (2011) show how CDS lead to an empty creditor problem from a contract theory perspective. Lenders' incentives to rollover loans are reduced, leading to increased bankruptcy and default risk. Firms internalize this effect ex ante and have stronger commitment to repay debt. Parlour and Winton (2013) show that CDS can reduce the incentives to monitor loans, thereby increasing default and credit risk. Similarly, Morrison (2005) shows that banks ability to sell off the credit risk of their loan portfolio leads firms to substitute away from monitored bank lending and into issuing risky public debt. Danis and Gamba (2015) study the trade-offs between higher ex ante commitment to debt repayment and higher ex post probability of default in a dynamic model with debt and equity issuance. They calibrate the model to U.S. data and find the positive benefits of lower CDS spreads stemming from higher repayment commitment dominate and increase welfare. The implicit assumption in all of these models is that CDS are used to hedge credit risk, which is equivalent to our covered CDS economy. All told, the increased default risk when covered CDSs trade in our model is, thus, complementary. We propose a new mechanism, though. Quite simply, lower default premiums and limited liability raise the incentives to issue bonds that default. However, none of the papers evaluate the effect on investment and default risk of situations in which investors may take purely speculative positions in the CDS market.

The default risk implicit in the empty creditor problem is that firms face debt refinancing needs that may not be met when creditors buy CDS contracts to insure against default. Thus there is a maturity-mismatch argument underlying that story.

Our theory does not require debt refinancing or maturity mis-match. In fact, debt liabilities and asset cash flows are perfectly aligned. The increase in default risk stemming from covered CDS trading in our model operates entirely through lower default premiums, which incentivizes issuing risky debt rather than debt that is default-risk-free.

Empirically, Norden et. al (2014) find evidence of interest rate spillovers in syndicated bank lending markets. The authors attribute these spillovers to more effective portfolio risk management. Our model suggests an alternative explanation operating through how derivatives change the demand for bonds when firm cash flows are correlated. Li and Tang (2016) find that there are leverage and investment spillovers between CDS reference firms and their suppliers. They argue that the higher the concentration of CDS reference entities is among a firm's customers, the lower supplier leverage ratios and investment levels are. The authors interpret their finding as the CDS market providing superior information about the credit quality of supplier firm customers (see Acharya and Johnson (2007) and Kim et. al. (2014) for works on the insider trading role of CDSs). Our model provides a complementary explanation under the interpretation that firms in our model are in the same industry. Investor demand for debt at the industry level is altered when firms in the industry become named CDS reference entities. Introducing CDSs changes the demand for bonds of other firms in the industry, and subsequent bond prices and firm investment levels.

Our paper also provides the following new testable implication: The effect on corporate default risk of trading CDS depends on whether the CDS buyer has an insurable interest in the underlying reference entity. Current empirical studies on the effect of CDSs on default risk resulting from the empty creditor problem (see Subrahmanyam, Wang, and Tang (2014); Kim (2013); and Shan, Tang, and Winton (2015)) cannot distinguish between covered and naked CDS positions.[5] Furthermore, the model's spillover implications call into question the widely employed method of propensity score matching used to control for the endogeneity of CDS issuance. Firm borrowing costs in matched samples will not be exogenous to CDS introduction if CDS trading alters the cost of capital for non-CDS reference firms whose cash flows are correlated with the CDS obligors.

The organization of the paper is as follows: In Section 2, we describe firms, debt contracts and investors. We then solve the baseline economy with no CDS contracts,

[5]This is an ongoing project we and co-authors are undertaking.

and describe the relevant comparative statics. In Section 3, we introduce covered CDSs. In section 4 we allow for naked CDS trading. In Section 5 we close with discussion and concluding remarks.

2 Non-CDS economy

2.1 Model

2.1.1 Time and Uncertainty

The model is a two-period general equilibrium model, with time $t = \{0, 1\}$. Uncertainty is represented by a tree, $S = \{0, U, D\}$, with a root, $s = 0$, at time 0 and two states of nature, $s = \{U, D\}$, at time 1. Without loss of generality we assume there is no time discounting. There is one durable consumption good in this economy that is also the numeraire good. We will refer to this good as cash throughout the paper.

2.1.2 Agents

Firms

There are two firms, $i = \{G, B\}$, in the economy where firm G is the "good" type and firm B is the "bad" type. Each firm is owned and operated by a manager with access to a production technology. The managers run the firms and consume from firm profits. One could think of the positive profits earned in equilibrium as non-pledgeable control rents with which equity owners are compensated to invest in production. We use the terms *control rents* and *profits* interchangeably.[6] The only difference between the two firms is the production technology at the respective managers' disposal. The firms use the durable consumption good as an input at time 0 and produce more of this good for consumption at time 1. The respective firms have standard, decreasing returns to scale, production functions given by $f_i(I_i; \alpha_i, A^s) = A^s I_i^{\alpha_i}$ with the following properties: $f_i^{'} > 0$, $f_i^{''} < 0$. Firm G is more productive than firm B; that is, $I^{\alpha_g} > I^{\alpha_b}$, $\forall\, 0 < I < 1$.

The technology shock, A^s, takes on binary values at time 1, with $A^U > A^D$. The technology parameter is identical for both firms. Consequently, the only type of

[6] We can abstract away from any agency problem between equity holders and firm managers. Alternatively, one could think of the managers as running the firm and being paid through equity.

uncertainty in our model is aggregate.[7] Idiosyncratic risk would not have an effect with two firms and two states, because agents would be able to perfectly insure themselves. The only way idiosyncratic risk would have an effect is if we considered more than two states (in which case there would still be non-aggregate risk remaining even after agents trade with each other).[8]

For simplicity we normalize the technology shock, A^U, to 1. Both firms have identical knowledge about the quality of their production process, where each firm knows that $s = U$ arrives with probability γ and $s = D$ with probability $(1 - \gamma)$. Lastly, firms are competitive price takers in the market for the durable consumption good.

Investors

We consider a continuum of uniformly distributed risk neutral investors, $h \in H \sim U(0,1)$, who do not discount the future. The absence of time discounting allows us to focus only on default premiums without loss of generality. Investors are characterized by linear utility for the single consumption good, x_s, at time 1. Each investor is endowed with one unit of the consumption good, $e^h = 1$, and assigns probability h to the up state U and $(1 - h)$ to the down state D. Thus, a higher h denotes more optimism and agents agree to disagree. The von Neumann–Morgenstern expected utility function for investor h is given by

$$U^h(x_U, x_D) = hx_U + (1 - h)x_D. \tag{1}$$

We assume a uniform distribution for tractability. The results will hold in general as long as the beliefs are continuous and monotone in h. In terms of investor preferences, we assume risk-neutrality but the results are also qualitatively preserved with common investors beliefs and state contingent endowments.

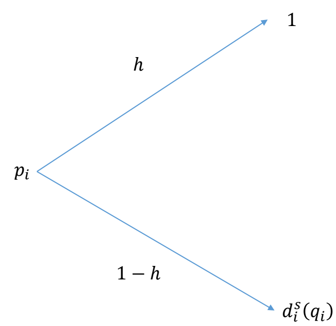
2.1.3 Firm financing

We assume the firms are highly leveraged and optimally raise capital from investors by issuing debt. Our aim is to better understand the effect of credit derivatives

[7]It may be natural to think of the model as one of intra-industry debt financing in which two firms in the same industry are equally affected by a industry specific technology shock.

[8]We leave this to further research. Also, it is interesting that in considering only aggregate risk, the firms differ only in the productivity and CDSs still have a big effect. With more dimensions of heterogeneity we predict the effect to be even bigger.

Figure 1: Bond payout



on the debt-issuance decision. The effect of credit derivatives on a more general capital allocation problem is an interesting natural extension of the model. At time 0, firms issue debt contracts that specify a fixed repayment amount (bonds) and are collateralized using the pledgeable proceeds from output, which we refer to as cash flows. The lender (investor) has the right to seize an amount of the collateral up to the value of the promise, but no more. This enforcement mechanism ensures that the firm will not simply default on all promises at time 1.

Each bond, priced p_i at time 0, promises a face value of 1 upon maturity. The two firms issue bonds denoted by q_i at time 0. In the up state, each bond returns full face value.[9] In the down state bonds pay creditors a deliverable, $d_i^s = \min\left[1, \frac{A^s I_i^{\alpha_i}}{q_i}\right]$. If debt obligations are not honored, creditors become the residual claimants of the firm and consume from the cash flows generated from production. Firm borrowing costs, which are equivalent to bond default premiums because there is no time discounting, are denoted by r_i, and equal to the difference in what the firm owes on maturity and the amount of capital they receive at the time of issuance, $r_i = 1 - p_i$. Figure 1 depicts the bond payouts.

2.1.4 Firm maximization problem

Each firm chooses an investment amount, I_i, given the market price of bonds to maximize profits. Since firms have no initial endowment, all their investment for

[9] We assume the bonds repay in full at $s = U$ to make the model interesting; otherwise, the firm does not invest in production.

production will have to be financed by issuing debt. Hence $I_i = p_i q_i$. Let π_i^s denote state-contingent firm profits. Firms solve the following program:

$$\left\{ \begin{array}{c} \max_{I_i} E\left[\pi_i\right] \equiv \Pi_i = \left\{ \gamma \left[A^U I_i^{\alpha_i} - q_i\right] + (1-\gamma)\left[A_i^D I_i^{\alpha_i} - q_i d_i^D\left(q_i\right)\right]\right\} \\ s.t.\, I_i = p_i q_i \end{array} \right\} \tag{2}$$

2.1.5 Investor Maximization Problem

We can now characterize each agents' budget set. Given bond prices p_i, each investor, $h \in H$, chooses cash holdings, $\left\{x_0^h\right\}$, and bond holdings, $\left\{q_i^h\right\}$, at time 0 to maximize utility given by (1) subject to the budget set defined by:

$$\begin{aligned} B^h\left(p_i\right) &= \Big\{ \left(x_0^h, q_i^h, x_s^h\right) \in R_+ \times R_+ \times R_+ : \\ & x_0^h + \sum_i p_i q_i^h = e^h, \\ & x_s^h = \left(1 - \sum_i p_i q_i^h\right) + \sum_i d_i^s q_i^h \Big\},\ s = \{U, D\}. \end{aligned}$$

Each investor consumes from two potential sources in either state of nature: consumption based on risk-less asset holdings and consumption based on their total bond portfolio. In the up state, consumption from bond holdings is equal to the quantity of bonds an investor owns in his portfolio because each bond has a face value of 1. In the down state, firms may default on their debt, in which case investors take ownership of the firm and consume from the firms' available assets on a per0-bond basis. Furthermore, note that we rule out short sales of bonds by assuming $q_i \in R_+$.[10]

2.1.6 Equilibrium

An equilibrium in the non-CDS economy is a collection of bond prices, firm investment decisions, investor cash holdings, bond holdings and final consumption decisions, $p_i, I_i, \left(x_0, q_i, x_s\right)_{h \in H} \in R_+ \times R_+ \times \left(R_+ \times R_+ \times R_+\right)$ such that the following are

[10] We believe this is not a terribly unreasonable assumption given the known difficulty in borrowing corporate bonds. CDS markets are far more liquid than secondary bond markets. Furthermore, in the spirit of Banerjee and Graveline (2014) derivative pricing in our model contains no noise, as will be clear in the following section, because the investors know technology fundamentals. Banerjee and Graveline show in proposition 6 that imposing a short-sale ban will have no effect on bond prices when investors can trade in derivatives with no noise.

satisfied:

1. $\int_0^1 x_0^h dh + \sum_i \int_0^1 p_i q_i^h dh = \int_0^1 e^h dh$

2. $\sum_i \int_0^1 q_i^h d_i^s dh + \sum_i \pi_i^s = \sum_i A^s I_i^{\alpha_i},\ s = \{U, D\}$

3. $I_i = \int_0^1 p_i q_i^h dh$

4. $\pi_i\left(I_i\right) \geq \pi_i(\hat{I}_i),\ \forall \hat{I}_i \geq 0$ for $i = \{G, B\}$

5. $\left(x_0^h, q_i^h, x_s^h\right) \in B^h\left(p_i\right) \Rightarrow U^h\left(x\right) \leq U^h\left(x^h\right),\ \forall h$

Condition (1) says that at time 0 the entire initial cash endowment is held by investors for consumption or used to purchase bonds from firms. Condition (2) says the goods market clears at time 1 such that all firm output is consumed either by firm managers via profits or by creditors via bond payments. Condition (3) corresponds with the capital market clearing conditions. Condition (4) says that firms choose investment to maximize profits, and condition (5) states that investors choose optimal portfolios given their budget sets.

2.2 Bond pricing

We begin by presenting how equilibrium is characterized, which will lay the foundation for analyzing the spillover effects in subsequent sections. We then analyze the firm investment/production problem in more detail to show how the choice to issue debt with or with out a positive default premium will also be affected by derivative trading.

Equilibrium in heterogeneous investor belief models, such as the one employed here, is based on marginal investors. In equilibrium, as a result of linear utilities, the continuity of utility in h, and the connectedness of the set of agents, $H = (0, 1)$, there will be marginal buyers, $h_1 > h_2$, at state $s = 0$. Every agent $h > h_1$ will buy bonds issued by firm B, every agent $h_2 < h < h_1$ will purchase type-G bonds, and every agent $h < h_2$ will remain in cash. This regime is shown in figure 2. The marginal buyer indifference between the returns of the various assets available in the economy will to be crucial in subsequent sections for understanding why introducing derivatives affects the default premium for all debt contracts in the economy and not just for those contracts on which the derivatives are based. With this, we now characterize the relationship between the respective firms' bond prices.

Proposition 1 *In any equilibrium without derivatives where both firm types receive funding and pay positive default premiums, type G bonds are priced higher than type B bonds.*

Proof. See appendix A.

The intuition is simple, in that the asset value of the more productive firm is always higher than the less productive firm. Debt holders will therefore be willing to pay a higher price for those bonds, since they are the residual claimants of the assets given default. Since both bonds pay 1 in full at $s = U$, more optimistic investors would rather hold the cheaper of the two assets, which are firm-B bonds. The two marginal investor indifference equations can be written as:

$$\frac{h_1 + (1 - h_1)\, d_g^D}{p_g} = \frac{h_1 + (1 - h_1)\, d_b^D}{p_b} \tag{3}$$

$$h_2 + (1 - h_2)\, d_g^D = p_g. \tag{4}$$

Equation (3) says that the more optimistic marginal buyer, h_1 will be indifferent to the expected returns on type-B and -G bonds. Equation (4) says that the less optimistic marginal buyer, h_2, will be indifferent between the return on type-G bonds and cash. The bond market clearing conditions for type-B and -G bonds require that the two bond prices be determined by the purchasing power of the respective sets of investors buying the bonds. That is $\frac{1-h_1}{p_b} = q_b$ for type-B bonds and $\frac{h_1-h_2}{p_g} = q_g$ for type-G bonds. This characterization clearly shows that bond prices are jointly determined based on how the marginal investors view the respective returns. A change in the price of one bond necessarily changes the expected relative return any investor places on the two assets. For example, an exogenous decrease in the price makes it more attractive and means it will become strictly preferred to the more expensive asset unless there is a corresponding change in the relative expected cash flows to which bond holders are entitled. In equilibrium, as more capital moves to the cheaper bond from the more expensive bond, the latter's price must also fall and will be marginally priced by a less optimistic investor.

2.3 Default premium

We now turn to the issue of when it is optimal to issue debt with positive rather than zero default premium. The maximization problem boils down to choosing an investment, I_i, that at $s = D$ ensures either repayment or, through limited liability, that the firm always defaults. In the former, bonds will be priced without risk: $p_i^f = 1$ where the super-script f denotes default risk-free pricing. In the latter case, bonds will carry a positive risk premium, $p_i^\rho < 1$, where the super-script ρ denotes a positive risk premium. Let the corresponding profit levels from each of these investment decisions be denoted by Π_i^f and Π_i^ρ. The firm thus chooses $I_i^*\left(\alpha_i, \gamma, A^D\right) \equiv \arg\max_{I_i}\left[\Pi_i^f, \Pi_i^\rho\right]$. The first-order conditions for the two investment levels are

$$I_i^\rho : \alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i^\rho} \tag{5}$$

$$I_i^f : \alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i^f}\left(\frac{1}{\gamma + (1-\gamma) A^D}\right). \tag{6}$$

These are the standard marginal products of capital that must equal the marginal costs of capital conditions. The default-risk-free condition in (6) takes into account that all debt is fully repaid at $s = U, D$ through the expected value of the technology shock, $\gamma + (1-\gamma) A^D$.

We begin by analyzing bonds with positive risk premiums. Using (5) and $I_i^{\rho *} = p_i^{\rho *} q_i^{\rho *}$, the bond repayment function, $d_i^s\left(q_i^{\rho *}\right)$, given $s = D$ can be written as

$$d_i^D\left(q_i\right) = \frac{A^D}{\alpha_i}. \tag{7}$$

Intuitively, recoverable firm asset values are proportional to the technology shock parameter, $0 < A^D < 1$, and the productivity parameter, $0 < \alpha_i < 1$. The recovery value also places a natural restriction on the relationship between the two parameters. Specifically, $A^D < \alpha_i$ ensures that there is, at least *fundamentally*, default risk in the economy. Since $\alpha_g < \alpha_b$ there are different values of A^D for which there is fundamental default risk for the two firms, $0 < A_G^D < \alpha_G < A_B^D < \alpha_B < 1$. As we show in subsequent discussion, for a given (α_G, α_B)-pair, whether or not firms issue bonds with positive default premiums depends on the given $\left(A^D, \gamma\right)$-tuple.

Turning now to risk-free bonds. Clearly, $p_i^{f*} = 1$ if firms always repay in all states. The risk free bond price gives the optimal investment level:
$I_i^{f*} = \left[\alpha_i\left(\gamma + (1-\gamma) A^D\right)\right]^{\frac{1}{(1-\alpha_i)}} = q_i^{f*}$. Note that the risk-free bond repayment

function, $d_i^s\left(q_i^{f*}\right) = \min\left[1, \frac{A^s\left(I_i^{f*}\right)^{\alpha_i}}{q_i^{f*}}\right] = 1$, implies that $\frac{A^D\left(I_i^{f*}\right)^{\alpha_i}}{q_i^{f*}} \geq 1$. Using (6) and $I_i^{f*} = q_i^{f*}$ gives a relationship between the good state probability, γ, and the value of the technology shock, A_i^D, for which issuing risk-free debt is *possible* even with fundamental default risk, $\alpha_i > A^D$. Formally,

Proposition 2 *There is a threshold value of the technology shock, $\underline{A}_i^D(\gamma)$, for any given state probability, γ. Technology shocks above this threshold allow for risk free bond pricing. The threshold value of $\underline{A}_i^D(\gamma)$ is an increasing function of γ.*

Proof. See appendix A.

The intuition underlying proposition 2 is that the cash flows from production must be sufficiently high in bad states (high A_i^D) for the firm to always honor its debt obligations. Additionally, the more likely it is that good states will arrive (high γ) the more firms invest because doing so increases their expected equity value given that the face value of debt is always equal to one. This means that cash flows in the down state must be even higher as γ increases if the firm is to fully honor its debt obligations.

We must still determine $I_i^*\left(\alpha_i, \gamma, A^D\right)$ in equilibrium. We can easily calculate the different profit levels, $\left\{\Pi_i^f\left(I_i^{f*}\right), \Pi_i^\rho\left(I_i^{\rho*}\right)\right\}$, that determine I_i^*. Plugging $q_i^{f*} = I_i^{f*}$ into (2), we obtain

$$\Pi_i^{f*} = \frac{(1-\alpha_i)}{\alpha_i} q_i^{f*}\left(\underline{A}_i^D(\gamma), \gamma\right).$$

Because debt is always repaid, profits are only affected by state probabilities, γ, through the affect on bond quantities, q_i^{f*}. Moreover, by proposition 2, an equilibrium in which risk-free bond pricing exists must be characterized by $A_i^D > \underline{A}_i^D(\gamma)$. If $A_i^D < \underline{A}_i^D(\gamma)$, borrowing with a zero default premium is not possible, and profits are calculated using the optimality condition (5) along with (2):

$$\Pi_i^{\rho*} = \gamma \times \frac{(1-\alpha_i)}{\alpha_i} q_i^{\rho*}\left(A^D\right).$$

Note from (5) that good state probabilities, γ, do not influence the investment levels, $I_i^{\rho*}$, when $p_i < 1$ because of limited liability. The firm always invests as if $\gamma = 1$ because they always default at $s = D$; only expected profits are influenced by γ. Comparing the two expected profit functions, it is clear that if the bond issuances are the same size $\left(q_i^{f*} = q_i^{\rho*}\right)$ firms would always prefer the risk-free investment level.

Equilibrium debt contracts characterized by positive default premiums must allow the firm to invest more than debt contracts that are risk free do, and the difference in the two amounts is proportional to the likelihood that the equity claimants retain positive equity γ, $\gamma > \frac{q_i^{f*}}{q_i^{\rho*}}$. Lastly, note that the only interesting range of technology shocks for analyzing a tradeoff between the two different debt issuances is $A^D \in \left[\underline{A}_i^D, \alpha_i\right)$. All shocks below $\underline{A}_i^D$ result in positive default premiums, $q_i^* = q_i^{\rho*}$, and all shocks above α_i result in risk free bond pricing, $q_i^* = q_i^{f*}$. The following proposition characterizes the parameter regions over which firms issue the two different types of bonds.

Proposition 3 *Firms will always issue bonds with positive default premium when $A_i^D < \underline{A}_i^D$ and will always issue bonds risk free for $A_i^D \geq \alpha_i$. For $A^D \in \left[\underline{A}_i^D, \alpha_i\right)$ firms issue bonds risk free if and only if $\gamma < \bar{\gamma}$.*

This result says that for intermediate values of stochastic cash flows, firms must optimally choose between debt for which they can always repay and debt that carries a positive default premium and will not be repaid when cash flows are low. Firms may only issue debt without (with) credit risk in this region if the likelihood that cash flows are good is sufficiently low (high) because when good cash flow states are likely, the default premium to issue debt will be lower, which incentivizes raising more capital that cannot be repaid if bad states arrive. Alternatively, when good cash flow states are less likely, the default premium will be higher, which incentivizes issuing smaller amounts of debt that can repaid given bad news.

Example 1 $\left\{A^D = 0.2, \gamma = 0.5, \alpha_G = 0.5, \alpha_B = 0.65\right\}$

This example gives results for default premiums, marginal buyers, and the remainder of the endogenous variables in the economy for the listed parameters. Figure 2 is the marginal buyer equilibrium characterization and table 1 gives the values of the endogenous variables. The values of the technology thresholds for determining whether default risk free bonds may be issued for $\gamma = 0.5$ are $\underline{A}_G^D = .3\bar{3}$ and $\underline{A}_B^D = .4814$, and clearly $p_i < 1$, $i = G, B$. The comparative statics for bond pricing and investment are not particular to the parameters chosen so long as $A^D < A_G^D$. Higher values of the technology shock naturally raise bonds prices, investment and control rents/profits. However, increases in the technology shock above the respective thresholds, $\underline{A}_i^D$, will change the comparative statics since the respective bond prices will be $p_i = 1$.

Figure 2: Non-CDS economy

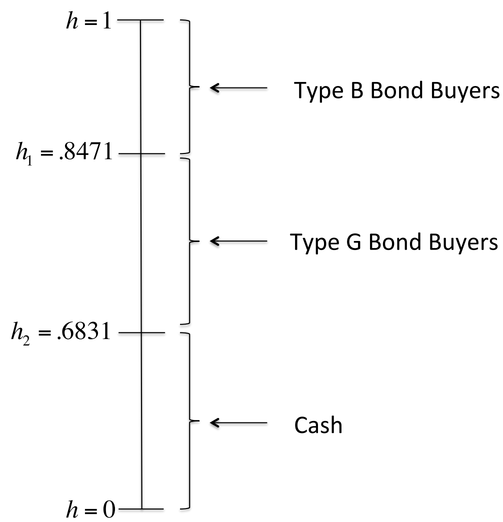


Table 1: Equilibrium values: *Non-CDS economy*

non-CDS economy		
	$i = G$	$i = B$
Price: p_i	.8099	.7973
Quantity: q_i	.2025	.1918
Investment: I_i	.1640	.1529
Output: Y_i^U	.4049	.2950
Exp.Profit: $E[\pi_i]$	.1012	.0516

3 Covered CDS economy

In this section, we incorporate CDSs into the baseline model. A CDS is a financial contract in which the CDS seller compensates the buyer for losses to the value of an underlying asset for a specified credit event or default. The underlying assets in this economy are firm bonds. CDS contracts compensate buyers with the difference between a bond's face value at maturity and its recovery value at the time of the credit event. Thus, CDS allow investors to hedge against idiosyncratic default risk.[11]

We first consider covered CDSs, in which buyers are required to also hold the underlying asset (that is, that bond for whom the CDS is written). We assume the seller must post enough collateral to cover the payment in the worst case scenario to rule out any counter-party risk. Let q_{ic}^h be the number of CDSs that investor h can sell, and let p_{ic} be the CDS price. Therefore, the total cash that investor h holds to collateralize CDS contracts, including payments received for selling CDSs, will equal the maximum possible CDS payout in the event of firm default, times the number of CDS contracts sold:

$$1 + p_{ic}\, q_{ic} = q_{ic}^h \left(1 - d_i^s\right). \tag{8}$$

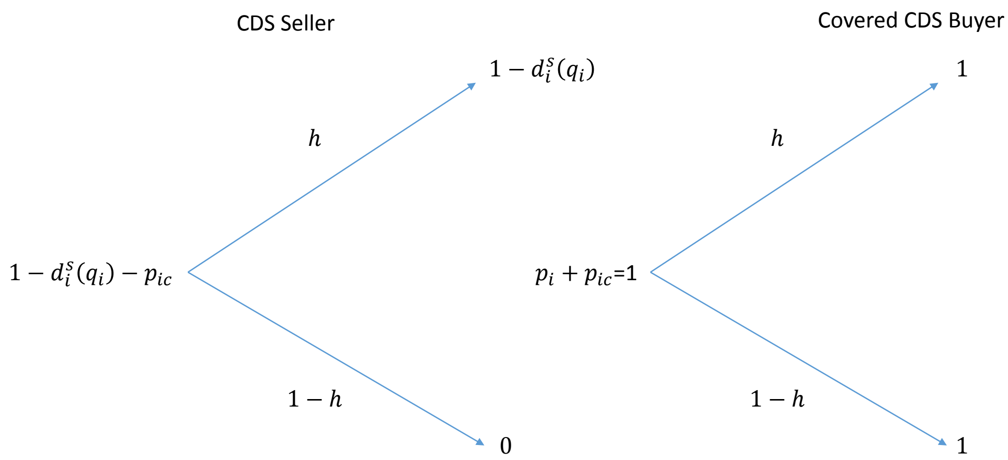
Solving for the total number of CDS contracts gives

$$q_{ic}^h = \frac{1}{1 - d_i^s - p_{ic}}. \tag{9}$$

Figure 3 shows the payout to the CDS seller and buyer. At time 0, a CDS seller must post a portion of his own collateral, $1 - d_i^s - p_{ic}$, to insure each CDS. At $s = U$, the CDS seller consumes the collateral, as the bond pays in full. At $s = D$, all of the collateral is used to compensate CDS buyers. Thus, selling CDSs is equivalent to holding an Arrow-Up security, as it pays out only when $s = U$.

[11] CDS do not allow investors to insure away aggregate risk.

Figure 3: Covered CDS payout



3.1 Investor maximization problem

Given bond and CDS prices, (p_i, p_{ic}), each investor h decides on cash, bond and CDS holdings, $\{x_0^h, q_i^h, q_{ic}^h\}$, to maximize utility (1) subject to the following budget set:

$$\begin{aligned} B^h(p_i, p_{ic}) \quad = \quad & \Big\{ \left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in R_+ \times R_+ \times R \times R_+ : \\ & x_0^h + \sum_i p_i q_i^h + \sum_i p_{ic} q_{ic}^h = e^h, \\ & x_s^h = x_0^h - \sum_i p_{ic} q_{ic}^h + \sum_i q_i^h d_i^s + \sum_i q_{ic}^h \left(1 - d_i^s\right), \ s = \{U, D\} \\ & max\left\{0, q_{ic}^h\right\} \leq q_i^h \Big\}. \end{aligned}$$

The first two equations are analogous to the investor budget set in the non-CDS economy. The third equation states that since CDS buyers are required to hold the underlying asset, the maximum number of CDS contracts that can be purchased cannot exceed the number of bonds owned. Note that there is no sign restriction on q_{ic}^h. Selling CDS implies that $q_{ic}^h < 0$, while $q_{ic}^h > 0$ implies purchasing CDS. Short selling of bonds is still ruled out by the restriction $q_i \in R_+$ as in the non-CDS economy.

3.2 Equilibrium

An equilibrium in the covered-CDS economy is a collection of bond prices, CDS prices, firm investment decisions, investor cash holdings, bond holdings, CDS holdings and final consumption decisions,

$p_i, I_i, (x_0, q_i, q_{ic}, x_s)_{h\in H} \in R_+ \times R_+ \times (R_+ \times R_+ \times R \times R_+)$, such that the following are satisfied:

1. $\int_0^1 x_0^h dh + \sum_i \int_0^1 p_i q_i^h dh = \int_0^1 e^h dh$
2. $\sum_i \int_0^1 q_i^h d_i dh + \sum_i \pi_i^s = \sum_i A^s I_i^{\alpha_i},\ s = \{U, D\}$
3. $\int_0^1 q_{ic}^h = 0$
4. $I_i = \int_0^1 p_i q_i^h dh$
5. $\pi_i \left(I_i\right) \geq \pi_i(\hat{I}_i),\ \forall \hat{I}_i \geq 0$
6. $\left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in B^h \left(p_i, p_{ic}\right) \Longrightarrow U^h \left(x\right) \leq U^h \left(x^h\right),\ \forall h$

Condition (1) states that all of the initial endowment is either held by investors or used to purchase bonds. Condition (2) says the goods market clears such that total firm output is consumed by firm managers via profits and used to repay bond holders. Condition (3) says that the CDS market is in zero net supply, while (4) states that the capital market clears. Condition (5) says firms choose investment to maximize profits. Lastly, (6) states that investors choose a portfolio that maximizes their utility, given their budget set.

We make use of the following lemma to characterize equilibrium in the covered CDS economy.

Lemma 1 *If* $0 < d_i^D \left(q_i\right) < 1$*, then no bonds for which CDS are sold will be purchased without CDS.*

Proof: See appendix B.

The intuition behind lemma 1 is that any investor optimistic enough to buy a bond without a CDS will be better off selling CDSs on that bond. Additionally, if the recovery value of the bond is zero, then CDSs and bonds pay the same amount in both states, thus making CDSs redundant assets. Finally, if the recovery value of the bond is 1, then bonds are risk free and no CDSs will trade in equilibrium.

3.3 Borrowing costs and spillovers

We introduce CDSs only on one firm's debt to establish the borrowing cost spillover result. For expositional purposes, let investors issue CDSs on firm-B debt.[12] As in the non-CDS economy, there will be *marginal buyers*, $h_1 > h_2$. In equilibrium, every agent $h > h_1$ will sell CDSs on type-B debt, every agent $h_2 < h < h_1$ will purchase type-G debt, and every agent $h < h_2$ is indifferent to holding covered CDS and cash. More specifically, agents $h < h_2$ hold a portfolio of covered positions on type-B and cash.[13] Compared with the non-CDS economy, covered CDSs lower borrowing costs for the firm on which CDSs are traded—firm B, because the derivative allows the optimists who believe firms will always repay their debt to hold all of the credit risk. Because CDS sellers only have to hold a portion of their collateral to cover the expected loss given default, which is always less than the price of the bond, each individual CDS seller can hold more credit risk than they could when just buying bonds. The resulting marginal buyers are therefore more optimistic than in the economy with out CDSs. The marginal buyer indifferent equations are now

$$\frac{h_1\left(1-d_b^D\right)}{1-p_{bc}-d_b^D} = \frac{h_1+(1-h_1)\,d_g^D}{p_g} \tag{10}$$

$$h_2+(1-h_2)\,d_g^D = p_g. \tag{11}$$

The CDS sellers' aggregate purchasing power to price CDSs is now $\frac{(1-h_1)}{1-p_{bc}-d_b^D} = q_b$. The purchasing power of the marginal type-G bond buyers takes the same form as the non-CDS economy: $\frac{h_1-h_2}{p_g} = q_g$, with different marginal buyers. Lastly, because some agents hold a portfolio of covered CDS and cash, a non-arbitrage pricing condition between the CDS and bond market links the price of credit risk in the CDS market with the default premium in the bond market: $p_{bc} + p_b = 1$. Consistent with Che

[12] The qualitative results for borrowing cost spillovers and endogenous positive default premiums are not particular to the firm's debt from which the CDSs derive their value. Endogenizing CDS issuance is an interesting question, but beyond the scope of this paper. Work by Banerjee and Graveline (2014), Bolton and Oehmke (2011), and Oehmke and Zawadowski (2015b) provides nice reference points for thinking about why we see CDSs emerge on some firms and not others.

[13] In equilibrium, every agent $h < h_2$ will be indifferent between cash and a covered position and each agent will hold a portfolio consisting of 30.43% in covered positions and the rest in cash, for the same parameters chosen in example 1. To compute this portfolio allocation we simply divide the number of type B bonds investors hold by their total cash endowment, $\frac{q_b}{h_2}$.

and Sethi (2015), but with a twist, we have the following bond pricing implications:

Proposition 4 *Covered CDSs raise bond pricing for the firm for whom CDSs are written.*

Proof. See appendix B.

More interestingly, because our model has an additional asset with a different fundamental valuation based on heterogeneous but correlated cash flows, there are general equilibrium borrowing cost implications for firm G even though investors do not trade CDSs on firm-G debt.

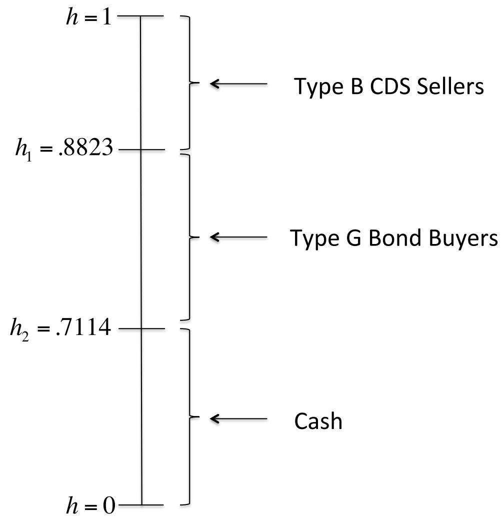
Corollary 1 *The default premium for a firm without CDSs trading is affected by CDSs trading on other firms with correlated cash flows.*

Proof. See appendix B.

This result follows directly from proposition 4 because the marginal CDS seller who prices the credit risk for firm B is more optimistic than the corresponding marginal bond buyer in the non-CDS economy. Because each CDS seller buys credit risk using collateral in the amount equal to the loss given default of the underlying bond contract, a given set of investors of size $(1 - h_1)$ in the non-CDS economy have more purchasing power to buy credit risk in the covered CDS economy. Firm G, competing with firm B to raise capital from optimistic investors, can therefore issue debt to more optimistic set of investors than in the non-CDS economy, as well. This increase in the set of investors financing both firms' debt issuances raises bond prices for firm G.

The fact that cash flows are correlated across good and bad states is also important. Take the limiting example in which firm G's cash flows are high at $s = D$ and low at $s = U$. The firm would then find it optimal to issue debt to those investors most willing to pay for an asset whose payouts are most correlated with their subjective beliefs. Firm G would then prefer to issue debt to pessimists who would pay a higher price for an asset that pays more at t $s = D$ with certainty than to optimists who would pay high prices for high payouts in the exact opposite states. It is this interpretation of the model that leads naturally to an intra-industry or geographically limited model of debt financing. Firms in the same industry or geographic area may

Figure 4: Type-B covered CDS economy

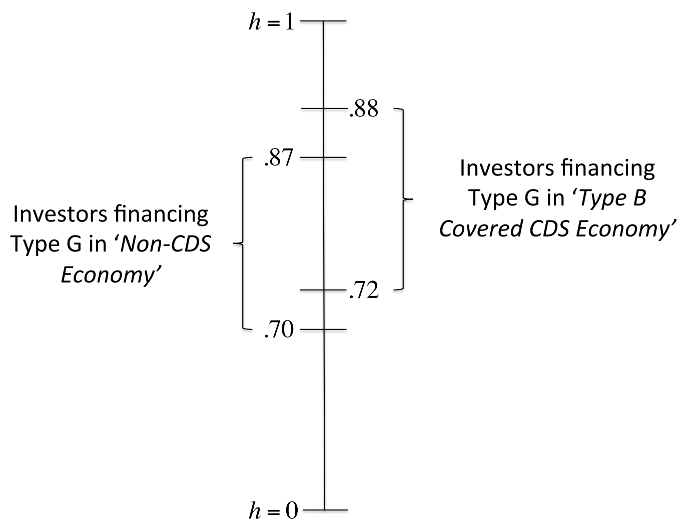


be more likely to default in similar states than firms in completely different industries or regions. Moreover, the recent empirical findings in Li and Tang (2016) on the leverage and borrowing of intra-industry firms are consistent with this interpretation of the model. Figure 4 and table 2 show the corresponding marginal buyer regimes and endogenous variables in the economy for the same parameters as in example 1. Note that even though the CDS trades on firm B, the price of firm G debt rises from $p_G = .8099$ to $p_G = .8269$. For the same parameters as in example 1, Figure 5 shows the equilibrium shift in the set of investors financing firm G's debt issuance in the non-CDS economy and the covered CDS economy with CDSs only trading on firm B.

Table 2: Equilibrium values: *Type-B covered CDS economy*

Type B Covered CDS Economy		
	$i=G$	$i=B$
Price: p_i	.8269	.8511
Quantity: q_i	.2067	.2165
Investment: I_i	.1709	.1843
Output: Y_i^U	.4134	.3331
Exp.Profit: $E[\pi_i]$	.1034	.0583

Figure 5: Covered CDS economy: *Type-G positive spillover*



3.4 CDSs and credit risk

In this section we show that the increase in bond pricing attributed to CDS trading also induces firms to issue debt that carries a positive default premium for more values of the down state probability than in the non-CDS economy. As shown in appendix A for proposition 2, the threshold values of $\underline{A}_i^D$ above which issuing default-risk-free debt is possible are determined only by parameters γ and α_i. CDSs do not change any of the fundamentals of the economy, so the threshold values are the same as in the non-CDS economy. What does change is the cost of issuing debt with a positive default premium and hence the relative costs and benefits of issuing such debt claims.

To see how CDSs affect the extensive default premium choice margin, recall that the necessary and sufficient condition to issue debt with a positive default premium is that the debt must yield higher control rents to equity claimants, which can be simply expressed through the optimal debt issuance levels of the two debt contracts: $\gamma\hat{q}_i^{\rho*} > q_i^{f*}$, where the hat corresponds to the non-CDS economy quantities. The left-hand side is the expected control rent associated with issuing debt with positive default premium and the right-hand side is the control rent from issuing debt that is always repaid. Because the default-risk-free contract is purely a function of fundamentals, the right-hand side of the inequality remains unchanged. From proposition 4 we know that firms borrow at lower borrowing costs and issue more bonds when CDSs trade. Thus, if there is a $\bar{\gamma} < 1$ for which $\bar{\gamma}\hat{q}_i^{\rho*} = q_i^{f*}$ in the non-CDS economy, then $\bar{\gamma}\tilde{q}_i^{\rho*} > q_i^{f*}$ in the covered CDS economy and $\bar{\gamma} > \tilde{\gamma} \equiv \gamma\tilde{q}_i^{\rho*} = q_i^{f*}$, where the tildes denote covered CDS economy variables. The decrease in the threshold γ in covered CDS economies above which firms issue risky debt establishes the first result of the trade-off between borrowing costs and default risk in CDS economies.

Proposition 5 *Give bad states, CDS contracts that lower the default premium of the underlying bond contract will tend to increase the likelihood that default occurs in equilibrium. For any value of $A_i^D \in \left[\underline{A}_i^D, \alpha_i\right)$ firms will issue default-risk free debt contracts if and only if $\gamma \leq \tilde{\gamma} < \bar{\gamma}$.*

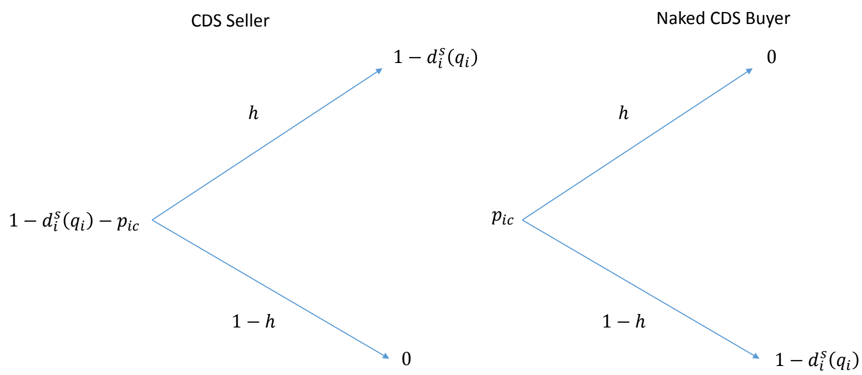
The endogeneity of production in the economy helps establish this result. In Che and Sethi (2015), covered CDS also lower borrowing costs, but they increase the ability to repay debt for any given investment level. The positive relationship between covered CDSs and ability to repay debt would be true in our model if I_i were fixed. When endogenized however, firms respond to bond pricing by issuing more debt. Because production rents are only captured in the up state when the debt raised to produce carries a positive default premium, a decrease in default premiums raises the resulting profitability from issuing these debt contracts, even though fundamentals do not change. If, in expectation, there was a state probability in the non-CDS economy for which the firm was indifferent between issuing either debt contract, the debt contract with a positive default premium would be strictly preferred. The decrease in default premiums and increase in investment are offset by the increase in default incidence at $s = D$.

4 Naked CDSs

In this section, we extend the model by allowing investors to hold naked CDS positions: investors do not need to hold the underlying asset to purchase a CDS. A naked CDS buyer expects to receive the difference between the face value of the bond and its value at the time of default. The naked CDS payout structure is given in figure 6. Furthermore, notice that buying a naked CDS is equivalent to buying the Arrow-down security, since it pays out only when $s = D$.

At time 0. an investor can purchase a naked CDS by paying p_{ic}. The buyer believes with probability h that the up-state will occur at time 1, the firm will not

Figure 6: Naked CDS payout



default, and the CDS will not payout. The buyer believes with probability $(1-h)$ that the downstate will occur at time 1. In this case, the buyer expects to receive the difference between the face value of the bond and its recovery value, $(1-d_i^s)$. We continue to assume that CDS sellers post enough collateral to cover payments in the worst-case scenario. Therefore the maximum CDS payout carries over from previous economies and is given by equation (9). Moreover, the implications of lemma 1 still hold and no bond is bought without credit protection if CDSs trade on that bond.

4.1 Investor maximization problem

Given bond and CDS prices (p_i, p_{ic}), each investor chooses cash, bond, and CDS holdings,$\{x_0^h, q_i^h, q_{ic}^h\}$, to maximize utility (1) subject to the budget set:

$$\begin{aligned} B^h(p_i, p_{ic}) \;\; = \;\; & \Big\{ \left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in R_+ \times R_+ \times R \times R_+ \} : \\ & x_0^h + \sum_i p_i q_i^h + \sum_i p_{ic} q_{ic}^h = e^h, \\ & x_s^h = x_0^h - \sum_i p_{ic} q_{ic}^h + \sum_i q_i^h d_i^s + \sum_i q_{ic}^h (1 - d_i^s) \Big\}. \end{aligned}$$

The investor's budget set is exactly the same as the one described in the covered CDS economy except that investors can now buy CDS without holding the underlying asset. Hence there is no restriction tying the maximum number of CDS contracts bought to the number of bonds held.

Equilibrium existence and institutional investor

Fostel and Geanakoplos (2014) identify an existence problem in production economies wherein collateral equilibrium may breakdown in the presence of naked CDS. In an earlier working paper version of their paper, Che and Sethi (2010) assume that a retail investor will always demand bonds so that equilibrium does not break down. We follow this assumption to compute equilibrium. Lastly, the presence of the institutional investor would not affect equilibrium in the non-CDS and covered-CDS economies. Limiting the investor's risk exposure of the investor will prevent it from purchasing bonds without any CDS protection. Thus the investor will remain in cash in an economy without CDS. Moreover, in the covered CDS economy, agents who did not sell CDS held a portfolio of cash and covered CDS positions. The institutional investor could just as easily replace the agents' covered CDS positions, leaving them only holding cash and the equilibrium unperturbed.[14]

4.2 Equilibrium

An equilibrium in the naked-CDS economy is a collection of bond prices, CDS prices, firm investment decisions, and investor consumption decisions,

$p_i, p_{ic}, I_i, (x_0, q_i, q_{ic}, x_s)_{h \in H}, \left(x_0^M, x_s^M\right) \in R_+ \times R_+ \times R_+ \times (R_+ \times R_+ \times R \times R_+) \times (R_+ \times R_+)$,

[14] A formal presentation of the retail investor problem is presented in appendix C.

such that the following are satisfied:

1. $\int_0^1 x_0^h dh + x_0^M + \sum_i p_i q_i^M + \sum_i p_{ic} q_{ic}^M = \int_0^1 e^h dh + e^M$
2. $\sum_i q_i^M + \sum_i \pi_i^s = \sum_i A^s I_i^{\alpha_i},\ s = \{U, D\}$
3. $\int_0^1 q_{ic}^h dh + q_i^M = 0$
4. $I_i = p_i q_i^M$
5. $\pi_i\left(I_i\right) \geq \pi_i(\hat{I}_i),\ \forall \hat{I}_i \geq 0,$
6. $\left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in B^h\left(p_i, p_{ic}\right) \Rightarrow U^h\left(x\right) \leq U^h\left(x^h\right),\ \forall h$
7. $\left(c_i^M\right) \in B^M\left(p_i, p_{ic}\right) \Rightarrow U^M\left(c\right) \leq U^h\left(c^M\right)$

Condition (1) states that all endowments, including the *retail investor's,* go to one of three uses: (a) held as collateral to issue CDSs, (b) held by the *retail investor* for consumption, or (c) used by the *retail investor* to purchase bonds and covered CDSs. Condition (2) says the goods market clears such that total firm output is consumed by firm managers in the form of profits and is used to repay bond holders. Condition (3) says that the CDS market is in zero net supply because all CDSs purchased as naked investments or by the retail investor as covered investments will be equal to all of the CDS issued. Condition (4) is the capital market clearing condition. Condition (5) says firms choose investment to maximize expected profits. Condition (6) states that investors choose a portfolio that maximizes their utility given their budget sets, and condition (7) says that the *retail investor* holds a portfolio that maximizes his utility given his budget set.

4.3 Borrowing costs and spillovers revisited

In this section, we show that the borrowing cost spillovers are also present in the naked CDS economy. To keep the examples comparable across economies, we continue to examine an economy for which CDSs trade only on firm-B debt. As before, there will be *marginal buyers*, $h_1 > h_2$. In equilibrium, every agent $h > h_1$ will sell CDSs on type$B$ debt, every agent $h_2 < h < h_1$ will purchase bonds issued by type G, and every agent $h < h_2$ will buy naked CDSs on type-B debt. Naked CDSs raise type B's borrowing costs because pessimists are able to purchase Arrow-Down securities created through the CDS contract, which increases the price a CDS seller receives for selling such contracts. As the price increases, investors can further leverage their cash endowments because they have to hold less of their own capital to issue CDSs. The resulting marginal CDS sellers are now more pessimistic than the bond buyers in the non-CDS economy. The marginal buyer indifference equations in the naked CDS economy are:

$$\frac{h_1\left(1-d_b^D\right)}{p_b-d_b^D}=\frac{h_1+(1-h_1)d_g^D}{p_g} \tag{12}$$

$$\frac{(1-h_2)\left(1-d_b^D\right)}{1-p_b}=\frac{h_2+(1-h_2)d_g^D}{p_g}. \tag{13}$$

Note here that firm-G bonds are priced by two separate investors. The more optimistic investor, h_1, is indifferent between selling a CDS on firm-B debt and buying firm-G bonds in (12). The more pessimistic investor, h_2, is indifferent between buying a CDS on firm-B debt and buying firm-G bonds in (13). This wedge between the Arrow-Up and the Arrow-Down securities means even though the arrow claims are traded, the Arrow-Debreu equilibrium is not established in this economy, as in Fostel and Geanakoplos (2012). The market-clearing conditions that establish the total purchasing power investors have to clear the markets are $\frac{(1-h_1)}{1-p_{bc}-d_b^D}=\left(q_b+\frac{h_2}{p_{bc}}\right)$ for the CDS market on firm B and $\frac{(h_1-h_2)}{p_g}=q_g$ for the bond market on firm G . Lemma 1 from the covered CDS economy continues to hold, implying that no bonds on which CDS are traded will be purchased without insurance. Furthermore, we derive the following lemma to solve for equilibrium in the naked CDS economy.

Lemma 2 *Only the institutional investor holds cash or covered assets in equilibrium.*

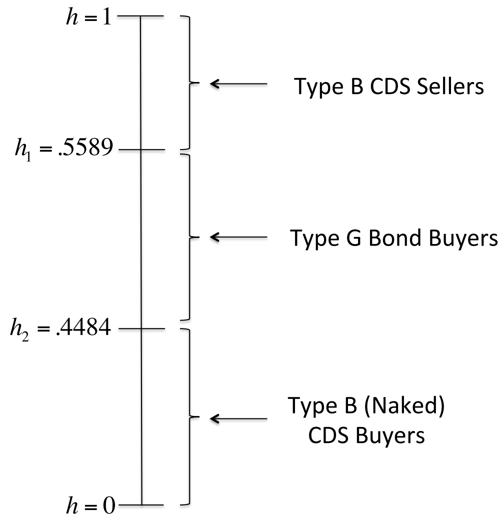
Proof. See appendix A. ■

The intuition for the optimists holds from lemma 1. The for pessimists, any investor pessimistic enough to remain in cash will be better off buying a naked CDS. Thus, only the institutional investors hold risk-free assets. The borrowing cost implications for the economy are similar to those in Fostel and Geanakoplos (2012 and 2016) and Che and Sethi (2015) and are summarized in the following proposition.

Proposition 6 *Naked CDS lower bond prices for the firm for whom CDS are written.*

Proof. See appendix B.

Figure 7: Type-B naked CDS economy

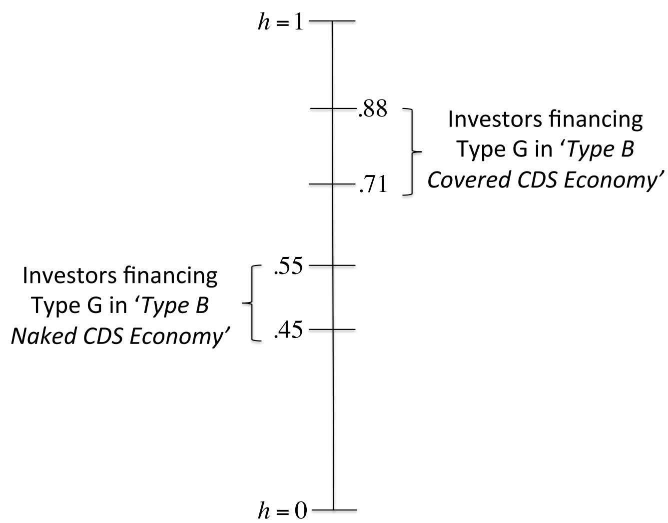


As described above, the derivative markets attracts more optimists to sell CDS as pessimists bid up the prices of derivative contracts. This "capital reallocation channel" is common to a class of heterogeneous agent models used to analyze CDS and bond market pricing (see Fostel and Geanakoplos (2012 and 2016), Che and Sethi (2015), and Oehmke and Zawadowski (2015) for other examples). What is new in our model in terms of borrowing cost implications, is, once again, the fact that firm G will also be affected by the movement of capital from bond market purchases to collateralizing derivative contracts because firm G's cash flows from production are correlated with firm B's. Figure 7 and table 3 show the marginal buyer characterization of equilibrium and the values of the endogenous variables for the same parameters used in the non-CDS and covered CDS Economies. Figure 8 shows the shift in marginal investors financing firm-G debt in the naked CDS and covered CDS economies.

Table 3: Equilibrium values: *Type-B naked CDS economy*

Type B Naked CDS Economy		
	$i = G$	$i = B$
Price: p_i	.6341	.6381
Quantity: q_i	.1585	.1268
Investment: I_i	.1005	.0809
Output: Y_i^U	.3170	.1950
Exp.Profit: $E[\pi_i]$	.0793	.0341

Figure 8: Naked CDS economy: *Good firm negative spillover*



4.4 CDS and default risk revisited

As with the covered CDS economy, the default-risk-free contract is unchanged. The debt levels that are sustainable and permit firms to fully honor debt obligations in all states are driven by economy fundamentals, $\left(\alpha_i, \gamma, A^D\right)$. Therefore, the change in the default premium associated with issuing a risky debt contract alters the extensive margin between risky and risk-free contracts. Using proposition 6, we know that default premiums increase in the naked CDS economy and firms respond by issuing less debt, $\dot{q}_i^\rho < \hat{q}_i^\rho$, where hats denote the risky bond quantity in the non-CDS economy and dots denote the equivalent in the naked CDS economy. Thus, for a given state realization in which $\bar{\gamma}\hat{q}_i^\rho = q_i^{f*}$ holds in the non-CDS economy, $\bar{\gamma}\dot{q}_i^\rho < q_i^{f*}$ will hold in the naked CDS economy.

Proposition 7 *Given bad states, CDS contracts that raise the default premium of the underlying bond contract will tend to decrease the likelihood that default occurs in equilibrium. For any value of $A_i^D \in \left[\underline{A}_i^D, \alpha_i\right)$, firms will issue default-risk-free debt contracts if and only if $\gamma \geq \dot{\gamma} > \bar{\gamma}$.*

Propositions 5 and 7 show that the effect on default premiums and endogenous investment of trading CDSs can be counterbalanced by the effect on the decision to issue risky rather than risk-free debt. On the one hand, CDS contracts allow the leveraging of cash, which changes the demand for bonds and the default premiums at which firms issue risky debt. Risk-free debt contracts, on the other hand, are determined by fundamental technology processes and state probabilities, and they are therefore unaffected by the presence of derivative trading. The fact that derivative trades affect default premiums when firms issue risk debt implies that the relative incentives to choose between debt contracts that do or do not contain default risk are altered even when fundamentals do not change. Thus, financial innovation through derivative trading may effect not only the intensive margin and how much debt to issue, but the extensive margin of whether debt contracts contain default risk.

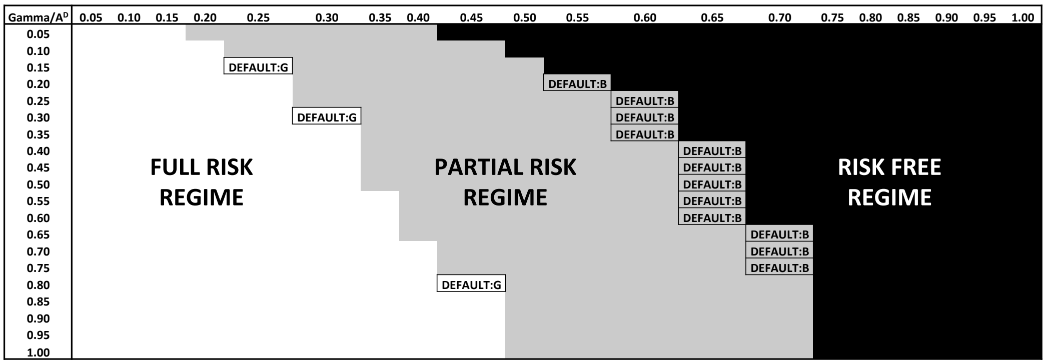
5 Discussion and concluding remarks

Credit derivatives can induce a tradeoff between firm borrowing costs and default probability. Lower default premiums incentivize investment for which debt obligations are not met in states characterized by negative productivity shocks. Alternatively, higher default premiums lead to debt financing that firms are more likely to repay for given productivity shocks. Financial instruments that effect default premiums can induce such a tradeoff even when nothing fundamental changes in the economy. In terms of our model, the set of parameters $\left(\gamma,\, A^D\right)$ in the different economies for which each firm defaults changes as CDS are introduced.

Figure 9 illustrates the effect covered CDSs have on default relative to the non-CDS economy. The parameters are those chosen for example 1 throughout the paper. The diagram shows the different possible default regimes. The area labeled "full risk regime" is the set of parameter values for which both firms default when $s = D$. Similarly, "partial risk regime" corresponds to the region where only type-B defaults, and neither firm defaults in the "risk free regime." The individual cells outlined and labeled "default G" and "default B" highlight the impact of introducing covered CDSs. For example, relative to the non-CDS economy, all of the default B cells indicate that firm B defaults on its debt obligations. Likewise, relative to the non-CDS economy, all of the default G cells indicate that firm G defaults on its debt obligations. The likelihood of $s = U$ given by γ is on the vertical axis (increasing γ moving downward) and the value of the technology shock, A^D, on the horizontal (increasing A^D going from left to right).

You can see that for a given A_i^D in which borrowing risk free is possible, firm B defaults in the down state for more values of γ (the gray default B cells would

Figure 9: Increased default risk: *Non-CDS versus covered CDS*

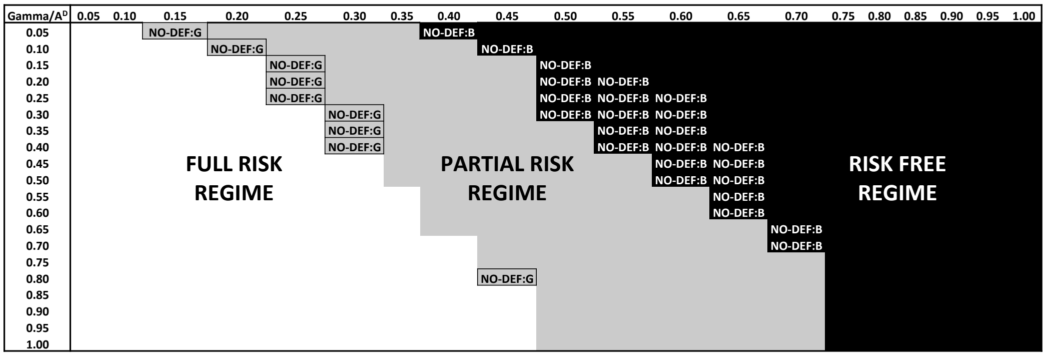


be black in the non-CDS economy).[15] The associated increase in default risk in the covered CDS economy is consistent with the empty creditor story. However, our mechanism is novel and does not rely on a maturity mis-match between firm assets and liabilities or rollover risk and coordination problems. Covered CDS can increase default risk through the pricing incentives to issue debt that defaults in bad times even when assets and liabilities are perfectly aligned and no maturity-mismatch is present. Additionally, the model shows that having an insurable interest and owning the CDS on that interest (covered) leads to very different default implications than taking a speculative position (naked). Current empirical studies cannot distinguish between covered and naked positions at the CDS-counterparty level and thus only test the covered story of the empty creditor problem.

Figure (10) shows the equilibrium comparing the covered CDS and naked CDS economies for the parameters used in example 1. It is the analogue to figure 9 in that it shows the effect of introducing naked CDSs when CDSs already exist. We make this comparison to show the effect on default of banning naked CDS; presumably, covered CDS would still be allowed. The cells labeled “No Def G” indicate that firm G switches from defaulting on debt obligations in covered CDS economies to fully repaying when naked CDS are introduced. Similarly, the cells labeled “No Def B” indicate that firm B switches from defaulting on debt obligations in covered CDS economies to fully repaying when naked CDS are introduced. In contrast to the

[15]The default effect on firm B is more pronounced in this example because the derivative is introduced on firm B. Still, there is a spillover default effect on firm G as well.

Figure 10: Decreased default risk: *Covered CDS versus naked CDS*



previous example, there are fewer values of γ for which the firms default. These results highlight the tradeoff between borrowing costs and default risk not identified in the extant CDS literature.

We further show, through the capital reallocation channel other models have highlighted, that when the productive assets on which CDSs trade have correlated cash flows, the returns between the debt contracts that finance the assets must be equal in equilibrium. This equality leads to spillover effects even as CDS markets do not exist on all underlying assets. The spillover effects include changes in bond pricing, investment demand, and default risk. Thus, our model provides a new mechanism through which to think about the empirical results on the leverage and investment externalities of CDS trading identified by Li and Tang (2016).

References

[1] Acharya, Viral and Timothy Johnson, (2007). "Insider Trading in Credit Derivatives." *Journal of Financial Economics.* 84, 110-141.

[2] Ashcraft, Adam and João Santos, (2009) "Has the CDS Market Lowered the Cost of Corporate Debt?", *Journal of Monetary Economics,* 56, 514-523.

[3] Augustin, Patrick, Marti Subramanyam, Dragon Yongjun Tang, and Sara Qian Wang, (2014). "Credit Default Swaps: A Survey." *Foundations and Trends® in Finance.* vol 9, No 1-2, 1-196.

[4] Banerjee, Snehal and Jeremy Graveline, (2014). "Trading in Derivatives when the Underlying is Scarce." *Journal of Financial Economics.* 111, 589-608.

[5] Bernanke, Ben, (2008), "The Future of Mortgage Finance in the United States.", *Address at* UC Berkeley/UCLA Symposium: The Mortgage Meltdown, the Economy and Public Policy, Berkeley, California.

[6] Bolton, Patrick and Martin Oehmke (2011), " Credit Default Swaps and the Empty Creditor Problem.", *Review of Financial Studies*, 24 (8), February 2011.

[7] Che, Yeon-Koo and Rajiv Sethi, (2015) "Credit Market Speculation and the Cost of Capital," *American Economic Journal: Microeconomics.* 6(4), 1-34.

[8] Danis, András and Andrea Gamba, (2015). "The Real Effects of Credit Default Swaps." Working Paper.

[9] Financial Times, (2011), "Bankers Fear Political Motives Will Kill Off CDS" David Oakley and Tracy Alloway. Capital Markets, October, 25, 2011

[10] Fostel, Ana and John Geanakoplos, (2016), "Financial Innovation, Collateral and Investment." *American Economic Journal: Macroeconomics.* 8(1), 242-84.

[11] Fostel, Ana and John Geanakoplos, (2012), "Tranching, CDS, and Asset Prices: How Financial Innovation Can Cause Bubbles and Crashes." *American Economic Journal: Macroeconomics*, Vol. 4, no 1, 190-225.

[12] Kim, Gi, (2013). "Credit Default Swaps, Strategic Default and the Cost of Corporate Debt." Working Paper.

[13] Kim, Jae, Pervin Dushyantkumar, Shroff Vyas ,and Regina Wittenberg-Moerman, (2014). "Active CDS Trading and Managers' Voluntary disclosure, Working Paper. University of Chicago.

[14] Geanakoplos, John, (2010), "The Leverage Cycle," In *NBER Macroeconomics Annual 2009.* Vol 24 edited by Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, 1-65. Chicago: University of Chicago Press.

[15] Li, Jay and Dragon Youngjun Tang, (2016). "The Leverage Externalities of Credit Default Swaps." Forthcoming: *Journal of Financial Economics.*

[16] Morrison, Alan, (2005). "Credit Derivatives, Disintermediation, and Investment Decisions." *Journal of Business.* 78(2), 621-648.

[17] Norden, Lars, Consuelo Silva Buston and Wolf Wagner (2014). "Financial Innovation and Bank Behavior: Evidence from Credit Markets." *Journal of Economic Dynamics and Control.* vol. 43, 130-145.

[18] Norden, Lars, and Kristina Radoeva, (2013). "What do We Know about Speculation in the CDS Market?" Working Paper. Midwest Finance Association 2013.

[19] Oehmke, Martin and Adam Zawadowski, (2015a). "Synthetic or Real? The Equilibrium Effects of Credit Default Swaps on Bond Markets" *Review of Financial Studies* 28(12), 3307-3337.

[20] Oehmke, Martin and Adam Zawadowski, (2015b). The Anatomy of the CDS Market" Forthcoming: *Review of Financial Studies.*

[21] Parlour, Christine and Andrew Winton, (2013). "Laying Off Credit Risk: Loan Sales Versus Credit Default Swaps." *Journal of Financial Economics.* vol. 107, 25-45.

[22] Shan, Susan, Dragon Yongjun Tang, and Andrew Winton, (2015). "Market Versus Contracting: Credit Default Swaps and Creditor Protection in Loans." Working Paper.

[23] Subrahmanyam, Marti, Dragon Yongjun Tang, and Sara Qian Wang, (2014). "Does the Tail Wag the Dog? The Effect of Credit Default Swaps on Credit Risk." *Review of Financial Studies.* 27(10). 2927-2960.

A Appendix

A.1 Partial-Risk

A.1.1 Non CDS Equilibrium

This equilibrium is characterized by the fact that Firm G is always able to fully repay its debt while the bad firm defaults when $s = D$. This equilibrium will be characterized as a combination of the *Risk-Free Regime* presented in the preceding section and the *Full-Risk Regime* described in Section 2.2.

The system of equations that characterizes equilibrium in the *non-CDS economy* includes eight endogenous variables $(p_i, q_i, I_i, h_1, h_2)$ and is as follows:

$$1. h_1 + (1 - h_1)\, d_b^D = p_b$$

$$2. h_2 + (1 - h_2) = p_g$$

$$3. \alpha_g I_g^{\alpha_g - 1} = \frac{1}{[\gamma A^U + (1 - \gamma)\, A^D]}$$

$$4. \alpha_b I_b^{\alpha_b - 1} = \frac{1}{p_b}$$

$$5. I_i = p_i q_i$$

$$6. 1 - h_1 = p_b q_b$$

$$7. h_1 - h_2 = p_g q_g$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equation (3) corresponds to type G's optimizing decision that takes into account firm profitability in the down-state while (4) corresponds to type B's optimizing decision by considering profits exclusively in the up-state. Equation (5) says that each firms' bond issuance q_i will be in accordance with the desired investment level given bond market prices. Finally, equations (6) & (7) correspond to the bond market clearing conditions for the respective firms.

A.1.2 Covered CDS Equilibrium

CDS allow investors to sell insurance on firm default. Note that in the *Covered-CDS Economy,* CDS buyers are required to hold the underlying asset. In equilibrium, CDS will only be traded on type B debt since type G does not default in the *Partial-Risk Regime*.

The system of equations that characterizes equilibrium in the *Covered-CDS Economy* includes ten endogenous variables $(p_i, p_{bc,}q_i, q_{bc,}I_i, h_1, h_2)$ and is as follows:

$$1.\frac{h_1\left(1-d_b^D\right)}{p_b-d_b^D}=1$$

$$2.h_2+(1-h_2)=p_g$$

$$3.\alpha_g I_g^{\alpha_g-1}=\frac{1}{p_g\left[\gamma A^U+(1-\gamma)A^D\right]}$$

$$4.\alpha_b I_b^{\alpha_b-1}=\frac{1}{p_b}$$

$$5.I_i=p_i q_i$$

$$6.1-h_1=p_b q_b-q_b d_b^D$$

$$7.h_1-h_2=p_g q_g$$

$$8.p_b+p_{bc}=1$$

$$9.q_{bc}=\frac{1}{1-p_{bc}-d_b^D}$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equations (3) & (4) corresponds to the firms' optimizing decisions. Equation (5) says that each firms' bond issuance q_i will be in accordance with the desired level given market price for bonds. Equations (6) & (7) correspond to the CDS market and bond market clearing conditions respectively. Equation (8) is a non-arbitrage pricing implication of assuming the return to holding a covered CDS position is equivalent to holding cash and will be priced accordingly. Equation (9) solves for the number of CDS contracts an individual investor can sell given their initial endowment.

A.1.3 Naked CDS Equilibrium

In the *Naked-CDS Economy* CDS buyers are not required to hold the underlying asset. In equilibrium, CDS will only be traded on type B debt since type G does not default in the *Partial-Risk Regime*.

The system of equations that characterizes equilibrium in the *Naked-CDS Econ-*

omy includes ten endogenous variables $(p_i, p_{bc,} q_i, q_{bc,} I_i, h_1, h_2)$ and is as follows.

$$1. \frac{h_1 \left(1 - d_b^D\right)}{p_b - d_b^D} = \frac{(1 - h_1) \left(1 - d_b^D\right)}{1 - p_b}$$

$$2. h_2 + (1 - h_2) = p_g$$

$$3. \alpha_g I_g^{\alpha_g - 1} = \frac{1}{\left[\gamma A^U + (1 - \gamma) A^D\right]}$$

$$4. \alpha_b I_b^{\alpha_b - 1} = \frac{1}{p_b}$$

$$5. I_i = p_i q_i$$

$$6. (1 - h_1) + p_{bc} q_b + h_2 = \left(q_b + \frac{h_2}{p_{bc}}\right) \left(1 - d_b^D\right)$$

$$7. h_1 - h_2 = p_g q_g$$

$$8. p_b + p_{bc} = 1$$

$$9. q_{bc} = \frac{1}{1 - p_{bc} - d_b^D}$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equations (3) & (4) corresponds to the firms' optimizing decisions. Equation (5) says that each firms' bond issuance q_i will be in accordance with the desired level given market price for bonds. Equations (6) & (7) correspond to the CDS market and bond market clearing conditions respectively. Equation (8) is a non-arbitrage pricing implication of assuming the return to holding a covered CDS position is equivalent to holding cash and will be priced accordingly. Equation (9) solves for the number of CDS contracts an individual investor can sell given their initial endowment.

B Appendix Omitted Proofs

Proposition 1. **Proof.** The two bond delivery functions are given by $d_i^s\left(q^i\right), i = G, B.$ Note that $d_g^U\left(q_g\right) = d_b^U\left(q_b\right)$, and $d_g^D\left(q_g\right) = \frac{A^D}{\alpha_g} > \frac{A^D}{\alpha_b} = d_b^D\left(q_g\right)$. This means that the bonds deliver the same in the up-state at time 1, and the type g bond returns strictly more than the type b bond in the down state, hence $d_1^g\left(q_0^g\right) > d_1^b\left(q_0^b\right)$. If $p_g = p_b$ every investor would strictly prefer the type g bond because the expected return would be higher and for no additional cost. Thus firm b would not receive debt financing. Moreover, for any investor to be indifferent between the two assets,

$p_g > p_b$ because $d_1^g\left(q_0^g\right) > d_1^b\left(q_0^b\right)$. ■

Proposition 2. **Proof.** Plugging in $I_i^{f*} = q_i^{f*}$ from (6) into the risk free bond delivery function $d_i^D\left(q_i^{f*}\right) : A_i^D\left(q_i^{f*}\right)^{\alpha_i - 1} \geq 1$ gives

$$\left\{\left[\alpha_i\left(\gamma + (1-\gamma)A_i^D\right)\right]^{\frac{1}{1-\alpha_i}}\right\}^{\alpha_i - 1} \geq 1.$$

Solving through for A_i^D yields the following expression:

$$\frac{\alpha_i \gamma}{1 - \alpha_i(1-\gamma)} \leq A_i^D. \tag{14}$$

Let $\underline{A}_i^D \equiv \frac{\alpha_i\gamma}{1-\alpha_i(1-\gamma)} = A_i^D$. Therefore, any $A_i^D > \underline{A}_i^D(\gamma)$ will allow the firm to issue risk free bonds when $\Pi^{f*} > \Pi^{\rho*}$. It is straightforward to verify $\frac{\partial \underline{A}_i^D}{\partial \gamma} > 0$ for $\alpha_i < 1$. ■

Proposition 3. **Proof.** The extreme cases where $A_i^D < \underline{A}_i^D$ implies $p_i < 1$ and $A_i^D \geq \alpha_i$ implies $p_i = 1$ follow directly from the discussion is the text. For $A_i^D \in \left[\underline{A}_i^D, \alpha_i\right)$ we know from Proposition 2 that $\underline{A}_i^D(\gamma)$ is increasing in γ. Let $\underline{\gamma}$ correspond to a value that determines a threshold of $\underline{A}_i^D\left(\underline{\gamma}\right)$, so that realizations of A_i^D above this can support $p_i = 1$. Now choose a value of $\hat{A}_i^D \in \left(\underline{A}_i^D\left(\underline{\gamma}\right), \alpha_i\right)$. The corresponding $\hat{\gamma}$ for which $\hat{A}_i^D$ is also a threshold for determining whether risk free debt is permissible must also be higher: $\hat{\gamma} > \underline{\gamma}$. This means that if $\hat{A}_i^D(\hat{\gamma})$ corresponds to risk free borrowing for $\hat{\gamma}$ it will also correspond to risk free borrowing for $\underline{\gamma}$ and $\forall \gamma \leq \hat{\gamma}$. Alternatively, if a value $\overline{\gamma}$ only corresponds to risk free borrowing for A_i^D such that $\alpha_i < A_i^D \leq \hat{A}_i^D < \bar{A}_i^D$, then the tuple $\left(\hat{A}_i^D, \bar{\gamma}\right)$ will not be risk free, nor will any tuple $\left(\hat{A}_i^D, \gamma\right)$ with $\gamma \geq \bar{\gamma}$. Thus if borrowing risk free is possible for any given $A_i^D \in \left[\underline{A}_i^D\left(\underline{\gamma}\right), \alpha_i\right)$, it will be the case that there is an upper-bound on the probability of good news $\underline{\gamma} < \overline{\gamma}$ that will push the firm into issuing bonds with a positive default premium. ■

Lemma 1. **Proof.** Suppose to the contrary that bonds are purchased unprotected. Then it must be the case that the utility of the agent who buys the unprotected bond is given by $u^b(h_1) = \frac{h_1 + (1-h_1)d_b^D}{p_b} > 1$, which can be written as $\frac{h_1\left(1-d_b^D\right)+d_b^D}{p_b} > 1$. Note that the utility of the CDS seller is given by $u^s(h_{cds}) = \frac{h_{cds}\left(1-d_b^D\right)}{p_b - d_b^D}$. Now suppose that the investor h_1 who purchases the bad bond unprotected instead writes the CDS. His utility would be given by $u^s(h_1) = \frac{h_1\left(1-d_b^D\right)}{p_b - d_b^D}$. To finish the proof it suffices to show that h_1 prefers to write CDS over buying unprotected bonds. Let $h_1\left(1-d_b^D\right) = X$, $p_b = Y$, and $d_b^D = \Lambda$. We can then rewrite the utilities in the following way:

$u^b(h_1) = \frac{X+\Lambda}{Y}$ and $u^s(h_1) = \frac{X}{Y+\Lambda}$. If $u^s(h_1) > u^b(h_1)$ then,

$$\begin{aligned}
&\Longrightarrow \quad \frac{X+\Lambda}{Y} < \frac{X}{Y-\Lambda} \\
&\Longrightarrow \quad (X+\Lambda)(Y-\Lambda) < XY. \\
&\Longrightarrow \quad -X\Lambda + \Lambda Y - \Lambda^2 < 0 \\
&\Longrightarrow \quad (X - Y + \Lambda) > 0.
\end{aligned}$$

Substituting back in for X, Y, and Λ we see that $h_1\left(1-d_b^D\right) - p_b + d_b^D > 0$, which is the same as $\frac{h_1+(1-h_1)d_b^D}{p_b} > 1$. Thus, any agent who would buy unprotected bonds would be better off selling CDS. ■

Proposition 4. **Proof.** This follows directly from Che & Sethi (2014) with two differences. 1) the marginal investor indifferent between selling CDS and having exposure to another risky asset does not directly price the bond for which the CDS is sold. The investor prices the return to the CDS relative to the return on the alternative risky asset. 2) investment is endogenous so that supply of bonds q_i is not fixed.

Suppose to the contrary that $\tilde{p}_i \leq \hat{p}_i, i = G, B$, where $\tilde{p}_i$ is firm is bond price in an economy with covered CDS and $\hat{p}_i$ is the bond price in a *Baseline Economy.* The expected return any given investor h places on selling a CDS is given by

$$\frac{h\left(1-d_i^D\right)}{\tilde{p}^i - d_i^D}.$$

In the up state, a CDS seller retains the value of the contract, $\left(1-d_i^D\right)$. The CDS seller must hold enough collateral to sell this contact. That collateral value is equal to $1 - d_i^D - p_i^c$. The fundamental recovery value of the bond d_i^D is not insured, and investors receive a price for selling CDS that they can use as collateral for additional contracts p_i^c. This is a decreasing function of bond prices because higher bond prices mean lower CDS prices. Higher bond prices mean investors need to post more of their own cash as collateral to sell a CDS, which lowers the expected return to their collateral. Thus, as bond prices decrease, the marginal investor indifferent to selling a CDS and buying an alternative risky asset is less optimistic because the returns to selling a CDS are increasing relative to the return on holding the alternative asset. Therefore, $h^1\left(\tilde{p}_i\left(\tilde{q}_i\right), d_i^s\right) \leq h^1\left(\hat{p}_i\left(\hat{q}_i\right), d_i^s\right)$. The market clearing conditions in the *Baseline Economy* and *Covered CDS Economy* for the firm for whom CDS are written are, respectively, given by

$$1 - h^1\left(\hat{p}_i\left(\hat{q}_i\right), d_i^s\right) = \hat{p}_i\hat{q}_i \tag{15}$$
$$1 - h^1\left(\tilde{p}_i\left(\tilde{q}_i\right), d_i^s\right) = \tilde{p}_i\tilde{q}_i - \tilde{q}_i d_i^s. \tag{16}$$

Note that bond quantities are different across economies as well. However, it can be shown that $q_i = \beta_i \times (p_i)^{-\Gamma_i}$ where $\beta_i \equiv \left(\frac{1}{\alpha_i}\right)^{\frac{1}{\alpha_i - 1}}$ and $\Gamma_i \equiv \frac{\alpha_i}{\alpha_i - 1} < 0$. This means that $\frac{\partial q_i}{\partial p_i} > 0$ and p_i and q_i move in the same direction. Thus the left hand side of (16) is weakly greater than that of (15) while the right hand side is strictly less. A contradiction. Hence $\tilde{p}_i > \hat{p}_i, i = G, B$. ■

Corollary 1. **Proof.** This follows directly from Proposition 4 in that $h^1\left(\tilde{p}_B\left(\tilde{q}_B\right), d_B^D\right) > h^1\left(\hat{p}_B\left(\hat{q}_B\right), d_B^D\right)$ where the tilde corresponds to bond pricing in the Covered Economy and the hat corresponds to bond pricing in the Non-CDS economy. The higher is the bond price, the lower is the CDS premium, which lowers the return to "buying the Arrow-Up." The resulting marginal CDS seller in the Covered CDS Economy must therefore be more optimistic than the marginal bond buyer in the non-CDS economy. Because the set of agents is connected, the next most optimistic agent h_-^1 in the covered CDS economy will be more optimistic than h_-^1 in the non-CDS economy. ■

Proposition 5. **Proof.** From Proposition 3 we know that if risk free borrowing exists in the range of A_i^D for which $p_i = 1$ is possible, it will only occur if γ is below the threshold $\gamma < \bar{\gamma} < 1$. From Proposition 4, we know that $\frac{\partial q_i}{\partial p_i} > 0$ so that $\tilde{q}_i^{\rho *} > \hat{q}_i^{\rho *}$, bond quantities rise in the covered CDS economy. This therefore implies that $\bar{\gamma}\tilde{q}_i^{\rho *} > \bar{\gamma}\hat{q}_i^{\rho *} = q_i^{f*}$. Lastly, we know that for the same A_i^D from the $\left(\bar{\gamma}, A_i^D\right)$-tuple that made the two debt contracts equivalent in the Non-CDS economy, if issuing risk free is chose in the covered CDS economy, there must be another $\tilde{\gamma} < \bar{\gamma}$ for which the two debt contracts yield the same profits; otherwise, the firm always issues risky debt in which case default always occurs at A_i^D. ■

Lemma 2: **Proof.** Suppose to the contrary that h_1 holds cash. It must be the case then that the investor prefers holding cash to any other instrument int the economy. Thus we can say

$$h_1 + (1 - h_1)\, d_b^D < p_b, \tag{17}$$

and

$$1 > \frac{(1-h_1)\left(1-d_b^D\right)}{1-p_b}. \tag{18}$$

Inserting (17) into the denominator of the *r.h.s* of (18) we do not perturb the inequality

$$1 > \frac{(1-h_1)\left(1-d_b^D\right)}{1-\left[h_1+(1-h_1)\,d_b^D\right]}.$$

Rearranging and regrouping we get

$$(1-h_1)\left(1-d_b^D\right) > (1-h_1)\left(1-d_b^D\right) \;\otimes$$

a contradiction. ■

Proposition 6. **Proof.** To see this consider the re-written equilibrium market clearing condition for firm i when naked CDS are permitted where dots above variables indicate the naked CDS values:

$$1-h^1\left(\dot{p}_i\left(\dot{q}_i\right),d_i^D\right) = \dot{p}_i\dot{q}_i - \dot{q}_i d_i^D + \left(\dot{p}_i - d_i^D\right)\frac{h^2}{1-\dot{p}_i}. \tag{19}$$

The only functional form difference between (19) and (16) is the final term on the right hand side, $\left(\dot{p}_i - d_i^D\right)\frac{h^2}{1-\dot{p}_i}$. This term is always greater than zero because $\dot{p}_i > d_i^D$ whenever a funding equilibrium exists. The left hand sides are the same form. Consider to the contrary that $\dot{p}_i \geq \hat{p}_i$ in which case the left hand side of (19) is less than or equal to the left hand side of (15). The right hand side of (19) is a monotonically decreasing function of d_i^D. Suppose the upper-limiting case $d_i^D = \dot{p}_i$. Then the right hand side of (19) is equal to zero and $h^1\left(\dot{p}_i\left(\dot{q}_i\right),d_i^D\right) = 1$ meaning there is no equilibrium funding for firm i. Suppose the lower-limiting case where $d_i^D = 0$, in which case the right hand side of (19) is strictly greater than that of (15), a contradiction. Thus $\dot{p}_i < \hat{p}_i$ for $\forall d_i^D \in \left[0,p_i^D\right)$. ■

Proposition 7. **Proof.** The logic is the same as Proposition 5. Use $\dot{q}_i^{\rho*} < \hat{q}_i^{\rho*}$ from Proposition 6 and $\frac{\partial q_i}{\partial p_i} > 0$ from Proposition 3. ■

C Retail Investor

Let his utility be given by $U^M\left(x_s,c_s\right)\epsilon$, $\epsilon > 0$, where x^s is time 1 consumption and c_i is a covered CDS package made up of a bond and CDS: $c_i = q_i + q_{ic}$.[16] Since he

[16] The micro-foundations of the investor's preferences are not explicitly modeled, though it is easy to do so. For example, one could assume the investor is a fund manager who invests in covered

holds no risk, a CDS contract will accompany every bond purchased. Furthermore, $U^{M\prime}(c_i) > 0$ so that the investor always prefers to invest when the opportunity exists. We can now characterize the *institutional investor's* budget set:

$$
\begin{aligned}
B^M(p_i, p_{ic}) \quad = \quad & \Big\{ \left(x_0^M, q_i^M, q_{ic}^M, x_s^M\right) \in R_+ \times R_+ \times R_+ \times R_+ : \\
& x_0^M + \sum_i (p_i + p_{ic})\, c_i^M = e^M, \\
& x_s^M = x_0^M + \sum_i c_i^M \Big\}.
\end{aligned}
$$

The investor uses his endowment to either purchase bonds with covered CDS or for consumption. The retail investor consumes the same amount in either state since covered CDS positions have identical payouts in both states. Final consumption is the sum of cash carried forward and the number of covered CDS packages purchased.

CDS for which he receives a fee given by ϵ.