Online Appendix for "Monetary Policy Strategies for a Low-Rate Environment"
by Ben S. Bernanke, Michael T. Kiley, and John M. Roberts

Part I. Specification of policy rules

A. Taylor rules

The baseline (non-inertial) Taylor (1993) rule is defined as follows:

(1) $i_t^{Tay} = r^* + \pi_t + 0.5(\pi_t - \pi^*) + \hat{y}_t$

(2) $i_t = \max\{0, i_t^{Tay}\}$

where i_t^{Tay} is the "notional" nominal interest rate implied by the Taylor rule, r^* is the real natural rate of interest (assumed here to be 1 percent), π^* is the central bank's inflation target (2 percent), the inflation rate π_t is defined to be the four-quarter percent change in core PCE prices, and $\hat{y}$ is the (GDP) output gap.[1] All data are at a quarterly frequency. The coefficients of (1) are consistent with the so-called balanced approach of Taylor (1999) and Yellen (2017). Equation (2) enforces the zero lower bound (ZLB) on the nominal interest rate.

The inertial variant of the Taylor rule, under which policymakers respond only gradually to deviation of inflation and output from target, is defined by:

(3) $i_t^{iTay} = \rho i_{t-1} + (1 - \rho)[r^* + \pi_t + 0.5(\pi_t - \pi^*) + \hat{y}_t]$

where i_t^{iTay} is the notional nominal interest rate for the inertial rule and i_t is the realized nominal interest rate. The inertia coefficient ρ is set to 0.85 in our simulations. An additional equation like (2) requires that the actual policy rate in each period be the greater of the notional rate and zero.

B. Flexible price-level targeting

We consider several variants of price-level targeting (Svensson, 1997). The first is a "flexible" price-level targeting rule, under which policy responds to inflation and output gaps, as in the Taylor rule, but also to deviations of the price level from trend. The notional policy rate in this case is defined by:

(4) $i_t^{FPLT} = r^* + \pi_t + 0.5(\pi_t - \pi^*) + \hat{y}_t + P_t$

where P_t is the deviation of the PCE price index from its target level, assumed to grow by 2 percent each year. Because this rule weights changes in output equally with changes in the price level, it may also be viewed as a trend-adjusted, nominal income target. We consider both the non-inertial formulation of the rule, given above, and an inertial formulation. Under the latter, in analogy to (3), the notional policy rate is defined by:

[1] The output gap is computed as 100*log(actual/potential), with potential defined as in FRB/US.

$$(5)\quad i_t^{iFPLT} = \rho i_{t-1} + (1-\ \rho)[r^* + \pi_t + 0.5(\pi_t - \pi^*) +\ \hat{y}_t + P_t]$$

We set ρ = 0.85. Again, in both formulations, in each period the actual policy rate is the maximum of the notional rate and zero.

Standard price-level targeting, flexible or otherwise, applies both during ELB episodes and away from the effective lower bound. An alternative discussed by Bernanke (2017), which we call here a flexible temporary price-level target, involves the use of a price-level target only during ELB periods. Our implementation of this flexible temporary price-level target is analogous to (5), but adds on a price-level term that only becomes non-zero at the ELB. Thus the notional policy rate is given by:

$$(6)\quad i_t^{FTPLT} = \rho i_{t-1} + (1-\ \rho)[r^* + \pi_t + 0.5(\pi_t - \pi^*) +\ \hat{y}_t + \alpha TP_t]$$

where α is the weight of the ELB price-level gap in the policy rule. Specifically, the price-level gap TP starts to accumulate in the first quarter of a ELB period, $t1$, and stops accumulating (and remains zero) when the accumulated shortfall of inflation since the start of the ELB period is made up (at period m):

$$(7)\quad TP_t = \ \sum_{j=t1}^{m} (\pi_j -\ \pi^*)$$

Note that, in this formulation, the period m for which the price-level gap equals or exceeds zero may come after the liftoff of the policy rate from zero. As usual, the actual policy rate is the maximum of the notional policy rate given by (6) and zero. Consistent with the flexible PLT rule, equation (5), in our simulations we set α = 1.[2]

C. Threshold rules

In flexible price-level targeting, of both the standard and temporary varieties, the price-level gap is one determinant of the policy rate, along with the output gap and the inflation gap. In the threshold rules we consider, the policy rate remains at zero until a threshold condition is met, irrespective of output and inflation gaps. Following a proposal by Bernanke (2017), we considered a threshold variant of temporary price-level targeting (TPLT). In this variant, away from the ELB, policymakers follow an inertial Taylor rule. However, once rates hit zero, they remain at zero until the accumulated deviations of inflation from target during the ELB period are equal to or greater than zero. Thus, the date k of the first interest rate increase after a period at zero satisfies:

$$(8)\quad \sum_{j=t1}^{k} (\pi_j -\ \pi^*) \geq 0$$

[2] Hebden and Lopez-Salido (2018) also consider this type of policy rule, but do not include an inertial interest-rate term. Incidentally, they also define their price-level gap as the average quarterly deviation rather than the sum of deviations, as we do. In their favored simulations, they choose a value of 8 for the parameter that multiplies the average price-level gap in their rule. Taking into account the difference in definition (average versus cumulative price-level gap) and their finding that ELB periods last on average about eight quarters under their rule, their parameter assumption seems roughly consistent with our choice of α = 1.

where $t1$ is the first period in which the inertial Taylor rule, equation (3), implies a policy rate of zero. Once the condition is satisfied, we assume that policy is determined by the inertial Taylor rule, plus the non-negativity condition.

Under the threshold version of TPLT, if the ELB period is extended and the cumulative inflation shortfall is large, the implied commitment to overshoot inflation may be correspondingly large. A substantial overshoot of inflation could pose its own problems, including the possible unanchoring of inflation expectations (Brainard, 2017). To mitigate that risk, policymakers might commit to TPLT with an "inflation lookback" shorter than the entire ELB period. For example, policymakers might commit to a liftoff date k that meets the condition:

$$(9)\quad \sum_{j=0}^{n}\left(\pi_{k-n+j} - \pi^*\right) \geq 0$$

where n is the "memory" of the rule, in quarters. (If the rule's "memory" is less than the time since the policy rate hit zero, then n is just the length thus far of the ELB period.) Note that this specification is equivalent to delaying the liftoff of rates from zero until the inflation target has been met, on average, over the past n quarters. Bernanke (2017) cites the communications benefits of being able to discuss TPLT in more standard inflation-targeting language.

For computational convenience, we approximate condition (9) by using a geometrically decaying coefficient on the lagged price-level gap term, so that the condition becomes

$$(10)\qquad \sum_{j=t1}^{k}\beta_n^{k-j}\left(\pi_j - \pi^*\right) \geq 0 \text{ , where } \beta_n \approx 1-\frac{1}{n}$$

We consider the lookback periods (and their corresponding decay factors) of 3 years ($\beta_{12} = 0.916$) and 1 year ($\beta_4 = 0.75$).

D. Shadow-rate rules

An alternative way to specify lower-for-longer policies makes use of so-called shadow policy rates. As actual policy rates cannot be (very) negative, shadow rates (which are not so constrained) provide a metric for keeping track of foregone accommodation through the ZLB period. This metric can help to guide the subsequent evolution of actual policy rates. Note that shadow-rate rules may provide similar policy prescriptions, or even be essentially equivalent, to rules expressed in terms of macroeconomic objectives such as output gaps or price-level gaps (Bernanke, 2017). However, in practice, the communications challenges of explaining policy between classes of policy rules may be quite different.

In this paper's simulations, we consider two variants of shadow-rate rules. Reifschneider and Williams (2000) proposed a rule that accumulates forgone interest rate cuts at the ELB, exiting from the ELB when the cumulative forgone accommodation equals zero. Following RW, we measure foregone accommodation relative to a non-inertial Taylor type rule (i_t^{Tay}), equation (1). The actual policy rate for the RW can then be compactly defined by:

$$(11)\qquad i_t = \max\{0, i_t^{Tay} - \sum_{j=t1}^{t-1}(i_j - i_j^{Tay})\}$$

where $t1$ is again the first period in which the Taylor rule, equation (1), implies a policy rate of zero.

The second shadow-rate variant we consider is due to Kiley-Roberts (2017), who describe a rule in which the *change* in the shadow, or notional, policy rate depends on the deviations of inflation and output from target:

$$(12) \qquad i_t^{KR} = i_{t-1}^{KR} + \alpha[(\pi_t - \pi^*) + \hat{y}_t]$$

where i_t^{KR} is the unobserved shadow rate. As can be seen in (12), the KR shadow rate rises when inflation and output are above target and falls when they are below target. The change formulation implies that the current level of the shadow rate depends on the whole history of inflation and output gaps. As usual, the actual policy rate is set equal to the shadow rate when it is non-negative, and is zero otherwise.

Following Reifschneider and Roberts (2006), we consider the case in which the parameter α, which determines the sensitivity of the shadow rate to current inflation and output gaps, is set equal to 0.4.

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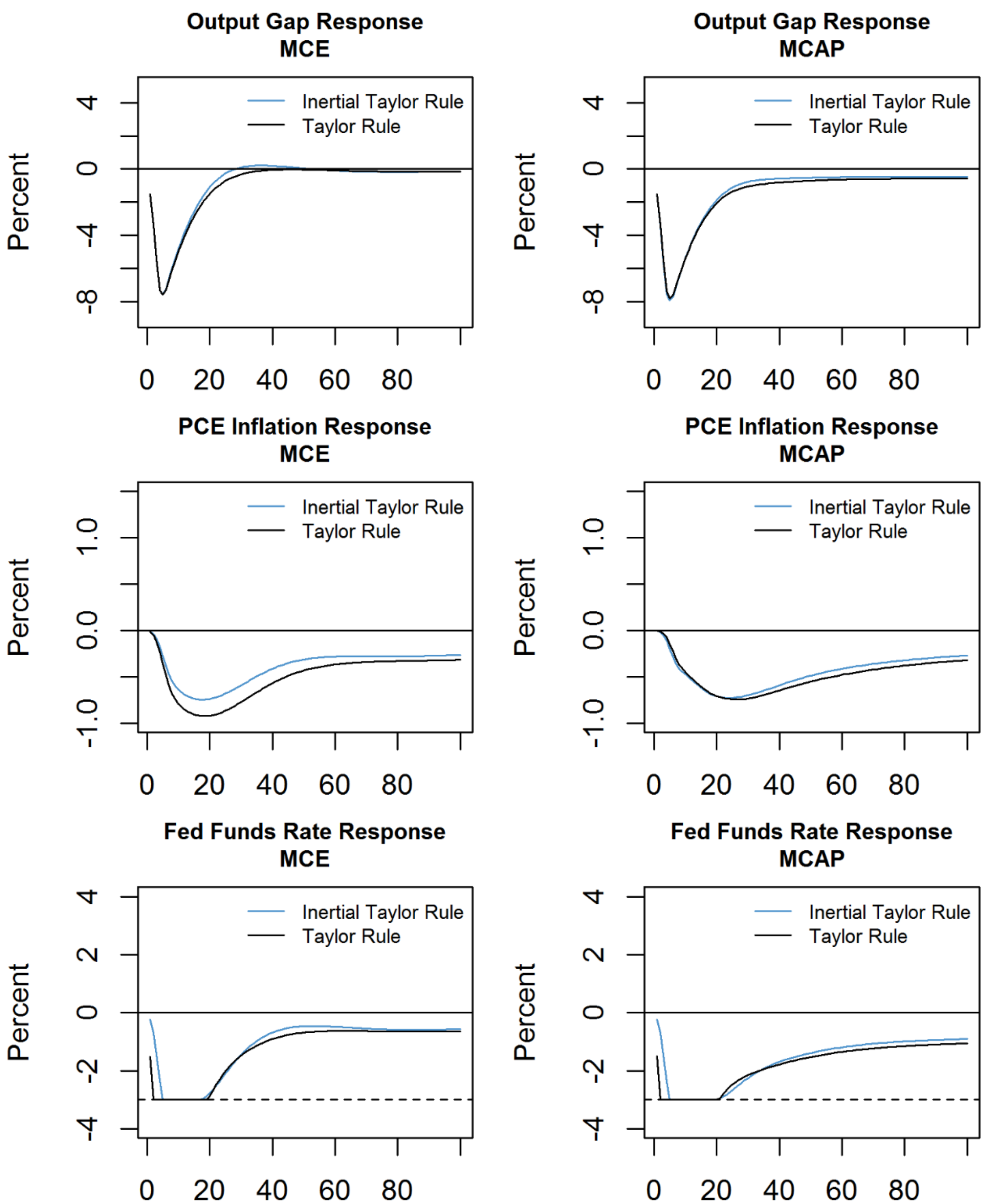
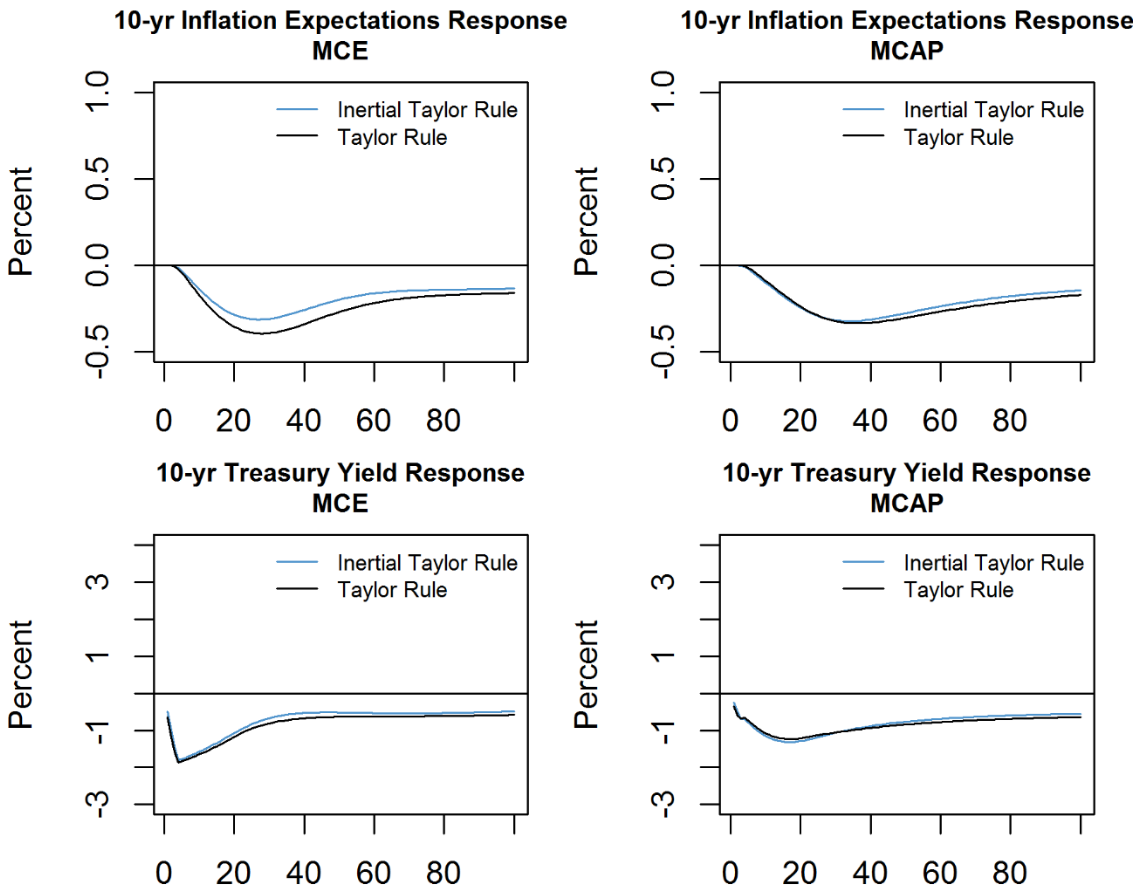
Part II. Model responses to an aggregate demand shock

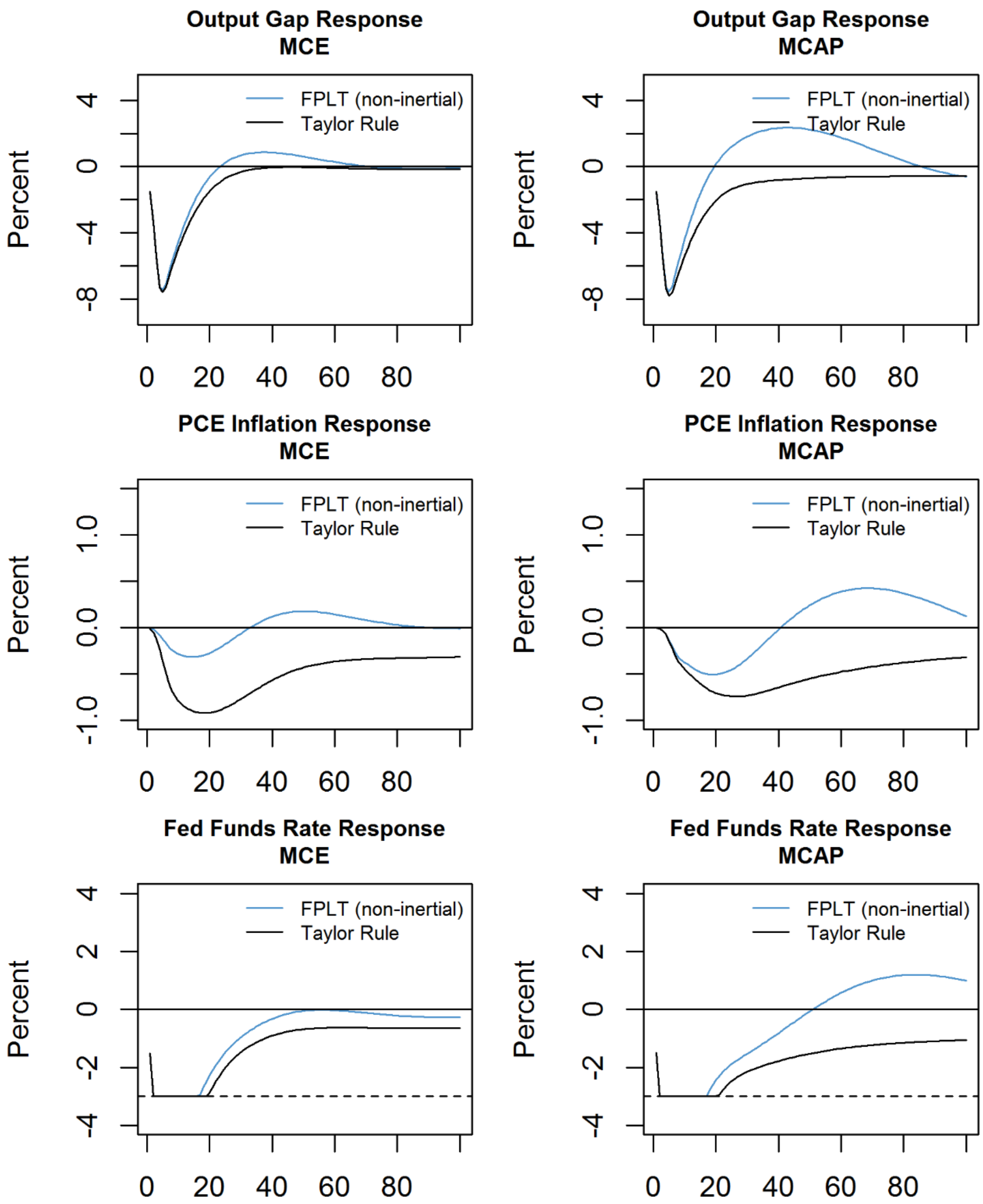
These figures illustrate model responses to a one-time aggregate demand shock. The shock involves a -1.5 percent shock in consumer expenditures on nondurables and non-housing services that persists for four quarters. As described in the text, this shock results in a protracted ELB episode. All simulations assume the neutral nominal interest rate is three percent. The left column displays responses assuming model-consistent expectations (MCE); the right column displays repsonses under the assumption that only financial markets have model-consistent expectations (MCAP).

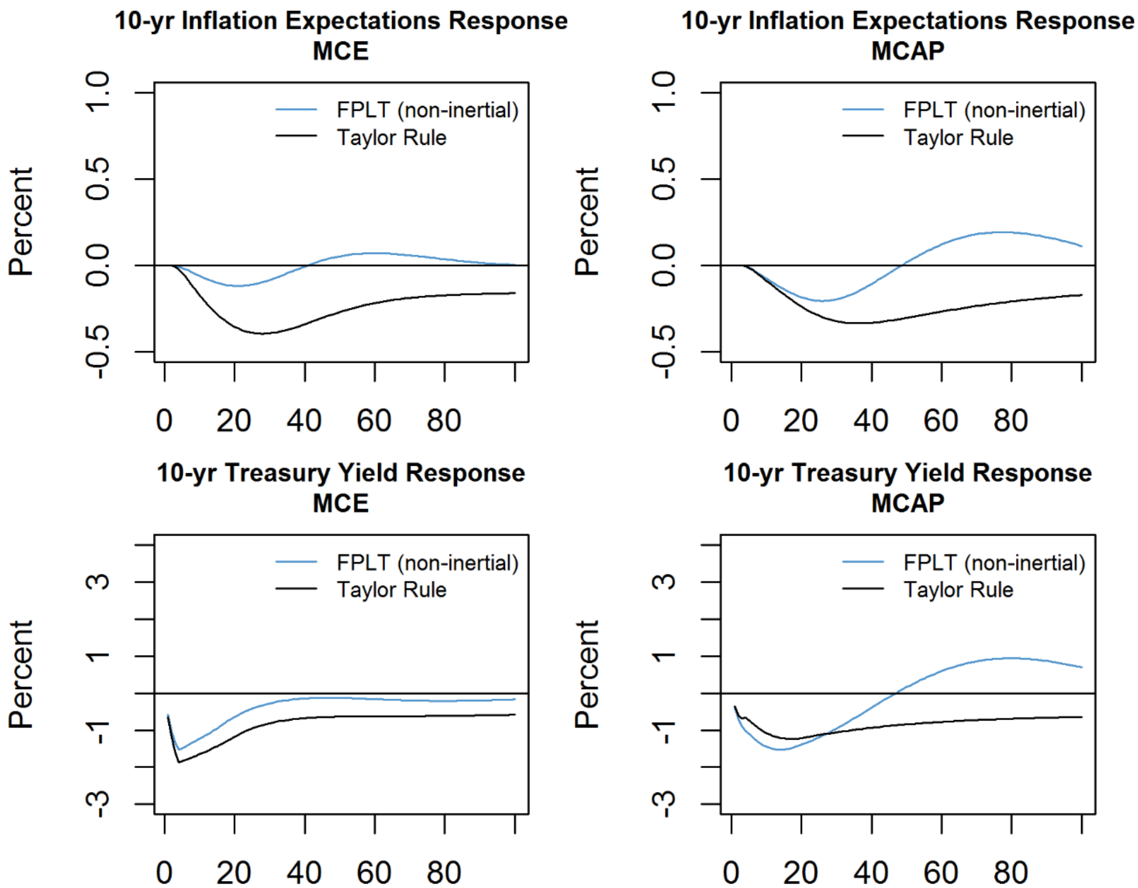
1-2. Taylor rules (inertial and non-inertial)



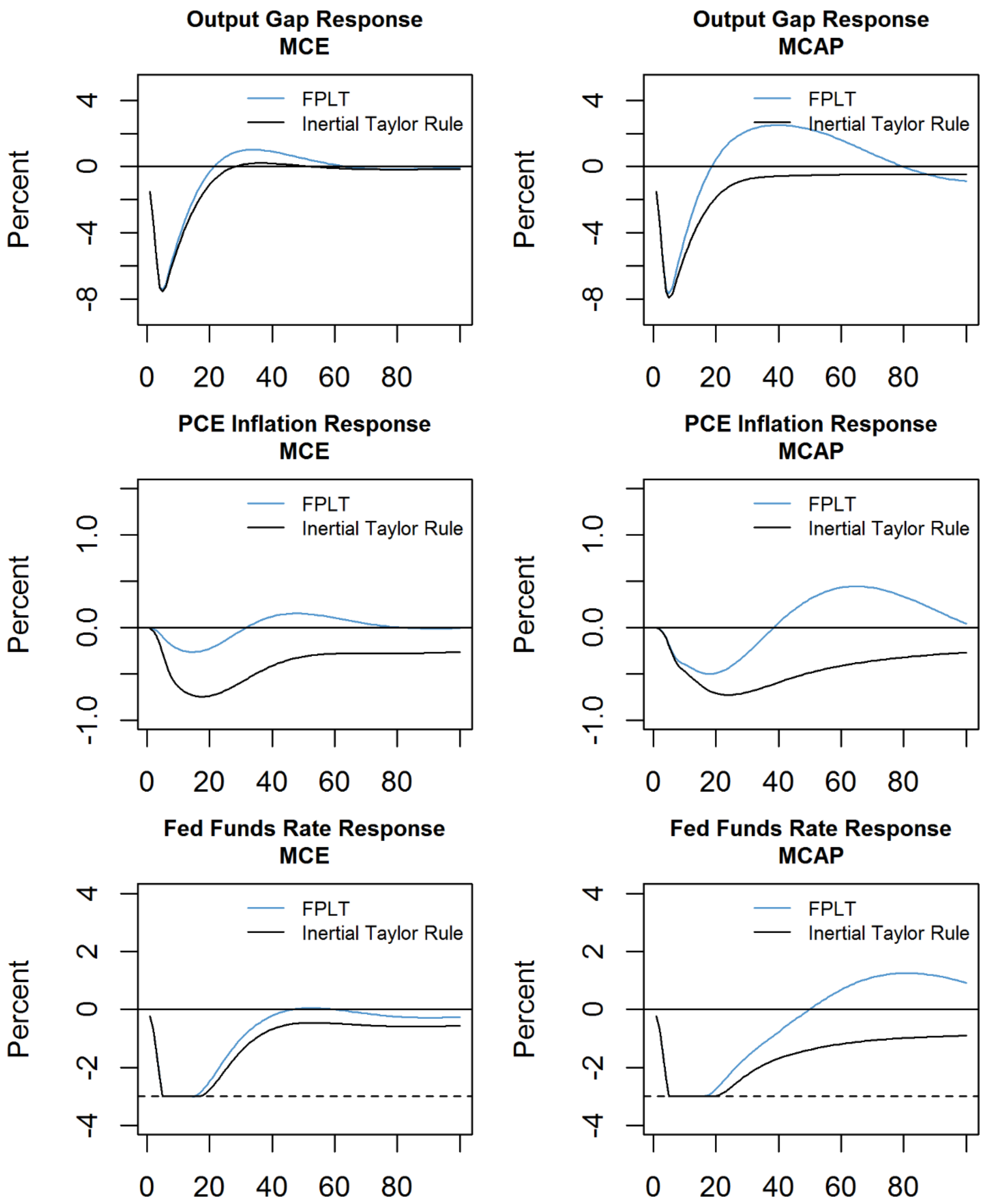

10-yr Inflation Expectations Response
MCE
Inertial Taylor Rule
Taylor Rule
Percent
-0.5
0.0
0.5
1.0
0
20
40
60
80
10-yr Inflation Expectations Response
MCAP
Inertial Taylor Rule
Taylor Rule
Percent
-0.5
0.0
0.5
1.0
0
20
40
60
80
10-yr Treasury Yield Response
MCE
Inertial Taylor Rule
Taylor Rule
Percent
-3
-1
1
3
0
20
40
60
80
10-yr Treasury Yield Response
MCAP
Inertial Taylor Rule
Taylor Rule
Percent
-3
-1
1
3
0
20
40
60
80

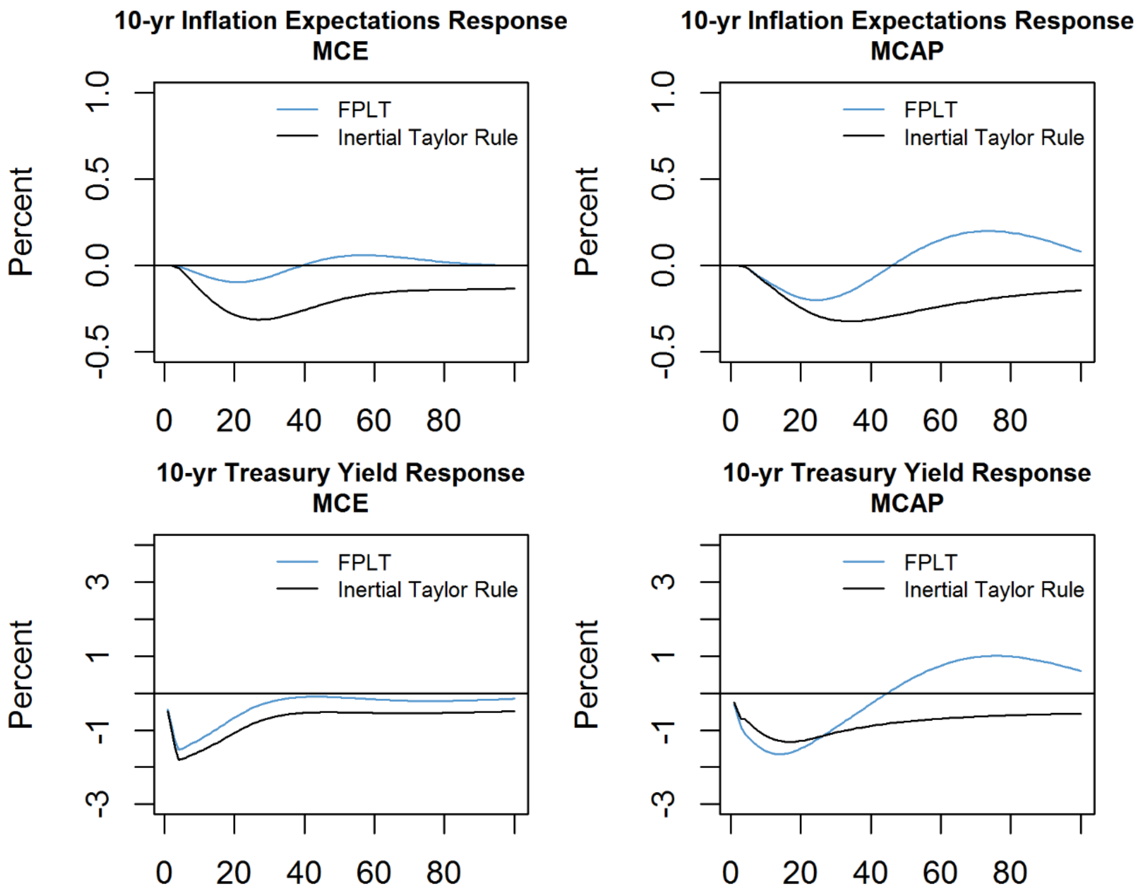
3. Flexible price-level target (non-inertial)



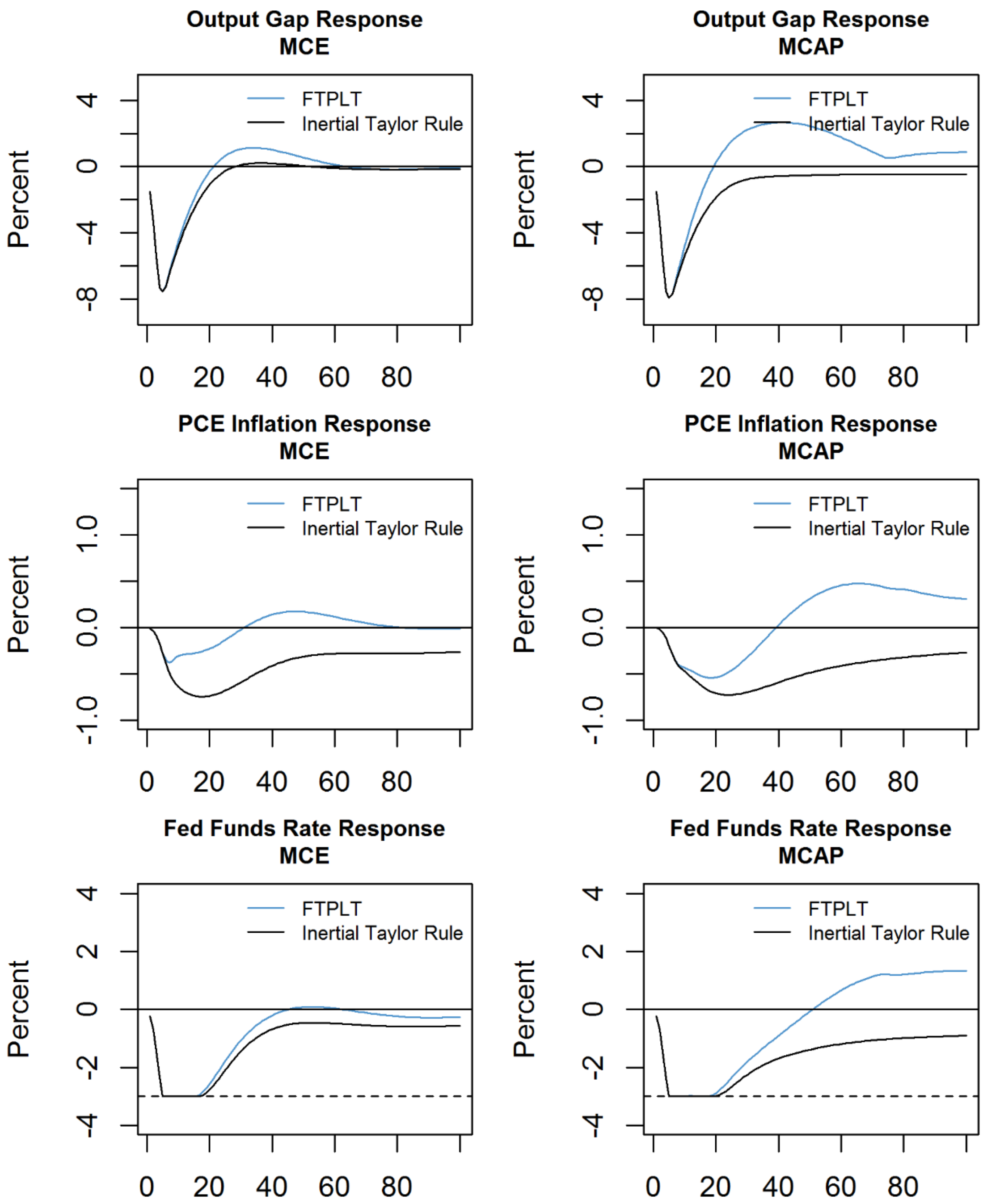

10-yr Inflation Expectations Response
MCE
10-yr Inflation Expectations Response
MCAP
10-yr Treasury Yield Response
MCE
10-yr Treasury Yield Response
MCAP
FPLT (non-inertial)
Taylor Rule
Percent
-0.5
0.0
0.5
1.0
-3
-1
1
3
0
20
40
60
80

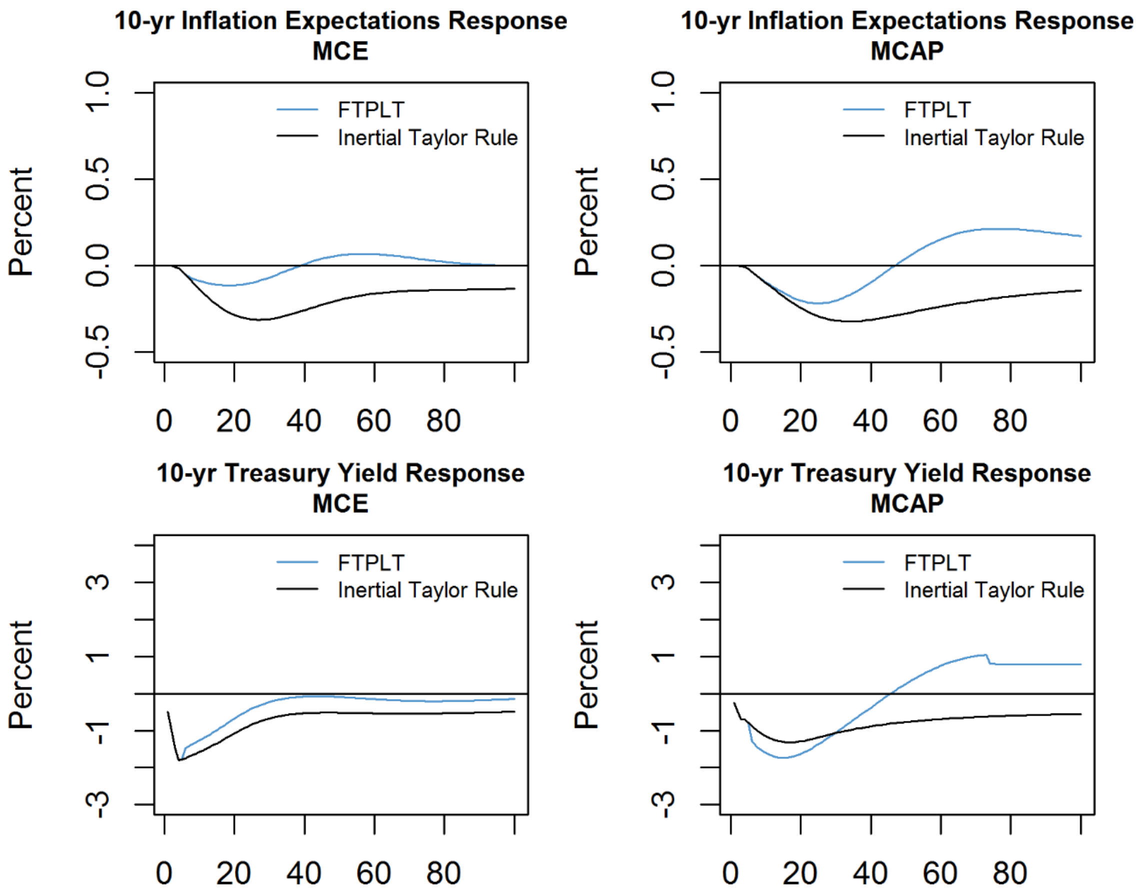
4. Flexible price-level target (inertial)



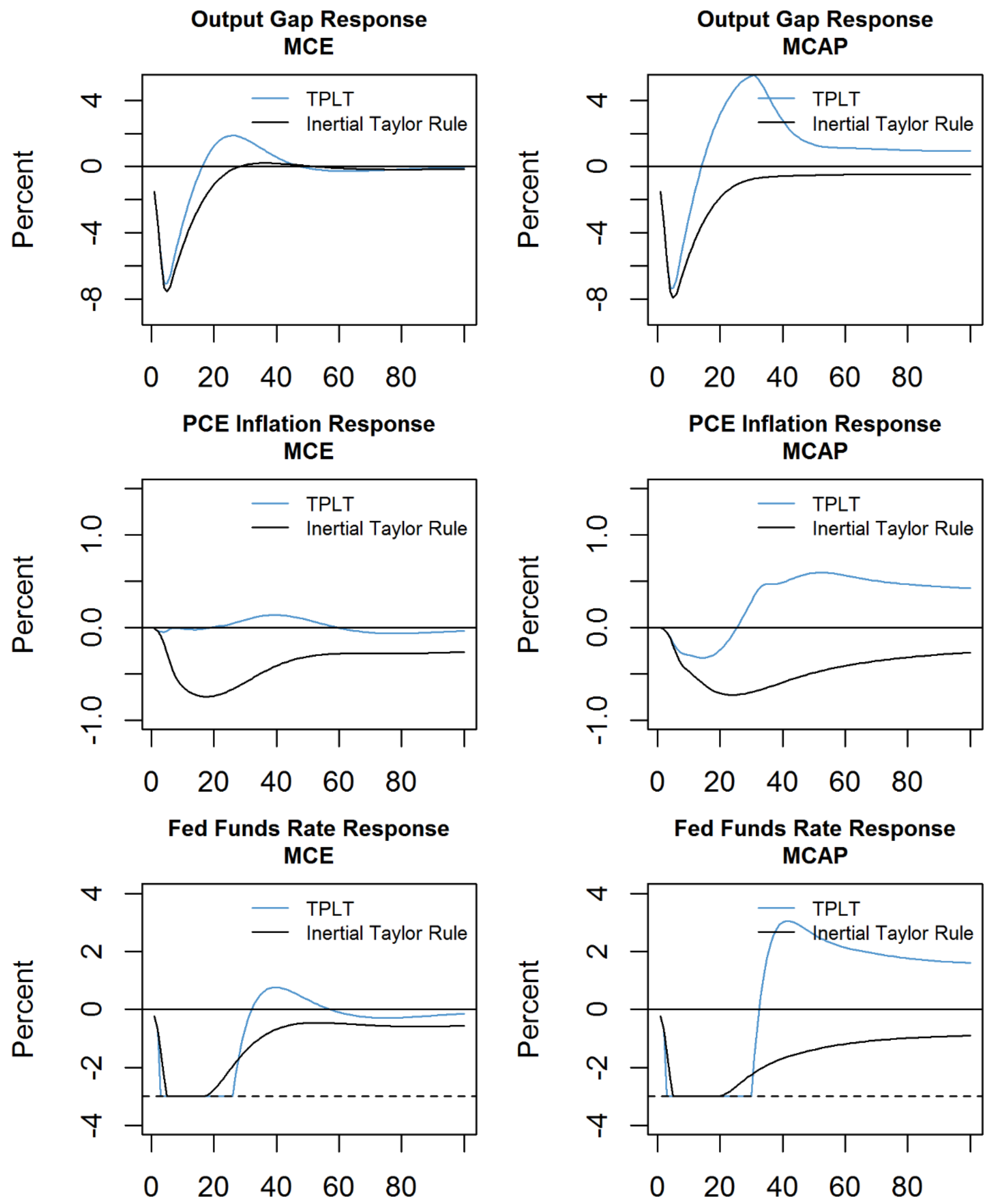

10-yr Inflation Expectations Response
MCE
10-yr Inflation Expectations Response
MCAP
10-yr Treasury Yield Response
MCE
10-yr Treasury Yield Response
MCAP
FPLT
Inertial Taylor Rule
Percent
-0.5
0.0
0.5
1.0
-3
-1
1
3
0
20
40
60
80

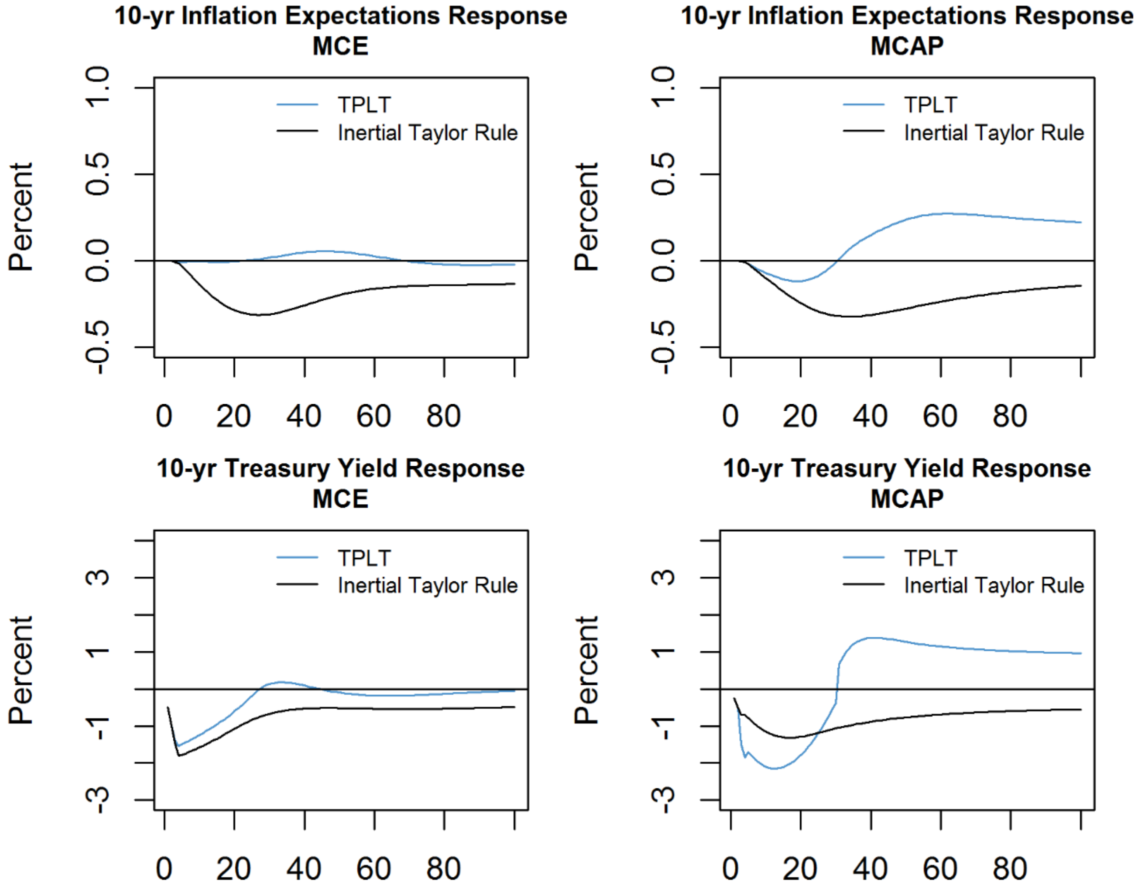
5. Flexible temporary price-level target (inertial)



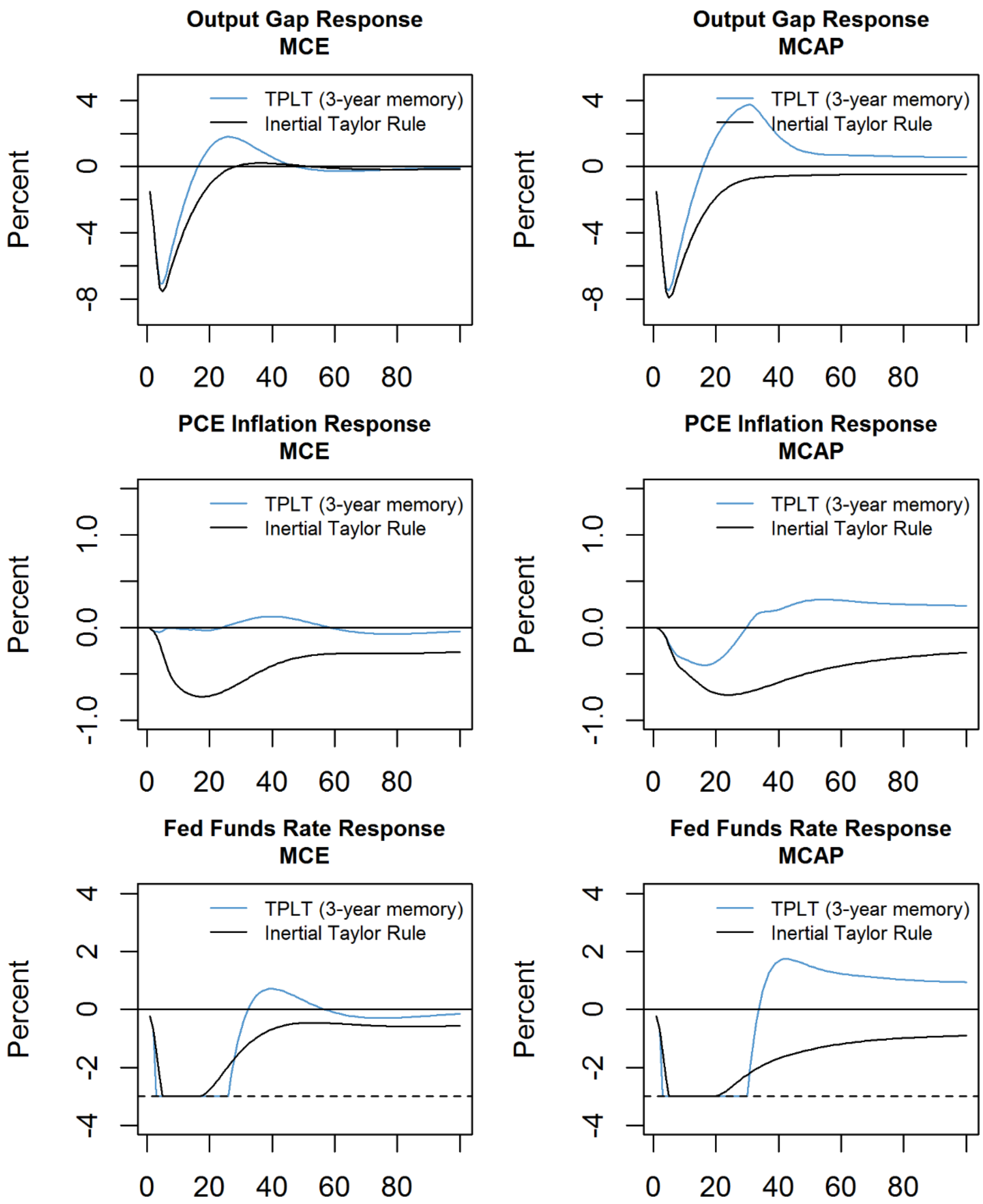

10-yr Inflation Expectations Response
MCE
Percent
-0.5
0.0
0.5
1.0
FTPLT
Inertial Taylor Rule
0
20
40
60
80
10-yr Inflation Expectations Response
MCAP
Percent
-0.5
0.0
0.5
1.0
FTPLT
Inertial Taylor Rule
0
20
40
60
80
10-yr Treasury Yield Response
MCE
Percent
-3
-1
1
3
FTPLT
Inertial Taylor Rule
0
20
40
60
80
10-yr Treasury Yield Response
MCAP
Percent
-3
-1
1
3
FTPLT
Inertial Taylor Rule
0
20
40
60
80

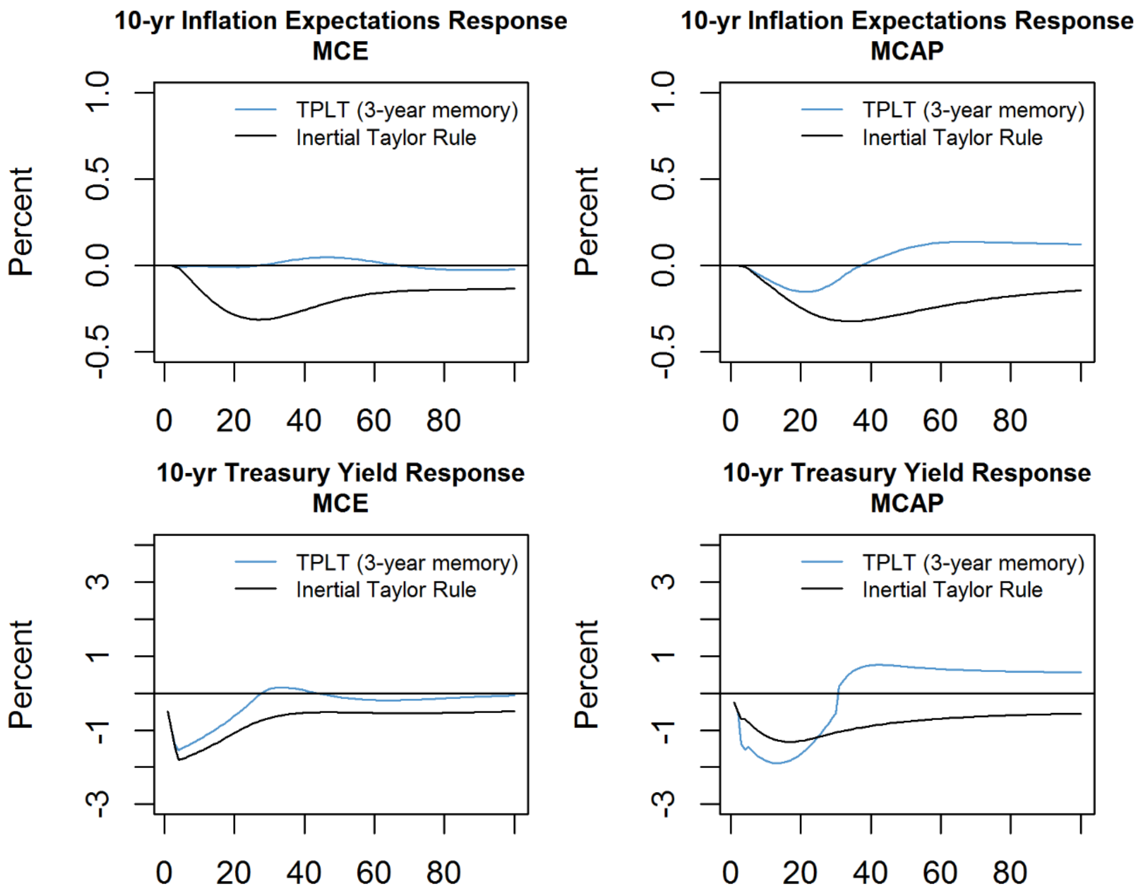
6. Temporary price-level target (threshold rule)



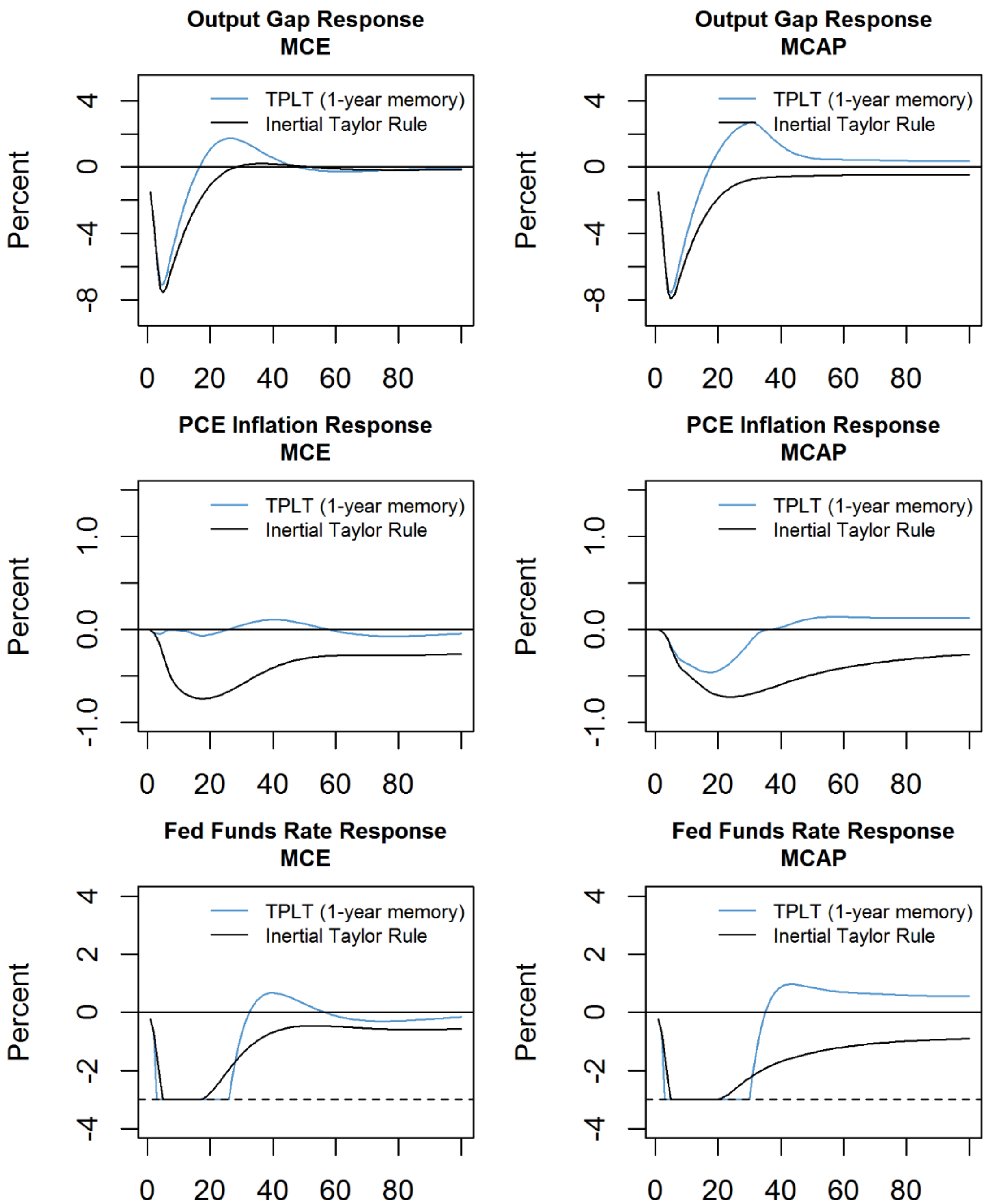

10-yr Inflation Expectations Response
MCE
10-yr Inflation Expectations Response
MCAP
10-yr Treasury Yield Response
MCE
10-yr Treasury Yield Response
MCAP
TPLT
Inertial Taylor Rule
Percent
0 20 40 60 80
-0.5 0.0 0.5 1.0
-3 -1 1 3

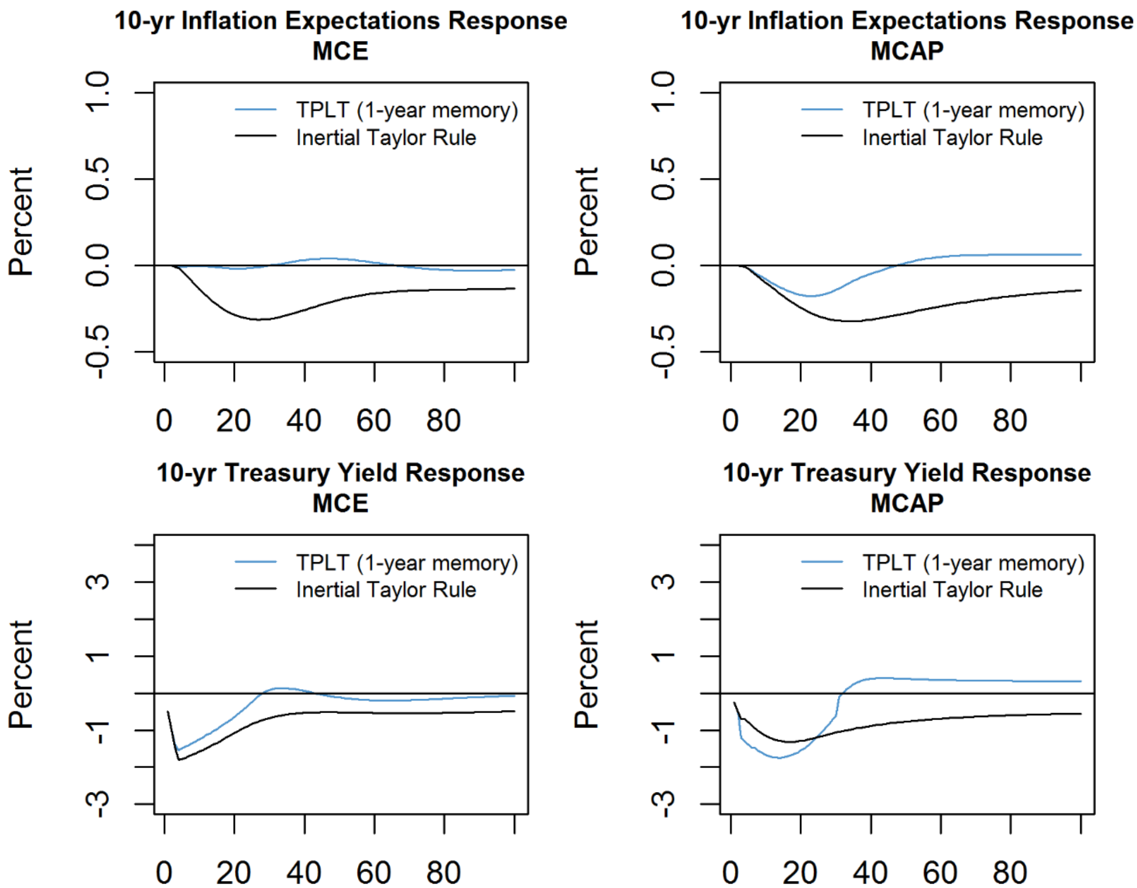
7. Temporary price-level target (threshold rule, 3-year memory)




10-yr Inflation Expectations Response
MCE
10-yr Inflation Expectations Response
MCAP
10-yr Treasury Yield Response
MCE
10-yr Treasury Yield Response
MCAP
TPLT (3-year memory)
Inertial Taylor Rule
Percent
-0.5
0.0
0.5
1.0
-3
-1
1
3
0
20
40
60
80

8. Temporary price-level target (threshold rule, 1-year memory)





10-yr Inflation Expectations Response
MCE
10-yr Inflation Expectations Response
MCAP
10-yr Treasury Yield Response
MCE
10-yr Treasury Yield Response
MCAP
TPLT (1-year memory)
Inertial Taylor Rule
Percent
-0.5
0.0
0.5
1.0
-3
-1
1
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20
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60
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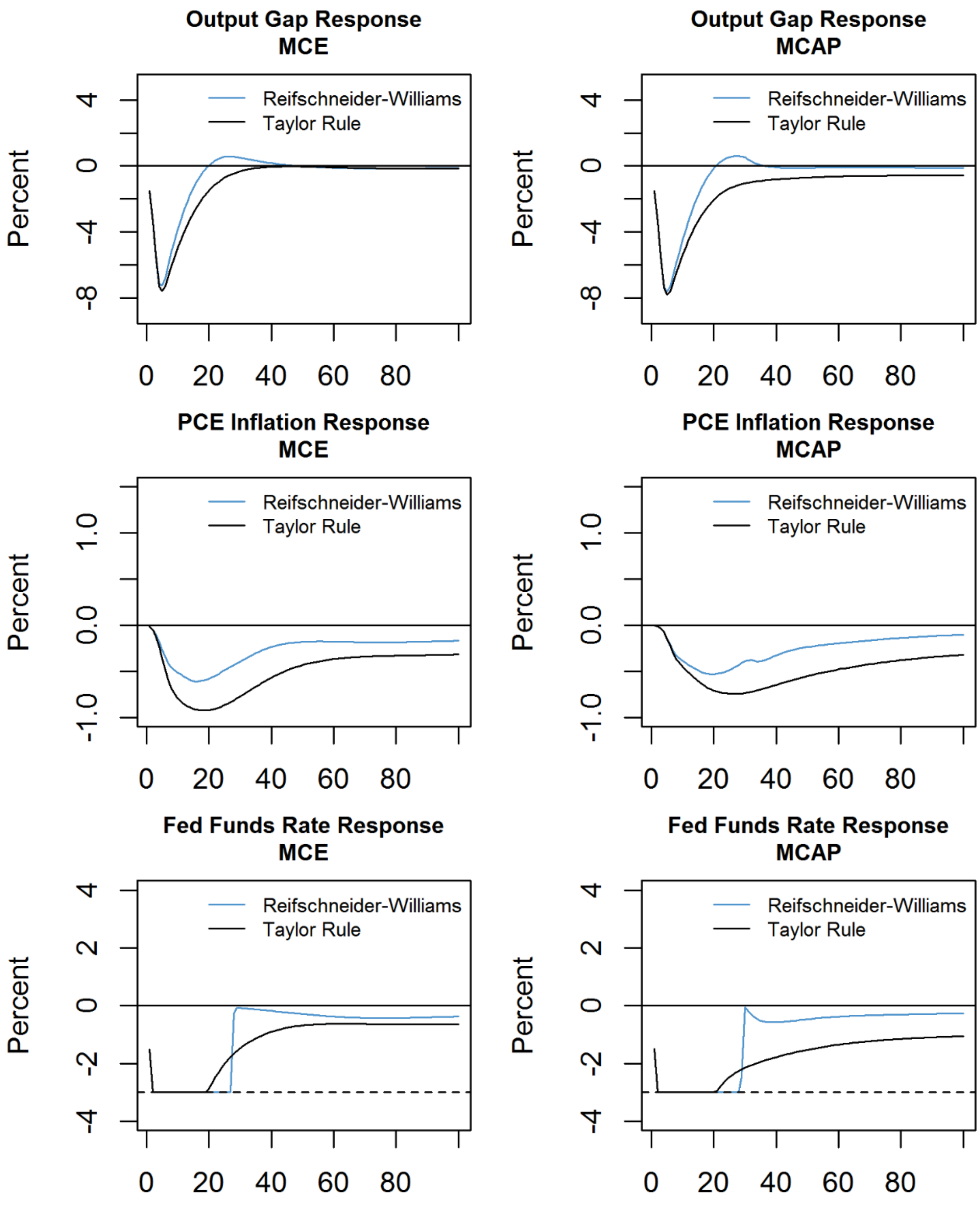
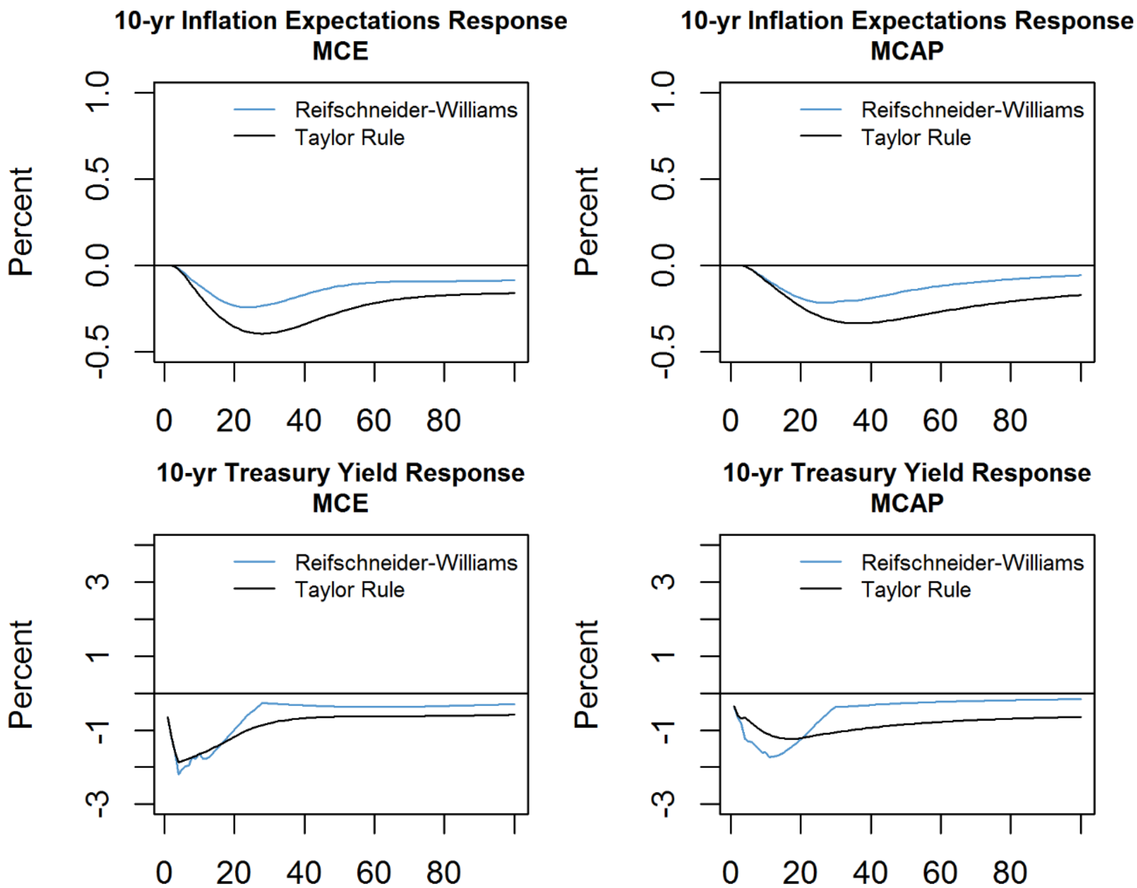
9. Reifschneider-Williams





10-yr Inflation Expectations Response
MCE
10-yr Inflation Expectations Response
MCAP
10-yr Treasury Yield Response
MCE
10-yr Treasury Yield Response
MCAP
Reifschneider-Williams
Taylor Rule
Percent
-0.5
0.0
0.5
1.0
-3
-1
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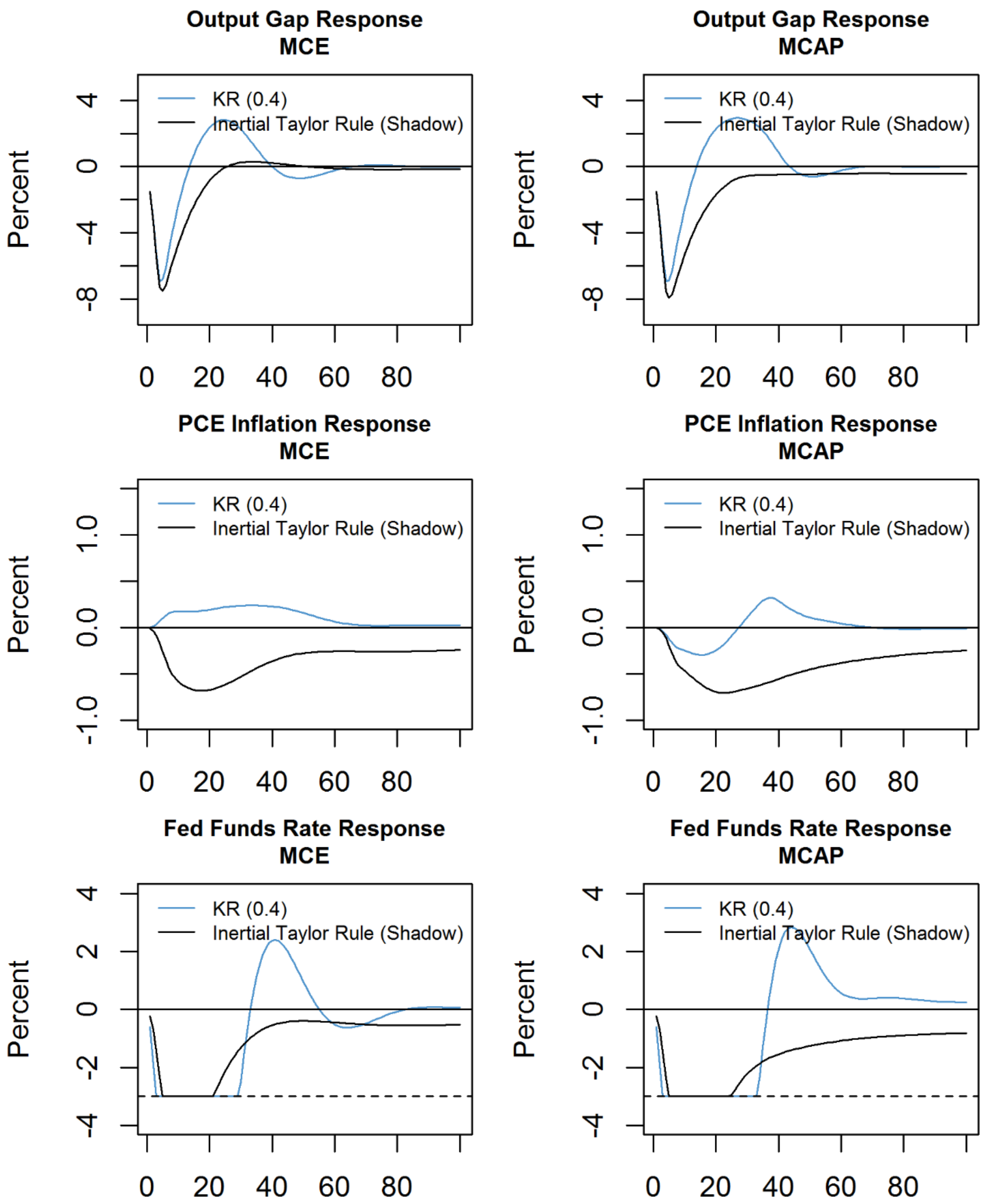
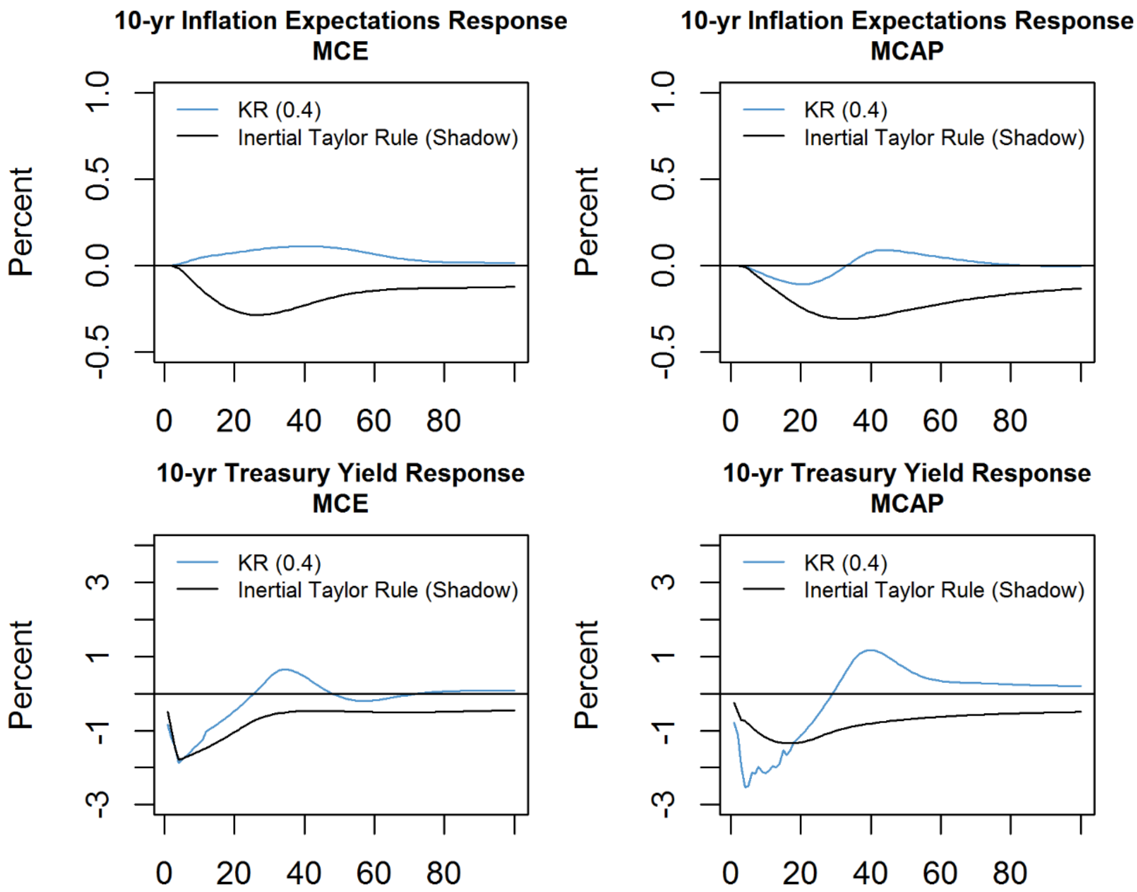
80

10. Kiley-Roberts change rule ($\alpha = 0.4$)




10-yr Inflation Expectations Response
MCE
10-yr Inflation Expectations Response
MCAP
10-yr Treasury Yield Response
MCE
10-yr Treasury Yield Response
MCAP
KR (0.4)
Inertial Taylor Rule (Shadow)
Percent
-0.5
0.0
0.5
1.0
-3
-1
1
3
0
20
40
60
80