Technical Appendix to: ‘Employment, Wages and Optimal Monetary Policy’

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A Two reference models with labor market frictions

This Appendix provides more details on the two models laid out in the main text.

A.1 NK model with search and matching frictions

A.1.1 Household

Households are modeled as in Andolfatto (1996) and Merz (1995). At any point in time n_t agents of the household are employed (w) and $1 - n_t$ agents are unemployed (u). As in Walsh (2005) and Christiano, Eichenbaum, and Trabandt (2013), we assume that each household member has the same concave preferences over consumption and that the household provides perfect consumption insurance. The household maximizes the inter-temporal utility of the members

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - \mu c_{t-1})^{1-\sigma}}{1-\sigma} - n_t \phi_0 \frac{(h_t)^{1+\phi}}{1+\phi} \right] \tag{A.1}$$

subject to the budget constraint

$$c_t + \frac{B_{t+1}}{P_t} \leqslant [w_t h_t n_t + b^u(1-n_t)] + \frac{\mathrm{Pr}_t}{P_t} + \frac{T_t}{P_t} + \frac{R_{t-1}B_t}{P_t}. \tag{A.2}$$

$\mathbb{E}_0$ is the expectations operator conditional on all the information available up to period 0. β is the time discount factor. Consumption is denoted by c_t, and the hours worked by the n_t employed household members are measured by h_t. Unemployed household members do not experience disutility from working. The real wage is given by w_t and unemployment benefits are measured by b^u. Bond holdings B_t, taxes and transfers T_t, and profits Pr_t are measured in nominal terms and are converted into real units through division by the price level P_t. R_t is the nominal interest rate on bonds. We denote by λ_t the Lagrange multiplier attached to the budget constraint when solving the household's problem. As in Walsh (2005) we assume that total consumption c_t consists of a manufactured good c_t^m and home production $b^u(1-n_t)$, i.e., $c_t = c_t^m + b^u(1-n_t)$. This assumption guarantees that it is in principle possible under the conditions in Hosios (1990) for the outcomes of the search and matching process to be efficient.[1]

[1] If unemployment benefits are modeled as tax-financed, imposing the conditions in Hosios (1990) is not sufficient to achieve efficiency for $b^u > 0$. The exact way of modeling unemployment benefits is of little consequence for us as for empirical reasons we are not interested in parameterizations that satisfy the conditions in Hosios (1990). However, the modeling choice matters in our companion paper Bodenstein and Zhao (2016) from which we draw in this paper.

A.1.2 Employment and the labor market

The labor market is characterized by search and matching frictions. In this economy, the presence of search and matching frictions impedes people who are seeking jobs from finding one and wholesale firms that are posting vacancies from filling them. At the beginning of each period, a share ρ of matches that existed in the last period n_{t-1} breaks up. The share $(1-\rho)$ of matches survives. With the labor force normalised to unity, the total number of job seekers in period t is the sum of unemployed workers in period $t-1$ and the newly fired workers. Let u_t denote the total number of job seekers,

$$u_t = 1 - n_{t-1} + \rho n_{t-1} = 1 - (1-\rho)\, n_{t-1} \tag{A.3}$$

The unemployment rate differs from u_t as some job seekers may be matched and become employed. We define the unemployment rate

$$\tilde{u}_t = 1 - n_t. \tag{A.4}$$

Firms post vacancies v_t to be filled with job-seeking workers. Unemployed workers are matched to vacant jobs according to the constant returns to scale matching function

$$m_t = \chi u_t^{\zeta} v_t^{1-\zeta}. \tag{A.5}$$

χ determines the degree of matching efficiency, ζ captures the curvature of Beveridge curve, indicating the substitutability between vacancies and job seekers. Newly formed matches m_t result immediately in employment. The latter evolves according to

$$n_t = (1-\rho)\, n_{t-1} + m_t. \tag{A.6}$$

Finally, we define the job finding rate s_t as the probability of an unemployed worker being matched to a vacant job

$$s_t = \frac{m_t}{u_t} = \chi \theta_t^{1-\zeta}. \tag{A.7}$$

The vacancy filling rate q_t is the probability for a vacancy being filled

$$q_t = \frac{m_t}{v_t} = \chi \theta_t^{-\zeta}. \tag{A.8}$$

Labor market tightness θ_t is defined as

$$\theta_t = \frac{v_t}{u_t}. \tag{A.9}$$

We are now in a position to define the marginal value of employment to the household H_t consistent with the household's optimization problem

$$H_t = \frac{W_t}{P_t} h_t - b^u - \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t} + (1-\rho) E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - s_{t+1}) H_{t+1}. \tag{A.10}$$

Moving one household member into employment affects the household in three ways. First, total household resources rise by the difference between wages and unemployment benefits. Second, the utility of the agent changing employment status falls by the disutility from labor (divided by the marginal utility of wealth λ_t to turn it into monetary terms). Finally, the gains from matching a household member with a firm also occur in future periods.

A.1.3 Wholesale firms

Wholesale firms employ labor as the only factor of production. Their output is sold at the competitive market price P_t^w. Firms post vacancies at the flow vacancy posting cost κ^v. A wholesale firm's optimization problem is

$$\begin{aligned} &\max_{\{y_t^w, v_t, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left(\frac{P_t^w}{P_t} y_t^w - \frac{W_t}{P_t} n_t h_t - \kappa^v v_t \right) \\ &s.t. \quad n_t = (1-\rho) n_{t-1} + q_t v_t \\ &\qquad\quad y_t^w = a_t n_t h_t. \end{aligned} \tag{A.11}$$

The technology shock a_t follows an exogenous AR(1) process

$$\log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_t^a \tag{A.12}$$

with ε_t^a given by $N(0, \sigma_a^2)$.

Let J_t denote the marginal value of employment to the wholesale firm (the lagrange multiplier associated with the first constraint). The first order condition with respect to vacancy postings implies

$$q_t J_t = \kappa^v. \tag{A.13}$$

Using the envelop theorem J_t itself is defined as

$$J_t = \left(\frac{P_t^w}{P_t} a_t h_t - \frac{W_t}{P_t} h_t \right) + (1 - \rho) E_t \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}. \tag{A.14}$$

Employing one additional worker raises the firm's profits in the current period by the increment between marginal product of labor and wage payment. Furthermore, if the match survives future the firm also enjoys a continuation value.

Combining equations, the wholesale firm's vacancy posting condition equation (A.14) can be rewritten as

$$mc_t a_t h_t = \frac{W_t}{P_t} h_t + \frac{\kappa^v}{q_t} - (1 - \rho) E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa^v}{q_{t+1}}. \tag{A.15}$$

The wholesale firms' real revenue $\frac{P_t^w}{P_t}$ is in effect the intermediate firms' real marginal cost mc_t. The left hand side of equation (A.15) indicates the marginal benefit of hiring an additional worker. The right hand side of equation (A.15) captures the marginal cost of hiring a new worker, involving wage payments for hours worked, vacancy posting costs associated with a new match, and the present value of saved future vacancy posting costs if the match survives in following periods.

Notice that the search and matching frictions work through the presence of vacancy posting costs. Absent vacancy posting costs, wholesale firms would post infinitely many vacancies. All the unemployed workers seeking jobs will find one. The NK model with search and matching frictions reduces to the standard NK model and marginal costs would be given by $mc_t = \frac{w_t}{a_t}$.

A.1.4 Wage bargaining

The real wage w_t and hours worked are determined by Nash bargaining between the worker and the firm after forming a match. The total surplus of the match is given by

$$J_t + H_t. \tag{A.16}$$

Under Nash bargaining the solution to the bargaining game is obtained from

$$\max_{w_t, h_t} J_t^{1-\xi} H_t^{\xi} \tag{A.17}$$

subject to equations (A.14) and (A.10). ξ stands for the bargaining power of the household, and $1-\xi$ indicates the bargaining power of the firm.

The sharing rule for this Nash bargaining mechanism as derived from the first order condition with respect to w_t implies

$$\xi J_t = (1-\xi) H_t. \tag{A.18}$$

Combining equations (A.14), (A.10) and (A.18) yields an expression for the bargained wage

$$w_t h_t = \xi \left(h_t mc_t a_t + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} s_{t+1} J_{t+1} \right] \right) + (1-\xi) \left(b^u + \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t} \right). \tag{A.19}$$

Combining equation (A.15) and equation (A.19), we obtain the equilibrium condition for vacancy posting

$$\frac{\kappa^v}{q_t} = (1-\xi) \left(h_t mc_t a_t - b^u - \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t} \right) + (1-\rho) E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \xi s_{t+1}) \frac{\kappa^v}{q_{t+1}}. \tag{A.20}$$

The first order condition associated with hours worked in the Nash bargaining problem can be written as

$$mc_t a_t = \frac{\phi_0 h_t^{\phi}}{\lambda_t}. \tag{A.21}$$

A.1.5 Retailers

Retail good producers purchase wholesale goods to produce differentiated intermediate good varieties. The retailers have monopoly power over their variety. The retailer's cost minimization problem is then given by

$$\begin{aligned} &\min_{y_t^w(i), y_t(i)} P_t^w y_t^w(i) \\ &s.t. \quad y_t(i) = y_t^w(i) \end{aligned} \tag{A.22}$$

with the first order condition for $y_t^w(i)$ being

$$P_t^w - \lambda_t^w = 0 \tag{A.23}$$

where λ_t^w is the Lagrange multiplier for the production function and thus represents the marginal cost. Therefore, real marginal costs satisfy

$$\frac{P_t^w}{P_t} = mc_t. \tag{A.24}$$

The prices of intermediate goods $P_t(i)$ are determined by Calvo-style staggered contracts, see Calvo (1983). Each period, a firm faces a constant probability $1 - \xi^p$ to re-optimize its price $P_t(i)$. The probability is independent across firms and across time. For those firms that do not re-optimize their price, prices will be updated as a weighted average of $\Pi_t = \frac{P_t}{P_{t-1}}$ the nominal price inflation in the last period and $\bar{\Pi}$ the steady state inflation rate. The relative importance of Π_t and $\bar{\Pi}$ is governed by price indexation parameter ι^p.[2] More specifically,

$$P_{t+1}(i) = \tilde{P}_t(i) \left({\pi_t}^{\iota_p} \bar{\pi}^{1-\iota_p} \right). \tag{A.25}$$

Price setting behavior of intermediate good firm i is derived from

$$\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\left((1+\bar{\tau}^p) \tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right) - MC_{t+s} \right) \right] y_{t+s}(i)$$

[2] This price updating scheme avoids price dispersions in steady state if the steady state inflation rate is not zero.

$$s.t. \quad y_{t+s}(i) = \left(\frac{\tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right)}{P_{t+s}} \right)^{-\dfrac{\lambda^p}{\lambda^p - 1}} y_{t+s}. \tag{A.26}$$

$\bar{\tau}^p$ is the subsidy to intermediate firms. We assume $\bar{\tau}^p = \lambda^p - 1$ to remove the distortions arising from monopolistic competition between the retailers. We introduce markup shocks in the first order conditions for intermediate firms. We define $\theta^p = \lambda^p - 1$.

A.1.6 Final good producer

Differentiated intermediate products are combined to form the composite goods by a continuum of representative bundlers in a perfectly competitive environment based on the CES aggregator

$$y_t = \left[\int_0^1 y_t(i)^{\frac{1}{\lambda^p}} \mathrm{d}i \right]^{\lambda^p} \tag{A.27}$$

where $\frac{\lambda^p}{\lambda^p - 1}$ refers to the elasticity of substitution between intermediate varieties. Profit maximisation of a bundler is defined as

$$\begin{aligned} &\max_{y_t(i), y_t} P_t y_t - \int_0^1 P_t(i) y_t(i) di \\ &s.t. \qquad y_t = \left[\int_0^1 y_t(i)^{\frac{1}{\lambda^p}} \mathrm{d}i \right]^{\lambda^p}. \end{aligned} \tag{A.28}$$

The first order conditions can be recombined to obtain the demand function for intermediate good i

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\lambda^p}{\lambda^p - 1}} y_t \tag{A.29}$$

and the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda^p - 1}} \mathrm{d}i \right]^{-(\lambda^p - 1)}. \tag{A.30}$$

A.2 NK model with Calvo sticky wage

We only describe the parts of the model that are different from the search and matching model. More details are provided in Erceg, Henderson, and Levin (2000).

A.2.1 Household

Each Household maximizes preferences

$$E_0 \sum_{t=0}^{\infty} \beta^{t-t_0} \left[\frac{\left(c_t(j) - \mu c_{t-1}(j)\right)^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_t(j)^{1+\phi} \right] \tag{A.31}$$

subject to the budget constraint

$$P_t c_t(j) + B_{t+1}(j) = (1 + \bar{\tau}^w) W_t(j) h_t(j) + R_{t-1} B_t(j) + Pr_t(j) + T_t(j). \tag{A.32}$$

$\mathbb{E}_0$ is the expectations operator conditional on all the information available up to period 0. β is the time discount factor. The variable $c_t(j)$ stands for household consumption. μ indicates the degree of internal consumption habits. P_t is the price of consumption goods, and R_t denotes the gross return for the one period risk free bond $B_t(j)$. The Household earns income by supplying labor services $W_t(j)h_t(j)$, receives payments from last period bond holding $R_{t-1}B_t(j)$, and $Pr_t(j)$ which consists of an aliquot share of profits distributed. Finally, the household receives the government transfer $T_t(j)$. ϕ represents the inverse Frisch labor supply elasticity. Labor income $W_t(j)h_t(j)$ is subsidized at a fix rate $\bar{\tau}^w$.

A.2.2 Labor bundler

Labor bundlers package differentiated labor services supplied by each individual into an aggregate labor service with a CES technology resold to the intermediate good producers in perfectly competitive markets. The labor bundling technology is specified as

$$h_t = \left[\int_0^1 h_t(j)^{\frac{1}{\lambda^w}} \mathrm{d}j \right]^{\lambda^w} \tag{A.33}$$

where $\frac{\lambda^w}{\lambda^w - 1}$ refers to the elasticity of substitution between differentiated labor types. We define $\theta^w = \lambda^w - 1$.

Labor bundlers maximize profits in a perfectly competitive environment. Profit maximization for labor bundlers implies

$$\max_{h_t(j), h_t} W_t h_t - \int_0^1 W_t(j) h_t(j) dj$$

$$s.t. \quad h_t = \left[\int_0^1 h_t(j)^{\frac{1}{\lambda^w}} \mathrm{d}j\right]^{\lambda^w}.$$

The first order conditions imply that the demand for differentiated labor services satisfies

$$h_t(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\frac{\lambda^w}{\lambda^w - 1}} h_t \tag{A.34}$$

with the aggregate (nominal) wage being defined as

$$W_t = \left[\int_0^1 W_t(j)^{-\frac{1}{\lambda^w - 1}} \mathrm{d}j\right]^{-(\lambda^w - 1)}. \tag{A.35}$$

A.2.3 Wage setting

Households supply their differentiated labor services to the labor bundlers. There is a continuum of households, index by $j \in (0, 1)$. The imperfect substitutability of differentiated labor gives each individual household certain degree of market power in setting a nominal wage. Each monopolistic household chooses labor supply $h_t(j)$ and wage setting $W_t(j)$. In addition, wage setting is subject to nominal rigidities as in Calvo (1983). As in Erceg, Henderson, and Levin (2000), households can readjust nominal wages with probability $1 - \xi^w$ in each period. For those that cannot adjust wages, wages will increase by the weighted average of inflation in the last period Π_t and the steady state inflation rate $\bar{\Pi}$

$$W_{t+1}(j) = \tilde{W}_t(j) \left(\Pi_t^{\iota^w} \bar{\Pi}^{1-\iota^w}\right). \tag{A.36}$$

For those that can re-optimize, the problem is to choose a wage $\tilde{W}_t(j)$ that maximizes its utility in all states of nature where the household has to maintain that wage in the future

$$\begin{aligned}
&\max_{\tilde{W}_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^w \beta)^s \left[\frac{(c_{t+s} - \mu c_{t-1+s})^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_{t+s}(j)^{1+\phi}\right] \\
s.t. \quad & P_{t+s} c_{t+s} + B_{t+s+1} = (1 + \bar{\tau}^w) W_{t+s}(j) h_{t+s}(j) + R_{t+s-1} B_{t+s} + Pr_{t+s} + T_{t+s} \\
& h_{t+s}(j) = \left(\frac{W_{t+s}(j)}{W_{t+s}}\right)^{-\dfrac{\lambda^w}{\lambda^w - 1}} h_{t+s} \\
& W_{t+s}(j) = \tilde{W}_t(j) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^w} \bar{\Pi}^{1-\iota^w}\right)
\end{aligned} \tag{A.37}$$

Where $\bar{\tau}^w$ is the subsidy to households who supply differentiated labor varieties. We assume $\bar{\tau}^w = \lambda^w - 1$ to eliminate the distortions due to monopolistic competition among households.

B NK model with search and matching frictions: linear model

This section derives the linear model that approximates the NK model with search and matching model. We first derive the elasticity of labor market tightness with respect to shocks to understand the amplification of shocks in the presence of search and matching frictions. Subsequently, we show that the linear system of the NK model with search and matching frictions can be stated in terms of three equations. For simplicity, we abstract from price indexation and consumption habits from here on.

B.1 Simple analytics

To learn about the amplification of shocks in the framework with endogenous labor supply, we combine the wage bargaining equations to derive an expression for labor market tightness, θ_t.

Substituting the surplus sharing rule, $J_t = \frac{1-\xi}{\xi} H_t$ into the definition of the household's marginal value of employment

$$\begin{aligned} \xi J_t &= -(1-\xi)\frac{\phi_0}{1+\phi} h_t^{1+\phi}\frac{1}{\lambda_t} + (1-\xi)(w_t h_t - b^u) \\ &\quad + \xi(1-\rho)\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}(1-s_{t+1}) J_{t+1}\right). \end{aligned} \tag{B.1}$$

Combining with the marginal value of employment to the firm to eliminate the wage rate

$$\begin{aligned} & J_t + (1-\xi)\frac{\phi_0}{1+\phi} h_t^{1+\phi}\frac{1}{\lambda_t} + (1-\xi) b^u \\ = \quad & (1-\xi)\, mpl_t h_t mc_t + (1-\rho) E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}(1-\xi s_{t+1}) J_{t+1}\right] \end{aligned} \tag{B.2}$$

or recognizing that efficient bargaining over hours worked implies that the marginal product of labor is equal to the marginal rate of substitution between labor and consumption

$$J_t + (1-\xi) b^u = (1-\xi)\frac{\phi}{1+\phi} mpl_t h_t mc_t + (1-\rho) E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}(1-\xi s_{t+1}) J_{t+1}\right]. \tag{B.3}$$

Applying the definitions for s_t and q_t, and the condition

$$J_t = \frac{\kappa^v}{q_t} \tag{B.4}$$

we finally summarize the equations characterizing the wage bargaining process in a single equation

$$\begin{aligned}
& \frac{\kappa^v}{\chi}\theta_t^{\zeta} + (1-\xi)\, b^u \\
= \quad & (1-\xi)\,\frac{\phi}{1+\phi} mpl_t h_t mc_t + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(1 - \xi\chi\theta_{t+1}^{1-\zeta}\right) \left(\frac{\kappa^v}{\chi}\theta_{t+1}^{\zeta}\right)\right]
\end{aligned} \tag{B.5}$$

thereby eliminating H_t, J_t, w_t from the system of relevant equations.

In its log-linear form, the expression reduces to

$$\begin{aligned}
& \zeta\frac{\kappa^v}{\chi}\theta_{ss}^{\zeta}\hat{\theta}_t - (1-\rho)\beta\left(\frac{\zeta\kappa^v}{\chi}\theta_{ss}^{\zeta} - \xi\kappa^v\theta_{ss}\right) E_t\hat{\theta}_{t+1} \\
= \quad & (1-\xi)\frac{\phi}{1+\phi} mpl_{ss}h_{ss}mc_{ss}\widehat{mpl}_t + (1-\xi)\frac{\phi}{1+\phi} mpl_{ss}h_{ss}mc_{ss}\hat{h}_t \\
& +(1-\xi)\frac{\phi}{1+\phi} mpl_{ss}h_{ss}mc_{ss}\hat{mc}_t \\
& +(1-\rho)\beta\left(1-\xi\chi\theta_{ss}^{1-\zeta}\right)\left(\frac{\kappa^v}{\chi}\theta_{ss}^{\zeta}\right) E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t\right]
\end{aligned} \tag{B.6}$$

and after using the steady state relationship $q_{ss} = \chi\theta_{ss}^{-\zeta}$

$$\begin{aligned}
& \zeta\frac{\kappa^v}{q_{ss}}\hat{\theta}_t - (1-\rho)\beta\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right) E_t\hat{\theta}_{t+1} \\
= \quad & (1-\xi)\frac{\phi}{1+\phi} mpl_{ss}h_{ss}mc_{ss}\left(\widehat{mpl}_t + \hat{h}_t + \hat{mc}_t\right) \\
& +(1-\rho)\beta\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right) E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t\right].
\end{aligned} \tag{B.7}$$

To simplify the expression in equation (B.7), note that in the steady state equation (B.5) implies

$$\left(\frac{\kappa^v}{q_{ss}}\right)\left[1-(1-\rho)\beta\left(1-\xi q_{ss}\theta_{ss}\right)\right] = (1-\xi)\left[\frac{\phi}{1+\phi} mpl_{ss}h_{ss}mc_{ss} - b^u\right]. \tag{B.8}$$

Using the conditions involving the marginal value of employment to the firm J_t evaluated in the steady state and defining the replacement ratio as $b^u = r^u w_{ss} h_{ss}$

$$b^u = r^u w_{ss} h_{ss} = r^u \left(mpl_{ss} h_{ss} mc_{ss} - (1-(1-\rho)\beta) \frac{\kappa^v}{q_{ss}} \right). \tag{B.9}$$

Combining equations (B.8) and (B.9) defines the bargaining weight ξ in terms of the replacement ratio r^u and other parameters and steady state targets

$$\begin{aligned} & \left(\frac{\kappa^v}{q_{ss}}\right) [1-(1-\rho)\beta(1-\xi q_{ss}\theta_{ss})] \\ = \quad & (1-\xi) \left[\left(\frac{\phi}{1+\phi} - r^u \right) mpl_{ss} h_{ss} mc_{ss} + r^u (1-(1-\rho)\beta) \left(\frac{\kappa^v}{q_{ss}} \right) \right]. \end{aligned} \tag{B.10}$$

Assuming that changes in variables are small between two periods, we can approximate the response of labor market tightness as

$$\hat{\theta}_t \approx \frac{1}{\Upsilon} \frac{\frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss}}{\left[\left(\frac{\phi}{1+\phi} - r^u \right) mpl_{ss} h_{ss} mc_{ss} + r^u (1-(1-\rho)\beta) \left(\frac{\kappa^v}{q_{ss}} \right) \right]} \left(\widehat{mpl}_t + \hat{h}_t + \widehat{mc}_t \right) \tag{B.11}$$

where

$$\Upsilon = \zeta + \frac{(1-\rho)\beta\xi q_{ss}\theta_{ss}(1-\zeta)}{[1-(1-\rho)\beta(1-\xi q_{ss}\theta_{ss})]}. \tag{B.12}$$

B.2 Implications of negotiating over hours worked

In NK model with search and matching frictions and flexible hours worked, equation (A.21) resembles its counterpart in the standard NK model with flexible wages. Noticing that

$$c_t = y_t - \kappa^v v_t + b^u(1-n_t) \tag{B.13}$$

$$\Omega_t^p y_t = a_t n_t h_t \tag{B.14}$$

where Ω_t^p measures the dispersion of prices, negotiation over hours worked implies

$$\phi_0 \left(\frac{\Omega_t^p y_t}{n_t} \right)^{\phi} (y_t - \kappa^v v_t + b^u(1-n_t))^{\sigma} = \frac{P_t^w}{P_t} a_t^{1+\phi}. \tag{B.15}$$

In the model with a Walrasian labor market (as in the standard NK model), n_t is constant and search costs are zero

$$\phi_0 \left(\Omega_t^p y_t\right)^{\phi} \left(y_t\right)^{\sigma} = \frac{P_t^w}{P_t} a_t^{1+\phi}. \tag{B.16}$$

Relative to the standard NK model, we need to take into account the dynamics of n_t, v_t, and q_t. Or after log-linearizing, the two different models imply

$$(\phi + \sigma)\, \hat{y}_t = \widehat{\left[\frac{P^w}{P}\right]_t} + (1+\phi)\, \hat{a}_t \tag{B.17}$$

compared to

$$\left(\phi + \sigma \frac{\frac{y_{ss}}{y_{ss} + b^u(1-n_{ss})}}{1 - \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}}\right) \hat{y}_t - \Theta_t = \widehat{\left[\frac{P^w}{P}\right]_t} + (1+\phi)\, \hat{a}_t \tag{B.18}$$

with the correction term Θ_t being defined as

$$\Theta_t = \left(\phi + \sigma \frac{\frac{b^u n_{ss}}{y_{ss} + b^u(1-n_{ss})}}{1 - \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}}\right) \hat{n}_t + \sigma \frac{\frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}}{1 - \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}} \hat{v}_t. \tag{B.19}$$

The variables $\hat{v}_t$ and $\hat{q}_t$ can be expressed in terms of $\hat{n}_t$ using the (log-linearized) equations that describe the labor market

$$\hat{v}_t = \hat{\theta}_t + \hat{u}_t \tag{B.20}$$

$$\hat{q}_t = -\zeta \hat{\theta}_t \tag{B.21}$$

$$\hat{u}_t = -\frac{(1-\rho)n_{ss}}{1-(1-\rho)n_{ss}} \hat{n}_{t-1} \tag{B.22}$$

$$\hat{n}_t = (1-\rho)\hat{n}_{t-1} + \rho \hat{m}_t \tag{B.23}$$

$$\hat{m}_t = \hat{u}_t + (1-\zeta)\hat{\theta}_t \tag{B.24}$$

and therefore

$$\hat{v}_t = \nu_1 \hat{n}_t - \nu_2 \left(\nu_1 + \frac{n_{ss}}{1-n_{ss}}\right) \hat{n}_{t-1} \tag{B.25}$$

$$
\begin{aligned}
\hat{q}_t &= -\zeta\nu_1\hat{n}_t + \zeta\nu_1\nu_2\hat{n}_{t-1} && \text{(B.26)}\\
\hat{\theta}_t &= \frac{1}{\rho(1-\zeta)}\hat{n}_t - \frac{1}{\rho(1-\zeta)}\frac{(1-\rho)(1-n_{ss})}{1-(1-\rho)n_{ss}}\hat{n}_{t-1}\\
&= \nu_1\hat{n}_t - \nu_1\nu_2\hat{n}_{t-1}. && \text{(B.27)}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\Theta_t &= \left[\phi + \sigma\frac{\varpi^{b^u}}{1-\kappa^c} + \sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\right]\hat{n}_t - \sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\nu_2\left(1+\frac{n_{ss}}{1-n_{ss}}\frac{1}{\nu_1}\right)\hat{n}_{t-1}\\
&= \theta_1\hat{n}_t + \theta_2\hat{n}_{t-1} && \text{(B.28)}
\end{aligned}
$$

where

$$
\begin{aligned}
\nu_1 &= \frac{1}{\rho(1-\zeta)} && \text{(B.29)}\\
\nu_2 &= \frac{(1-\rho)(1-n_{ss})}{1-(1-\rho)n_{ss}} && \text{(B.30)}\\
\kappa^c &= \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})} && \text{(B.31)}\\
\varpi^{b^u} &= \frac{b^u n_{ss}}{y_{ss}+b^u(1-n_{ss})} && \text{(B.32)}\\
\varpi^{y_{ss}} &= \frac{y_{ss}}{y_{ss}+b^u(1-n_{ss})} && \text{(B.33)}\\
\theta_1 &= \left[\phi + \sigma\frac{\varpi^{b^u}}{1-\kappa^c} + \sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\right] && \text{(B.34)}\\
\theta_2 &= -\sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\nu_2\left(1+\frac{n_{ss}}{1-n_{ss}}\frac{1}{\nu_1}\right) && \text{(B.35)}
\end{aligned}
$$

The dynamics of real marginal costs satisfy

$$
\hat{mc}_t = \widehat{\left[\frac{P^w}{P}\right]_t} = \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}\right)\hat{y}_t - (1+\phi)\,\hat{a}_t - \left(\theta_1\hat{n}_t + \theta_2\hat{n}_{t-1}\right). \qquad \text{(B.36)}
$$

B.3 Implications of negotiating over the real wage

Combining the first order conditions of the firm with the bargaining outcome over wages, we arrive at the following relationship between real marginal costs of the wholesale retailers and labor market tightness

$$\begin{aligned}
& (1-\xi)\,\frac{\phi}{1+\phi}\frac{P_t^w}{P_t}a_t h_t \\
= \quad & (1-\xi)\,b^u + \frac{\kappa^v}{\chi}\theta_t^\zeta - (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1-\chi\theta_{t+1}^{1-\zeta}\right)\left(\frac{\kappa^v}{\chi}\theta_{t+1}^\zeta\right)\right]. \qquad \text{(B.37)}
\end{aligned}$$

Log-linearizing therefore delivers the following relationship between real marginal costs of the retailers and labor market tightness:

$$\begin{aligned}
\widehat{\left[\frac{P^w}{P}\right]}_t + \hat{y}_t - \hat{n}_t \quad = \quad & \frac{\zeta}{\nu}\frac{\kappa^v}{q_{ss}}\hat{\theta}_t - \frac{(1-\rho)\beta}{\nu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)E_t\hat{\theta}_{t+1} \\
& + \frac{(1-\rho)\beta}{\nu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t - \pi_{t+1}\right] \qquad \text{(B.38)}
\end{aligned}$$

where we have used the fact that $\hat{y}_t - \hat{n}_t = \hat{a}_t + \hat{h}_t$ and we defined

$$\nu = (1-\xi)\frac{\phi}{1+\phi}\left[\frac{P^w}{P}\right]_{ss} a_{ss}h_{ss}. \qquad \text{(B.39)}$$

Absent flexible hours worked, i.e., $\hat{h}_t = 0$, the above expression is used to substitute out for real marginal costs in the New Keynesian Phillips Curve, see Ravenna and Walsh (2011). Given the movements in marginal costs and the real interests rate, labor market tightness and therefore employment are pinned down.

In the case of flexible hours worked, we can combine equations (B.18) and (B.38) to

$$\begin{aligned}
(1+\theta_1)\hat{n}_t + \theta_2\hat{n}_{t-1} \quad = \quad & \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c} + 1\right)\hat{y}_t - (1+\phi)\,\hat{a}_t \\
& - \frac{\zeta}{\nu}\frac{\kappa^v}{q_{ss}}\hat{\theta}_t + \frac{(1-\rho)\beta}{\nu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)E_t\hat{\theta}_{t+1} \\
& - \frac{(1-\rho)\beta}{\nu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t - \pi_{t+1}\right]. \qquad \text{(B.40)}
\end{aligned}$$

and after substituting out for $\hat{\theta}_t$

$$\begin{aligned}
& -\frac{(1-\rho)\beta}{\nu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)\nu_1 E_t\hat{n}_{t+1} \\
& + \left[(1+\theta_1) + \frac{\zeta}{\nu}\frac{\kappa^v}{q_{ss}}\nu_1 + \frac{(1-\rho)\beta}{\nu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)\nu_1\nu_2\right]\hat{n}_t
\end{aligned}$$

$$
\begin{aligned}
&+\left[\theta_2 - \frac{\zeta}{\nu}\frac{\kappa^v}{q_{ss}}\nu_1\nu_2\right]\hat{n}_{t-1} \\
= \quad & \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c} + 1\right)\hat{y}_t - (1+\phi)\,\hat{a}_t \\
&-\frac{(1-\rho)\beta}{\nu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t - \pi_{t+1}\right]. \qquad \text{(B.41)}
\end{aligned}
$$

B.4 Aggregate demand equation

By taking account home production (i.e. unemployment benefits are not financed from any resources), the resource constraint in the economy is

$$
c_t = y_t - \kappa^v v_t + b^u(1-n_t). \qquad \text{(B.42)}
$$

Log-linearizing delivers

$$
\hat{c}_t = \frac{\varpi^{y_{ss}}}{1-\kappa^c}\hat{y}_t - \frac{\theta_1 - \phi}{\sigma}\hat{n}_t + \frac{\theta_2}{\sigma}\hat{n}_{t-1} \qquad \text{(B.43)}
$$

combined with log-linearized Euler equation for holding bonds

$$
-\sigma\left(\hat{c}_t - \hat{c}_{t+1}\right) = i_t - E_t\pi_{t+1} \qquad \text{(B.44)}
$$

we have the log-linearized aggregate demand equation

$$
\begin{aligned}
\hat{y}_t \quad = \quad & E_t\hat{y}_{t+1} - \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left(i_t - E_t\pi_{t+1}\right) \\
&-\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left[\left(\theta_1-\phi\right)\left(E_t\hat{n}_{t+1}-\hat{n}_t\right) + \theta_2\left(\hat{n}_t - \hat{n}_{t-1}\right)\right]. \qquad \text{(B.45)}
\end{aligned}
$$

B.5 Linear model

The policy rule not withstanding, the linear NK model with search and matching frictions is summarized by the following three equations

$$
\begin{aligned}
\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\Big[&\left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}\right)\hat{y}_t \\
&-(1+\phi)\,\hat{a}_t - \left(\theta_1\hat{n}_t + \theta_2\hat{n}_{t-1}\right)\Big] + \hat{\theta}_{p,t} \qquad \text{(B.46)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left(i_t - E_t\pi_{t+1}\right) \\
-\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left[\left(\theta_1-\phi\right)\left(E_t\hat{n}_{t+1}-\hat{n}_t\right)+\theta_2\left(\hat{n}_t-\hat{n}_{t-1}\right)\right]
\end{aligned} \tag{B.47}
$$

$$
\begin{aligned}
\gamma_1 E_t\hat{n}_{t+1} + \gamma_2\hat{n}_t + \gamma_3\hat{n}_{t-1} = \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}+1\right)\hat{y}_t - \left(1+\phi\right)\hat{a}_t \\
-\frac{(1-\rho)\beta}{\nu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t-\pi_{t+1}\right]
\end{aligned} \tag{B.48}
$$

with the coefficients

$$
\gamma_1 = -\frac{(1-\rho)\beta}{\nu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)\nu_1 \tag{B.49}
$$

$$
\gamma_2 = \left[(1+\theta_1) + \frac{\zeta}{\nu}\frac{\kappa^v}{q_{ss}}\nu_1 + \frac{(1-\rho)\beta}{\nu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)\nu_1\nu_2\right] \tag{B.50}
$$

$$
\gamma_3 = \left[\theta_2 - \frac{\zeta}{\nu}\frac{\kappa^v}{q_{ss}}\nu_1\nu_2\right] \tag{B.51}
$$

$$
\kappa^p = \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}. \tag{B.52}
$$

According to the NKPC (B.46), similar to the standard NK model, price inflation dynamics in search and matching models are determined by current and future real marginal costs which in turn are related to the ratio of real wage to marginal product of labor. However, the real wage in search and matching models is determined through a bargaining process rather than simply equal to marginal rate of substitution between leisure and consumption. Thus, labor market variables affect inflation dynamics directly through the NKPC. Furthermore, the real interest rate affects inflation dynamics the third equation (B.48). Ravenna and Walsh (2011) refers to this channel, which is absent in standard NK model, as the "cost-channel". In contrast to standard NK model, the aggregate demand equation (B.47) in search and matching models features not only forward looking behavior but also backward looking behavior even with standard household preferences that exhibit no habit persistence.

The standard NK model and the model in Ravenna and Walsh (2011) arise as special cases:

- Absent labor market frictions, $\hat{n}_t = 0$, $\kappa^c = 0$, $\varpi^{y_{ss}} = 1$ and equation (B.48) is taken out of the model, the standard NK model with flexible wages reemerges

$$
\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\left(\phi+\sigma\right)\left(\hat{y}_t - \frac{(1+\phi)}{(\phi+\sigma)}\hat{a}_t\right) \tag{B.53}
$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right). \tag{B.54}$$

- If there exists labor market frictions, but the individual labor supply is completely inelastic as in Ravenna and Walsh (2011), i.e. $\phi = \infty$, equation (B.48) reduces to $\hat{y}_t = \hat{a}_t + \hat{n}_t$. After substituting out for $\hat{y}_t$ by $\hat{n}_t$ in the aggregate demand equation, we obtain

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\Bigg[\gamma_1 E_t \hat{n}_{t+1} + (\gamma_2 - \theta_1)\hat{n}_t + (\gamma_3 - \theta_2)\hat{n}_{t-1} \\ &\quad - \hat{a}_t + \frac{(1-\rho)\beta}{\nu}\left(1 - \xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right) E_t\left(i_t - \pi_{t+1}\right)\Bigg] \qquad \text{(B.55)} \\ \hat{n}_t &= \gamma_n E_t \hat{n}_{t+1} + (1-\gamma_n)\hat{n}_{t-1} - \gamma_n^d \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left(i_t - E_t\pi_{t+1}\right) + \gamma_n^d(\rho_a - 1)\hat{a}_t \\ & \qquad \text{(B.56)} \end{aligned}$$

where

$$\begin{aligned} \gamma_n &= \frac{\varpi^{y_{ss}}\sigma - (1-\kappa^c)(\theta_1 - \phi)}{\varpi^{y_{ss}}\sigma + (1-\kappa^c)(\theta_2 - \theta_1 + \phi)} \qquad \text{(B.57)} \\ \gamma_n^d &= \frac{\varpi^{y_{ss}}\sigma}{\varpi^{y_{ss}}\sigma + (1-\kappa^c)(\theta_2 - \theta_1 + \phi)}. \qquad \text{(B.58)} \end{aligned}$$

C Optimal targeting rule for NK model with search and matching frictions

Having obtained the (linear) equations that describe the behavior of the private sector, we still need to derive the objective function of the policymaker as a purely quadratic approximation to the preferences of the representative household to formulate the linear-quadratic problem from which we derive the optimal targeting rule in the NK model with search and matching frictions. In this section, we first derive the correct quadratic loss function as the approximation to the preferences of the representative household. Then we obtain the first order conditions associated with the policymaker's problem of optimizing the (quadratic) objective function subject to the (linear) equations that describe the behavior of the private sector. Finally, the optimal targeting rule is then derived by combining the first order conditions to the policymaker's problem into a single equation without Lagrange multipliers.

C.1 Simplified nonlinear optimality conditions

Before retrieving a numerical representation of the quadratic loss function, we write the nonlinear model in terms of the variables that also enter the set of log-linear equations $\left\{n_t, i_t, y_t, \Pi_t\right\}$ as well as the variables $\left\{U_t^p, V_t^p, \Omega_t^p, \tilde{p}_t^{opt}\right\}$.

The number of job seekers is already expressed in terms of employment only

$$u_t = 1 - (1-\rho)n_{t-1} \tag{C.1}$$

and matches evolve thus according to

$$m_t = n_t - (1-\rho)n_{t-1}. \tag{C.2}$$

Using the matching technology $m_t = \chi u_t^{\zeta} v_t^{1-\zeta}$, the total number of vacancies satisfies

$$v_t = \left(\frac{m_t}{\chi u_t^{\zeta}}\right)^{\frac{1}{1-\zeta}} = \chi^{-\frac{1}{1-\zeta}} \left(n_t - (1-\rho)n_{t-1}\right)^{\frac{1}{1-\zeta}} \left(1-(1-\rho)n_{t-1}\right)^{-\frac{\zeta}{1-\zeta}} \tag{C.3}$$

while labor market tightness can be shown to follow

$$\theta_t = \frac{v_t}{u_t} = \chi^{-\frac{1}{1-\zeta}} (n_t - (1-\rho)n_{t-1})^{\frac{1}{1-\zeta}} (1-(1-\rho)n_{t-1})^{-\frac{1}{1-\zeta}}. \tag{C.4}$$

Finally, the vacancy filling rate is given by

$$q_t = \chi\theta_t^{-\zeta} = \chi^{\frac{1}{1-\zeta}} (n_t - (1-\rho)n_{t-1})^{-\frac{\zeta}{1-\zeta}} (1-(1-\rho)n_{t-1})^{\frac{\zeta}{1-\zeta}} \tag{C.5}$$

and the job finding rate can be written as

$$s_t = \frac{m_t}{u_t} = \frac{n_t - (1-\rho)n_{t-1}}{1-(1-\rho)n_{t-1}}. \tag{C.6}$$

Using the production technology, hours worked can be expressed as

$$h_t = \frac{\Omega_t^p y_t}{a_t n_t}. \tag{C.7}$$

The resource constraint implies for consumption that

$$c_t \quad = \quad y_t - \kappa^v v_t + b^u(1-n_t). \tag{C.8}$$

The equation governing vacancy postings (A.20) can be stated as

$$\left(\frac{\kappa^v}{q_t}\right) c_t^{-\sigma} = (1-\xi)\left(\frac{\phi}{1+\phi}\phi_0 h_t^{1+\phi} - b^u c_t^{-\sigma}\right) + (1-\rho)\beta E_t c_{t+1}^{-\sigma}(1-\xi s_{t+1})\left(\frac{\kappa^v}{q_{t+1}}\right) \tag{C.9}$$

whereas the wage bargaining equation is

$$w_t h_t = \xi\left(\phi_0 h_t^{1+\phi} c_t^{\sigma} + (1-\rho)\beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\theta_{t+1}\kappa^v\right) + (1-\xi)\left(b^u + \phi_0 \frac{h_t^{1+\phi}}{1+\phi} c_t^{\sigma}\right). \tag{C.10}$$

Finally, the nonlinear equations governing the evolution of prices in equilibrium are, the optimal price

$$\tilde{p}_t^{opt} = \frac{U_t^p}{V_t^p} \tag{C.11}$$

which is computed as the ratio of the recursively defined terms U_t^p and V_t^p

$$U_t^p = \frac{1+\theta_{p,t}}{\theta^p}\phi_0 h_t^{\phi}\frac{y_t}{a_t} + \xi^p \beta E_t \left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\left(\frac{1+\theta^p}{\theta^p}\right)} U_{t+1}^p \tag{C.12}$$

$$V_t^p = \frac{(1+\bar{\tau}^p)}{\theta^p} y_t c_t^{-\sigma} + \xi^p \beta E_t \left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\frac{1}{\theta^p}} V_{t+1}^p. \tag{C.13}$$

The definition of the price level implies

$$1 = \xi^p \left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\frac{1}{\theta^p}} + (1-\xi^p)\left(\tilde{p}_t^{opt}\right)^{-\frac{1}{\theta^p}} \tag{C.14}$$

and price dispersion evolves according to

$$\Omega_t^p = \xi^p \left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\frac{1+\theta^p}{\theta^p}} \Omega_{t-1}^p + (1-\xi^p)(\tilde{p}_t^{opt})^{-\frac{1+\theta^p}{\theta^p}}. \tag{C.15}$$

Recall, that we continue to abstract from price indexation and consumption habits.

C.2 Correct quadratic loss function

Following a large body of the literature, we compute the optimal monetary policy under commitment from *the timeless perspective* as the reference point to evaluate the performance of different policies. Optimality from the timeless perspective assumes that the policymaker can "pre-commit" at the beginning of time. This assumption converts the optimal policy problem into a recursive problem with time invariant functions as shown in detail in Benigno and Woodford (2012). As shown in Bodenstein, Guerrieri, and LaBriola (2014), the first-order approximation to the system of first order conditions associated with original nonlinear model can be mapped into the LQ problem

$$\begin{aligned} &\max_{\{\hat{x}_t\}_{t=t_0}^{\infty}} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2}\,\hat{x}_t' A(L)\hat{x}_t + \hat{x}_t' B(L)\zeta_{t+1}\right] \\ &s.t. \\ &E_t C(L)\hat{x}_{t+1} + D(L)\zeta_t = 0 \end{aligned}$$

$$
\begin{aligned}
&C(L)\hat{x}_{t_0} = d_{t_0} \\
&\zeta_t = \Gamma\zeta_{t-1} + \Upsilon\xi_t
\end{aligned} \tag{C.16}
$$

where $\hat{x}_{t_0}$ measures the (log-) deviation of variable "x" from its value assumed in deterministic steady state. The matrices $(A(L), B(L))$ capture the second-order approximation of the welfare function, where "L" denotes the lag-operator. The matrices $C(L)$ and $D(L)$ capture the linear approximation of the constraints. The linear constraints $C(L)\hat{x}_{t_0} = d_{t_0}$ implement the timeless perspective through the appropriate choice of d_{t_0}. The model description is completed by the evolution of the exogenous variables, the last equation in (C.16). The innovations ξ_t follow *iid* standard normal distributions.

All welfare relevant matrices in the above LQ problem can be retrieved using the numerical approach in Bodenstein, Guerrieri, and LaBriola (2014). After retrieving the welfare matrices $A(L)$ and $B(L)$ and accounting for all zero elements, the approximation to the preferences of the representative household is given by

$$
\begin{aligned}
\tilde{\mathcal{L}}^{s\&m} = \quad & \frac{1}{2}a_{2,2}\pi_t^2 + \frac{1}{2}a_{3,3}\hat{n}_t^2 + \frac{1}{2}a_{8,8}\hat{y}_t^2 + \frac{1}{2}a_{11,11}\hat{n}_{t-1}^2 + a_{3,8}\hat{n}_t\hat{y}_t + a_{3,11}\hat{n}_t\hat{n}_{t-1} + a_{8,11}\hat{y}_t\hat{n}_{t-1} \\
& + \frac{1}{2}a_{4,4}(\hat{\tilde{p}}_t^{opt})^2 + a_{2,6}\pi_t\hat{U}_t^p + a_{2,7}\pi_t\hat{V}_t^p + a_{6,7}\hat{U}_t^p\hat{V}_t^p + \frac{1}{2}a_{7,7}(\hat{V}_t^p)^2 \\
& + b_{3,3}\hat{n}_t\hat{n}_{t-1} + b_{8,3}\hat{y}_t\hat{n}_{t-1} + c_{3,1}\hat{n}_t\hat{a}_t + c_{3,2}\hat{n}_t\hat{\theta}_{p,t} + c_{8,1}\hat{y}_t\hat{a}_t + c_{8,2}\hat{y}_t\hat{\theta}_{p,t}
\end{aligned} \tag{C.17}
$$

where $\hat{y}_t$ is output, π_t refers to price inflation, and $\hat{n}_t$ stands for employment. $\hat{\tilde{p}}_t^{opt}$ is the optimal price set by re-optimizing firms. $\hat{U}_t^p$ and $\hat{V}_t^p$ are log-linear versions of the variables U_t^p and V_t^p. $\hat{a}_t$ is technology shock and $\hat{\theta}_{p,t}$ is price markup shock. $a_{i,j} = A_0(i,j)$, $b_{i,j} = A_1(i,j)$, and $c_{i,j} = B_1(i,j)$ for corresponding index (i,j) are the entries in $A(L)$ and $B(L)$. Besides terms that are already present in the standard NK model, labor market variables affect the loss function in the search and matching framework. Current and lagged employment enter the approximation.

When using a first order approximation, the nonlinear equations associated with Calvo sticky prices can be summarized in the NKPC for price inflation. Therefore, sticky price variables $\{\hat{\tilde{p}}_t^{opt}, \hat{U}_t^p, \hat{V}_t^p, \hat{\Omega}_t^p\}$ will only show up in the nonlinear system but not in the log-linearized system. To make the loss function work correspond to the linear structural equations, these sticky price variables have to be substituted out.

Log-linearizing the equation (C.14) delivers

$$\hat{\tilde{p}}_t^{opt} = \frac{\xi^p}{1-\xi^p}\pi_t. \tag{C.18}$$

Equation (C.18) can be used to substitute out $\hat{\tilde{p}}_t^{opt}$ in the loss function.

Log-linearizing the equation describing the evolution of price dispersion (C.15) provides

$$\begin{aligned}
\hat{\Omega}_t^p \quad &= \xi^p \hat{\Omega}_{t-1}^p + \xi^p \frac{1+\theta^p}{\theta^p}\pi_t - (1-\xi^p)\frac{1+\theta^p}{\theta^p}\hat{\tilde{p}}_t^{opt} \\
&= \xi^p \hat{\Omega}_{t-1}^p.
\end{aligned} \tag{C.19}$$

Thus, price dispersion can be ignored to the first order.

Applying the log-linearization for equations (C.11) and (C.14), we have

$$\begin{aligned}
& a_{2,6}\pi_t \hat{U}_t^p + a_{2,7}\pi_t \hat{V}_t^p + a_{6,7}\hat{U}_t^p \hat{V}_t^p + \frac{1}{2}a_{7,7}(\hat{V}_t^p)^2 \\
&= a_{2,6}\pi_t \left(\hat{\tilde{p}}_t^{opt} + \hat{V}_t^p\right) + a_{2,7}\pi_t \hat{V}_t^p + a_{6,7}\left(\hat{\tilde{p}}_t^{opt} + \hat{V}_t^p\right)\hat{V}_t^p + \frac{1}{2}a_{7,7}(\hat{V}_t^p)^2 \\
&= a_{2,6}\pi_t \hat{\tilde{p}}_t^{opt} + a_{2,6}\pi_t \hat{V}_t^p + a_{2,7}\pi_t \hat{V}_t^p + a_{6,7}\hat{V}_t^p \hat{\tilde{p}}_t^{opt} + \left(a_{6,7} + \frac{1}{2}a_{7,7}\right)(\hat{V}_t^p)^2 \\
&= a_{2,6}\pi_t \hat{\tilde{p}}_t^{opt} + a_{2,6}\pi_t \hat{V}_t^p + a_{2,7}\pi_t \hat{V}_t^p + a_{6,7}\hat{V}_t^p \hat{\tilde{p}}_t^{opt} \\
&= a_{2,6}\frac{\xi^p}{1-\xi^p}\pi_t^2 + \left(a_{2,6} + a_{2,7} + a_{6,7}\frac{\xi^p}{1-\xi^p}\right)\pi_t \hat{V}_t^p \\
&= a_{2,6}\frac{\xi^p}{1-\xi^p}\pi_t^2
\end{aligned} \tag{C.20}$$

The first identity comes from the relationship $\hat{U}_t^p = \hat{\tilde{p}}_t^{opt} + \hat{V}_t^p$; the third identity is true as $a_{6,7} + \frac{1}{2}a_{7,7} = 0$; plugging in equation (C.18) gives us the fourth identity; the fifth identity holds as $a_{2,6} + a_{2,7} + a_{6,7}\frac{\xi^p}{1-\xi^p} = 0$.

We convert the approximation to household preferences, $\tilde{\mathcal{L}}_t^{s\&m}$, into a loss function by defining $\mathcal{L}_t^{s\&m} = -\tilde{\mathcal{L}}_t^{s\&m}$. The loss function in the search and matching model is therefore written as

$$\begin{aligned}
\mathcal{L}_t^{s\&m} \quad = \quad & P_{\pi,\pi}\pi_t^2 + P_{y,y}\hat{y}_t^2 + P_{n,n}\hat{n}_t^2 + P_{n^-,n^-}\hat{n}_{t-1}^2 + P_{y,n}\hat{n}_t\hat{y}_t + P_{y,n^-}\hat{y}_t\hat{n}_{t-1} \\
& + P_{n,n^-}\hat{n}_t\hat{n}_{t-1} + P_{n,a}\hat{n}_t\hat{a}_t + P_{n,p}\hat{n}_t\hat{\theta}_{p,t} + P_{y,a}\hat{y}_t\hat{a}_t + P_{y,p}\hat{y}_t\hat{\theta}_{p,t}
\end{aligned} \tag{C.21}$$

where

$$
\begin{aligned}
P_{\pi,\pi} &= -\frac{1}{2}a_{2,2} - \frac{1}{2}a_{4,4}\left(\frac{\xi^p}{1-\xi^p}\right)^2 - a_{2,6}\frac{\xi^p}{1-\xi^p} \\
P_{y,y} &= -\frac{1}{2}a_{8,8} \\
P_{n,n} &= -\frac{1}{2}a_{3,3} \\
P_{n^-,n^-} &= -\frac{1}{2}a_{11,11} \\
P_{y,n} &= -\frac{1}{2}a_{3,8} \\
P_{y,n^-} &= -\left(a_{8,11} + b_{8,3}\right) \\
P_{n,n^-} &= -\left(a_{3,11} + b_{3,3}\right) \\
P_{n,a} &= -c_{3,1} \\
P_{n,p} &= -c_{3,2} \\
P_{y,a} &= -c_{8,1} \\
P_{y,p} &= -c_{8,2}.
\end{aligned}
$$

To sum up, the procedure for deriving the correct loss function in the search and matching models involves:

1. deriving the nonlinear equilibrium conditions for the original model;

2. simplifying the nonlinear system of equations such that it only involves variables that show up in the log-linearized model, together with sticky price variables $\{U_t^p, V_t^p, \tilde{p}_t^{opt}, \Omega_t^p\}$;

3. applying the numerical approach to retrieve welfare matrices based on the simplified equation system;

4. writing out the loss function by plugging in retrieved welfare matrices;

5. using the log-linearized structural equations to eliminate the sticky price variables $\{U_t^p, V_t^p, \tilde{p}_t^{opt}, \Omega_t^p\}$ in the loss function.

6. obtaining the correct quadratic loss function, even though we can only know numerically the values of the coefficients which in turn depend on the model's structural parameters.

C.3 First order conditions of the LQ problem

The correct LQ system is given by

$$\min_{\{\pi_t,i_t,\hat{n}_t,\hat{y}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ P_{\pi,\pi}\pi_t^2 + P_{y,y}\hat{y}_t^2 + P_{n,n}\hat{n}_t^2 + P_{n^-,n^-}\hat{n}_{t-1}^2 + P_{y,n}\hat{n}_t\hat{y}_t$$
$$+P_{y,n^-}\hat{y}_t\hat{n}_{t-1} + P_{n,n^-}\hat{n}_t\hat{n}_{t-1} + P_{n,a}\hat{n}_t\hat{a}_t + P_{n,p}\hat{n}_t\hat{\theta}_{p,t} + P_{y,a}\hat{y}_t\hat{a}_t + P_{y,p}\hat{y}_t\hat{\theta}_{p,t} \Bigg\}$$

$$\begin{aligned}
&s.t.\\
&\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\Big[\left(\phi+\sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}\right)\hat{y}_t \\
&\qquad -(1+\phi)\,\hat{a}_t - (\theta_1\hat{n}_t + \theta_2\hat{n}_{t-1})\Big] + \hat{\theta}_{p,t}\\
&\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\,(i_t - E_t\pi_{t+1})\\
&\qquad -\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\,[(\theta_1-\phi)\,(E_t\hat{n}_{t+1}-\hat{n}_t) + \theta_2\,(\hat{n}_t - \hat{n}_{t-1})]\\
&\gamma_1 E_t\hat{n}_{t+1} + \gamma_2\hat{n}_t + \gamma_3\hat{n}_{t-1} = \left(\phi+\sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}+1\right)\hat{y}_t - (1+\phi)\,\hat{a}_t\\
&\qquad\qquad -\frac{(1-\rho)\beta}{\nu}\,(1-\xi q_{ss}\theta_{ss})\left(\frac{\kappa^v}{q_{ss}}\right)E_t\,[i_t - \pi_{t+1}]\,.
\end{aligned} \tag{C.22}$$

The problem is to minimize the quadratic objective function subject to linear structural equations.

Taking first order conditions delivers

$$(i_t): \quad \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\Lambda_{2,t} + \frac{(1-\rho)\beta}{\nu}\,(1-\xi q_{ss}\theta_{ss})\left(\frac{\kappa^v}{q_{ss}}\right)\Lambda_{3,t} = 0 \tag{C.23}$$

$$\begin{aligned}
(\pi_t): \quad & 2P_{\pi,\pi}\pi_t + \Lambda_{1,t} - \Lambda_{1,t-1} - \frac{1}{\beta}\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\Lambda_{2,t-1}\\
& -\frac{1}{\beta}\frac{(1-\rho)\beta}{\nu}\,(1-\xi q_{ss}\theta_{ss})\left(\frac{\kappa^v}{q_{ss}}\right)\Lambda_{3,t-1} = 0
\end{aligned} \tag{C.24}$$

$$\begin{aligned}
(\hat{y}_t): \quad & 2P_{y,y}\hat{y}_t + P_{y,n}\hat{n}_t + P_{y,n^-}\hat{n}_{t-1} + P_{y,a}\hat{a}_t + P_{y,p}\hat{\theta}_{p,t}\\
& -\frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\left(\phi+\sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}\right)\Lambda_{1,t} + \Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1}\\
& -\left(\phi+\sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}+1\right)\Lambda_{3,t} = 0
\end{aligned} \tag{C.25}$$

$$\begin{aligned}
(\hat{n}_t): \quad & 2P_{n,n}\hat{n}_t + 2\beta P_{n^-,n^-}\hat{n}_t + P_{y,n}\hat{y}_t + \beta P_{y,n^-}E_t\hat{y}_{t+1} + P_{n,n^-}\hat{n}_{t-1}\\
& +\beta P_{n,n^-}E_t\hat{n}_{t+1} + P_{n,a}\hat{a}_t + P_{n,p}\hat{\theta}_{p,t} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\theta_1\Lambda_{1,t}\\
& +\beta\frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\theta_2 E_t\Lambda_{1,t+1} + \frac{1}{\varpi^{y_{ss}}}\frac{1}{\beta}\frac{1-\kappa^c}{\sigma}(\theta_1-\phi)\Lambda_{2,t-1}
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}(\theta_1-\phi)\Lambda_{2,t}+\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\theta_2\Lambda_{2,t}-\frac{1}{\varpi^{y_{ss}}}\beta\frac{1-\kappa^c}{\sigma}\theta_2E_t\Lambda_{2,t+1}\\
&+\frac{1}{\beta}\gamma_1\Lambda_{3,t-1}+\gamma_2\Lambda_{3,t}+\beta\gamma_3E_t\Lambda_{3,t+1}=0 \qquad \text{(C.26)}
\end{aligned}
$$

C.4 Substituting out Lagrange multipliers and the optimal targeting rule

To simplify notation, we define

$$
\begin{aligned}
\o_1 &= \phi+\frac{\varpi^{y_{ss}}\sigma}{1-\kappa^c}\\
\kappa^p &= \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\\
\nu^\Lambda &= -\frac{\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}}{\frac{(1-\rho)\beta}{\nu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)}\\
G_t^\Lambda &= 2P_{y,y}\hat{y}_t+P_{y,n}\hat{n}_t+P_{y,n^-}\hat{n}_{t-1}+P_{y,a}\hat{a}_t+P_{y,p}\hat{\theta}_{p,t}\\
H_t^\Lambda &= 2P_{n,n}\hat{n}_t+2\beta P_{n^-,n^-}\hat{n}_t+P_{y,n}\hat{y}_t+\beta P_{y,n^-}E_t\hat{y}_{t+1}+P_{n,n^-}\hat{n}_{t-1}+\beta P_{n,n^-}E_t\hat{n}_{t+1}\\
&\quad +P_{n,a}\hat{a}_t+P_{n,p}\hat{\theta}_{p,t}.
\end{aligned}
$$

Then the set of first order conditions simplifies to

$$
\begin{aligned}
(i_t):&\quad \Lambda_{3,t}=\nu^\Lambda\Lambda_{2,t} &&\text{(C.27)}\\
(\pi_t):&\quad 2P_{\pi,\pi}\pi_t+\Lambda_{1,t}-\Lambda_{1,t-1}=0 &&\text{(C.28)}\\
(\hat{y}_t):&\quad G_t^\Lambda-\kappa^p\o_1\Lambda_{1,t}+\Lambda_{2,t}-\frac{1}{\beta}\Lambda_{2,t-1}-\left(1+\o_1\right)\Lambda_{3,t}=0 &&\text{(C.29)}\\
(\hat{n}_t):&\quad H_t^\Lambda+\kappa^p\theta_1\Lambda_{1,t}+\kappa^p\beta\theta_2E_t\Lambda_{1,t+1}+\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi)\Lambda_{2,t-1}+\frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right)\Lambda_{2,t}\\
&\quad -\frac{1}{\o_1-\phi}\beta\theta_2E_t\Lambda_{2,t+1}+\frac{1}{\beta}\gamma_1\Lambda_{3,t-1}+\gamma_2\Lambda_{3,t}+\beta\gamma_3E_t\Lambda_{3,t+1}=0. &&\text{(C.30)}
\end{aligned}
$$

Substituting out $\Lambda_{3,t}$ in equation (C.29) and (C.30) by using equation (C.27),

$$
G_t^\Lambda-\kappa^p\o_1\Lambda_{1,t}+\left[1-\left(1+\o_1\right)\nu^\Lambda\right]\Lambda_{2,t}-\frac{1}{\beta}\Lambda_{2,t-1}=0 \qquad \text{(C.31)}
$$

and

$$
\begin{aligned}
&H_t^{\Lambda} + \kappa^p\theta_1\Lambda_{1,t} + \kappa^p\beta\theta_2 E_t\Lambda_{1,t+1} + \frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi)\Lambda_{2,t-1} + \frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right)\Lambda_{2,t} \\
&-\frac{1}{\o_1-\phi}\beta\theta_2 E_t\Lambda_{2,t+1} + \frac{1}{\beta}\gamma_1\nu^{\Lambda}\Lambda_{2,t-1} + \gamma_2\nu^{\Lambda}\Lambda_{2,t} + \beta\gamma_3\nu^{\Lambda}E_t\Lambda_{2,t+1} \\
&= H_t^{\Lambda} + \kappa^p\theta_1\Lambda_{1,t} + \kappa^p\beta\theta_2 E_t\Lambda_{1,t+1} + \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^{\Lambda}\right]\Lambda_{2,t-1} \\
&+ \left[\frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right) + \gamma_2\nu^{\Lambda}\right]\Lambda_{2,t} + \left[\beta\gamma_3\nu^{\Lambda} - \frac{1}{\o_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1} \\
&= 0. \qquad \text{(C.32)}
\end{aligned}
$$

Since price inflation is defined as the change in the price level (in terms of deviation from steady state) $\pi_t = \hat{P}_t - \hat{P}_{t-1}$, we can then express $\Lambda_{1,t}$ as proportional to the price level $\hat{P}_t$ from equation (C.28). At the same time, the equation $\pi_t = \hat{P}_t - \hat{P}_{t-1}$ has to be added into the model system. It is straightforward to show

$$
\Lambda_{1,t} = -2P_{\pi,\pi}\hat{P}_t. \qquad \text{(C.33)}
$$

Plugging the expression of $\Lambda_{1,t}$ into equation (C.31) and (C.32),

$$
\begin{aligned}
& G_t^{\Lambda} - \kappa^p\o_1\Lambda_{1,t} + \left[1-\left(1+\o_1\right)\nu^{\Lambda}\right]\Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1} \\
= \quad & G_t^{\Lambda} + 2\kappa^p\o_1 P_{\pi,\pi}\hat{P}_t + \left[1-\left(1+\o_1\right)\nu^{\Lambda}\right]\Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1} \\
= \quad & 0 \qquad \text{(C.34)}
\end{aligned}
$$

and

$$
\begin{aligned}
&H_t^{\Lambda} + \kappa^p\theta_1\Lambda_{1,t} + \kappa^p\beta\theta_2 E_t\Lambda_{1,t+1} + \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^{\Lambda}\right]\Lambda_{2,t-1} \\
&+ \left[\frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right) + \gamma_2\nu^{\Lambda}\right]\Lambda_{2,t} + \left[\beta\gamma_3\nu^{\Lambda} - \frac{1}{\o_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1} \\
&= H_t^{\Lambda} - 2\kappa^p\theta_1 P_{\pi,\pi}\hat{P}_t - 2\kappa^p\beta\theta_2 P_{\pi,\pi}E_t\hat{P}_{t+1} + \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^{\Lambda}\right]\Lambda_{2,t-1} \\
&+ \left[\frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right) + \gamma_2\nu^{\Lambda}\right]\Lambda_{2,t} + \left[\beta\gamma_3\nu^{\Lambda} - \frac{1}{\o_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1} \\
&= 0. \qquad \text{(C.35)}
\end{aligned}
$$

We further define

$$\begin{aligned}
\chi_{-1}^{\Lambda} &= \left[\frac{1}{\text{ø}_1-\phi}\frac{1}{\beta}(\theta_1-\phi)+\frac{1}{\beta}\gamma_1\nu^{\Lambda}\right] \\
\chi_{0}^{\Lambda} &= \left[\frac{1}{\text{ø}_1-\phi}\left(\theta_2+\phi-\theta_1\right)+\gamma_2\nu^{\Lambda}\right] \\
\chi_{1}^{\Lambda} &= \left[\beta\gamma_3\nu^{\Lambda}-\frac{1}{\text{ø}_1-\phi}\beta\theta_2\right] \\
\beta_{\delta} &= \frac{1}{\beta\left[1-\left(1+\text{ø}_1\right)\nu^{\Lambda}\right]} \\
\chi^{\Lambda 2} &= \left(\frac{\chi_{-1}^{\Lambda}}{\beta_{\delta}}+\chi_{0}^{\Lambda}+\chi_{1}^{\Lambda}\beta_{\delta}\right).
\end{aligned}$$

From equation (C.34), we get

$$\Lambda_{2,t}-\frac{1}{\beta\left[1-\left(1+\text{ø}_1\right)\nu^{\Lambda}\right]}\Lambda_{2,t-1}=-\frac{1}{1-\left(1+\text{ø}_1\right)\nu^{\Lambda}}\left(G_t^{\Lambda}+2\kappa^{p}\text{ø}_1P_{\pi,\pi}\hat{P}_t\right) \tag{C.36}$$

or

$$\Lambda_{2,t}-\beta_{\delta}\Lambda_{2,t-1}=-\beta\beta_{\delta}\left(G_t^{\Lambda}+2\kappa^{p}\text{ø}_1P_{\pi,\pi}\hat{P}_t\right). \tag{C.37}$$

This equation implies an expression for $\Lambda_{2,t}$

$$\begin{aligned}
&\Lambda_{2,t} \\
&= -\beta\beta_{\delta}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\left(G_{t-s}^{\Lambda}+2\kappa^{p}\text{ø}_1P_{\pi,\pi}\hat{P}_{t-s}\right) \\
&= -\beta\beta_{\delta}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\Big(2P_{y,y}\hat{y}_{t-s}+P_{y,n}\hat{n}_{t-s}+P_{y,n^-}\hat{n}_{t-1-s}+P_{y,a}\hat{a}_{t-s} \\
&\quad +P_{y,p}\hat{\theta}_{p,t-s}+2\kappa^{p}\text{ø}_1P_{\pi,\pi}\hat{P}_{t-s}\Big) \\
&= -\beta\beta_{\delta}2P_{y,y}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\hat{y}_{t-s}-\beta\beta_{\delta}P_{y,n}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\hat{n}_{t-s}-\beta\beta_{\delta}P_{y,n^-}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\hat{n}_{t-1-s} \\
&\quad -\beta\beta_{\delta}P_{y,a}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\hat{a}_{t-s}-\beta\beta_{\delta}P_{y,p}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\hat{\theta}_{p,t-s}-\beta\beta_{\delta}2\kappa^{p}\text{ø}_1P_{\pi,\pi}\sum_{s=0}^{\infty}\left(\beta_{\delta}\right)^{s}\hat{P}_{t-s} \\
&= -\beta\beta_{\delta}2P_{y,y}\hat{y}_t^{WA}-\beta\beta_{\delta}P_{y,n}\hat{n}_t^{WA}-\beta\beta_{\delta}P_{y,n^-}\hat{n}_{t-1}^{WA}-\beta\beta_{\delta}P_{y,a}\hat{a}_t^{WA} \\
&\quad -\beta\beta_{\delta}P_{y,p}\hat{\theta}_{p,t}^{WA}-\beta\beta_{\delta}2\kappa^{p}\text{ø}_1P_{\pi,\pi}\hat{P}_t^{WA} \\
&= -\beta\beta_{\delta}\Big[2P_{y,y}\hat{y}_t^{WA}+\left(P_{y,n}+\frac{P_{y,n^-}}{\beta_{\delta}}\right)\hat{n}_t^{WA}-\frac{P_{y,n^-}}{\beta_{\delta}}\hat{n}_t
\end{aligned}$$

$$+P_{y,a}\hat{a}_t^{WA} + P_{y,p}\hat{\theta}_{p,t}^{WA} + 2\kappa^p \o_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big].$$

Finally,

$$\Lambda_{2,t} = -\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} - \frac{P_{y,n^-}}{\beta_\delta}\hat{n}_t$$
$$+P_{y,a}\hat{a}_t^{WA} + P_{y,p}\hat{\theta}_{p,t}^{WA} + 2\kappa^p \o_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big] \tag{C.38}$$

where $\hat{y}_t^{WA}$, $\hat{n}_t^{WA}$, $\hat{a}_t^{WA}$, $\hat{\theta}_{p,t}^{WA}$, and $\hat{P}_t^{WA}$ are the weighted averages of historical realizations with

$$\hat{y}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{y}_{t-s} \tag{C.39}$$
$$\hat{n}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{n}_{t-s} \tag{C.40}$$
$$\hat{a}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{a}_{t-s} \tag{C.41}$$
$$\hat{\theta}_{p,t}^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{\theta}_{p,t-s} \tag{C.42}$$
$$\hat{P}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{P}_{t-s} \tag{C.43}$$

or written recursively

$$\hat{y}_t^{WA} = \beta_\delta\hat{y}_{t-1}^{WA} + \hat{y}_t \tag{C.44}$$
$$\hat{n}_t^{WA} = \beta_\delta\hat{n}_{t-1}^{WA} + \hat{n}_t \tag{C.45}$$
$$\hat{a}_t^{WA} = \beta_\delta\hat{a}_{t-1}^{WA} + \hat{a}_t \tag{C.46}$$
$$\hat{\theta}_{p,t}^{WA} = \beta_\delta\hat{\theta}_{p,t-1}^{WA} + \hat{\theta}_{p.t} \tag{C.47}$$
$$\hat{P}_t^{WA} = \beta_\delta\hat{P}_{t-1}^{WA} + \hat{P}_t. \tag{C.48}$$

Substituting equation (C.37) into equation (C.35) and using the newly defined coefficients,

$$H_t^\Lambda - 2\kappa^p\theta_1 P_{\pi,\pi}\hat{P}_t - 2\kappa^p\beta\theta_2 P_{\pi,\pi}E_t\hat{P}_{t+1} + \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^\Lambda\right]\Lambda_{2,t-1}$$
$$+\left[\frac{1}{\o_1-\phi}(\theta_2+\phi-\theta_1) + \gamma_2\nu^\Lambda\right]\Lambda_{2,t} + \left[\beta\gamma_3\nu^\Lambda - \frac{1}{\o_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1}$$

$$\begin{aligned}
&= && H_t^{\Lambda} - 2\kappa^p\theta_1 P_{\pi,\pi}\hat{P}_t - 2\kappa^p\beta\theta_2 P_{\pi,\pi}E_t\hat{P}_{t+1} + \chi_{-1}^{\Lambda}\left(\frac{1}{\beta_\delta}\Lambda_{2,t} + \beta\left(G_t^{\Lambda} + 2\kappa^p\emptyset_1 P_{\pi,\pi}\hat{P}_t\right)\right) \\
& && +\chi_0^{\Lambda}\Lambda_{2,t} + \chi_1^{\Lambda}\left(\beta_\delta\Lambda_{2,t} - \beta\beta_\delta\left(E_t G_{t+1}^{\Lambda} + 2\kappa^p\emptyset_1 P_{\pi,\pi}E_t\hat{P}_{t+1}\right)\right) \\
&= && H_t^{\Lambda} + \chi_{-1}^{\Lambda}\beta G_t^{\Lambda} - \chi_1^{\Lambda}\beta\beta_\delta E_t G_{t+1}^{\Lambda} + \left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\emptyset_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right)\hat{P}_t \\
& && -\left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\emptyset_1 P_{\pi,\pi}\right)E_t\hat{P}_{t+1} + \left(\frac{\chi_{-1}^{\Lambda}}{\beta_\delta} + \chi_0^{\Lambda} + \chi_1^{\Lambda}\beta_\delta\right)\Lambda_{2,t} \\
&= && H_t^{\Lambda} + \chi_{-1}^{\Lambda}\beta G_t^{\Lambda} - \chi_1^{\Lambda}\beta\beta_\delta E_t G_{t+1}^{\Lambda} + \left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\emptyset_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right)\hat{P}_t \\
& && -\left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\emptyset_1 P_{\pi,\pi}\right)E_t\hat{P}_{t+1} + \chi^{\Lambda 2}\Lambda_{2,t} \\
&= && \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta\frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t \\
& && +\left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right)\hat{n}_{t-1} + \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right)E_t\hat{n}_{t+1} \\
& && +\left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right)\hat{y}_t + \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right)E_t\hat{y}_{t+1} \\
& && +\left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a}\right)\hat{a}_t + \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p}\right)\hat{\theta}_{p,t} \\
& && -\chi_1^{\Lambda}\beta\beta_\delta P_{y,a}E_t\hat{a}_{t+1} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,p}E_t\hat{\theta}_{p,t+1} \\
& && +\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\emptyset_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right)\hat{P}_t - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\emptyset_1 P_{\pi,\pi}\right)E_t\hat{P}_{t+1} \\
& && -\chi^{\Lambda 2}\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} + P_{y,a}a_t^{WA} \\
& && +P_{y,p}\theta_{p,t}^{WA} + 2\kappa^p\emptyset_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big]. \qquad \text{(C.49)}
\end{aligned}$$

If the technology shock and the markup shock follow AR(1) process, then

$$E_t\hat{a}_{t+1} = \rho_a\hat{a}_t \qquad \text{(C.50)}$$

$$E_t\hat{\theta}_{p,t+1} = \rho_p\hat{\theta}_{p,t}. \qquad \text{(C.51)}$$

Together with the definition of price inflation $\pi_t = \hat{P}_t - \hat{P}_{t-1}$, we have

$$\begin{aligned}
0 = \quad & \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta\frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t \\
& +\left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right)\hat{n}_{t-1} + \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right)E_t\hat{n}_{t+1} \\
& +\left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right)\hat{y}_t + \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right)E_t\hat{y}_{t+1} \\
& +\left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,a}\rho_a\right)\hat{a}_t + \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,p}\rho_p\right)\hat{\theta}_{p,t} \\
& +\left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\emptyset_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\emptyset_1 P_{\pi,\pi}\right)\right]\hat{P}_t \\
& -\left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\emptyset_1 P_{\pi,\pi}\right)E_t\pi_{t+1}
\end{aligned}$$

$$
\begin{aligned}
& -\chi^{\Lambda 2}\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} + P_{y,a}a_t^{WA} \\
& +P_{y,p}\theta_{p,t}^{WA} + 2\kappa^p\o_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big] \\
= \quad & \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta\frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t \\
& + \left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right)\hat{n}_{t-1} + \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right)E_t\hat{n}_{t+1} \\
& + \left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right)\hat{y}_t + \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right)E_t\hat{y}_{t+1} \\
& + \left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,a}\rho_a\right)\hat{a}_t + \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,p}\rho_p\right)\hat{\theta}_{p,t} \\
& + \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)\right]\pi_t \\
& - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)E_t\pi_{t+1} \\
& + \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)\right]\hat{P}_{t-1} \\
& -\chi^{\Lambda 2}\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} + P_{y,a}a_t^{WA} \\
& +P_{y,p}\theta_{p,t}^{WA} + 2\kappa^p\o_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big]. \qquad \text{(C.52)}
\end{aligned}
$$

Hence, the *optimal targeting rule* is given by,

$$
\begin{aligned}
& \varpi_1\hat{n}_t + \varpi_2\hat{n}_{t-1} + \varpi_3\hat{n}_{t+1} + \varpi_4\hat{y}_t + \varpi_5\hat{y}_{t+1} + \varpi_6\hat{a}_t + \varpi_7\hat{\theta}_{p,t} + \varpi_8\pi_t + \varpi_9\pi_{t+1} \\
& +\varpi_{10}\hat{P}_{t-1} + \varpi_{11}\hat{y}_t^{WA} + \varpi_{12}\hat{n}_t^{WA} + \varpi_{13}\hat{a}_t^{WA} + \varpi_{14}\hat{\theta}_{p,t}^{WA} + \varpi_{15}\hat{P}_t^{WA} = 0 \qquad \text{(C.53)}
\end{aligned}
$$

where we define

$$
\begin{aligned}
\pi_t &= \hat{P}_t - \hat{P}_{t-1} && \text{(C.54)} \\
\hat{y}_t^{WA} &= \beta_\delta\hat{y}_{t-1}^{WA} + \hat{y}_t && \text{(C.55)} \\
\hat{n}_t^{WA} &= \beta_\delta\hat{n}_{t-1}^{WA} + \hat{n}_t && \text{(C.56)} \\
\hat{a}_t^{WA} &= \beta_\delta\hat{a}_{t-1}^{WA} + \hat{a}_t && \text{(C.57)} \\
\hat{\theta}_{p,t}^{WA} &= \beta_\delta\hat{\theta}_{p,t-1}^{WA} + \hat{\theta}_{p.t} && \text{(C.58)} \\
\hat{P}_t^{WA} &= \beta_\delta\hat{P}_{t-1}^{WA} + \hat{P}_t && \text{(C.59)}
\end{aligned}
$$

and

$$
\varpi_1 = \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta\frac{P_{y,n^-}}{\beta_\delta}\right)
$$

$$
\begin{aligned}
\varpi_2 &= \left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right)\\
\varpi_3 &= \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right)\\
\varpi_4 &= \left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right)\\
\varpi_5 &= \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right)\\
\varpi_6 &= \left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,a}\rho_a\right)\\
\varpi_7 &= \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,p}\rho_p\right)\\
\varpi_8 &= \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)\right]\\
\varpi_9 &= -\left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)\\
\varpi_{10} &= \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)\right]\\
\varpi_{11} &= -\chi^{\Lambda 2}\beta\beta_\delta 2P_{y,y}\\
\varpi_{12} &= -\chi^{\Lambda 2}\beta\beta_\delta\left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\\
\varpi_{13} &= -\chi^{\Lambda 2}\beta\beta_\delta P_{y,a}\\
\varpi_{14} &= -\chi^{\Lambda 2}\beta\beta_\delta P_{y,p}\\
\varpi_{15} &= -\chi^{\Lambda 2}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}
\end{aligned}
$$

with additional parameters being defined as

$$
\begin{aligned}
\o_1 &= \phi + \frac{\varpi^{y_{ss}}\sigma}{1-\kappa^c}\\
\kappa^p &= \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\\
\nu^{\Lambda} &= -\frac{\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}}{\frac{(1-\rho)\beta}{\nu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)}\\
\chi_{-1}^{\Lambda} &= \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^{\Lambda}\right]\\
\chi_0^{\Lambda} &= \left[\frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right) + \gamma_2\nu^{\Lambda}\right]\\
\chi_1^{\Lambda} &= \left[\beta\gamma_3\nu^{\Lambda} - \frac{1}{\o_1-\phi}\beta\theta_2\right]\\
\beta_\delta &= \frac{1}{\beta\left[1-\left(1+\o_1\right)\nu^{\Lambda}\right]}\\
\chi^{\Lambda 2} &= \left(\frac{\chi_{-1}^{\Lambda}}{\beta_\delta} + \chi_0^{\Lambda} + \chi_1^{\Lambda}\beta_\delta\right).
\end{aligned}
$$

D Optimal targeting rule for NK model with sticky nominal wages

To find the optimal targeting rule in the sticky wage model, we follow Giannoni and Woodford (2003). The linear quadratic problem can be shown to be

$$
\begin{aligned}
&\min_{\{\pi_t,\pi_t^w,x_t,i_t,\hat{w}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma+\phi}{2} x_t^2 + \frac{1+\theta^p}{2\theta^p \kappa^p} \pi_t^2 + \frac{1+\theta^w}{2\theta^w \kappa^w} (\pi_t^w)^2 \right\} \\
&s.t. \quad x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \hat{r}_t^*\right) && (\Lambda_{1,t}) \\
&\qquad \pi_t = \beta E_t \pi_{t+1} + \kappa^p \left(\hat{w}_t - \hat{a}_t\right) + \hat{\theta}_{p,t} && (\Lambda_{2,t}) \\
&\qquad \pi_t^w = \beta E_t \pi_{t+1}^w + \kappa^w \left(\sigma+\phi\right) x_t - \kappa^w \left(\hat{w}_t - \hat{a}_t\right) && (\Lambda_{3,t}) \\
&\qquad \hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t && (\Lambda_{4,t})
\end{aligned} \tag{D.1}
$$

where

$$
\theta^p = \lambda^p - 1 \tag{D.2}
$$

$$
\theta^w = \lambda^w - 1 \tag{D.3}
$$

$$
\kappa^p = \frac{(1-\xi^p)(1-\xi^p\beta)}{\xi^p} \tag{D.4}
$$

$$
\kappa^w = \frac{(1-\xi^w)(1-\xi^w\beta)}{\xi^w\left(1+\phi\frac{1+\theta^w}{\theta^w}\right)}. \tag{D.5}
$$

The first order conditions associated with the policymaker's preferences are

$$
(\pi_t): \quad \frac{1+\theta^p}{\theta^p\kappa^p}\pi_t + \frac{\beta^{-1}}{\sigma}\Lambda_{1,t-1} + \Lambda_{2,t-1} - \Lambda_{2,t} - \Lambda_{4,t} = 0 \tag{D.6}
$$

$$
(\pi_t^w): \quad \frac{1+\theta^w}{\theta^w\kappa^w}(\pi_t^w) + \Lambda_{3,t-1} - \Lambda_{3,t} + \Lambda_{4,t} = 0 \tag{D.7}
$$

$$
(x_t): \quad (\sigma+\phi)\,x_t + \beta^{-1}\Lambda_{1,t-1} - \Lambda_{1,t} + \kappa^w\left(\sigma+\phi\right)\Lambda_{3,t} = 0 \tag{D.8}
$$

$$
(i_t): \quad \frac{1}{\sigma}\Lambda_{1,t} = 0 \tag{D.9}
$$

$$
(w_t): \quad \kappa^p\Lambda_{2,t} - \kappa^w\Lambda_{3,t} + \beta\Lambda_{4,t+1} - \Lambda_{4,t} = 0. \tag{D.10}
$$

From equation (D.9), we obtain

$$
\Lambda_{1,t} = 0 \qquad \forall t. \tag{D.11}
$$

Accordingly, the optimality conditions simplify to

$$\begin{aligned}
(\pi_t): &\quad \frac{1+\theta^p}{\theta^p\kappa^p}\pi_t + \Lambda_{2,t-1} - \Lambda_{2,t} - \Lambda_{4,t} = 0 && \text{(D.12)}\\
(\pi_t^w): &\quad \frac{1+\theta^w}{\theta^w\kappa^w}(\pi_t^w) + \Lambda_{3,t-1} - \Lambda_{3,t} + \Lambda_{4,t} = 0 && \text{(D.13)}\\
(x_t): &\quad (\sigma+\phi)\, x_t + \kappa^w (\sigma+\phi)\, \Lambda_{3,t} = 0 && \text{(D.14)}\\
(w_t): &\quad \kappa^p\Lambda_{2,t} - \kappa^w\Lambda_{3,t} + \beta\Lambda_{4,t+1} - \Lambda_{4,t} = 0. && \text{(D.15)}
\end{aligned}$$

From equation (D.14),

$$\Lambda_{3,t} = -\frac{1}{\kappa^w}x_t. \tag{D.16}$$

Then substituting $\Lambda_{3,t}$ into equation (D.13), we get an expression of $\Lambda_{4,t}$

$$\Lambda_{4,t} = -\frac{1+\theta^w}{\theta^w\kappa^w}(\pi_t^w) - \frac{1}{\kappa^w}x_t + \frac{1}{\kappa^w}x_{t-1}. \tag{D.17}$$

Plugging the expressions for $\Lambda_{3,t}$ and $\Lambda_{4,t}$ into equation (D.15) delivers $\Lambda_{2,t}$

$$\begin{aligned}
\Lambda_{2,t} \quad &= \beta\frac{1+\theta^w}{\theta^w\kappa^w\kappa^p}(\pi_{t+1}^w) - \frac{1+\theta^w}{\theta^w\kappa^w\kappa^p}(\pi_t^w) + \frac{\beta}{\kappa^w\kappa^p}x_{t+1}\\
&+\frac{1}{\kappa^w\kappa^p}x_{t-1} - \left(\frac{\beta}{\kappa^w\kappa^p} + \frac{1}{\kappa^w\kappa^p} + \frac{1}{\kappa^p}\right)x_t.
\end{aligned} \tag{D.18}$$

After substituting the expressions for $\Lambda_{2,t}$ and $\Lambda_{4,t}$ into equation (D.12) and using the definition of the output gap ($x_t = \hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t$), the *optimal targeting rule* for sticky wage model is given by

$$\begin{aligned}
-\chi_1\pi_t \quad = \quad & \chi_2\left(\pi_{t+1}^w - \pi_t^w\right) + \chi_3\pi_t^w + \chi_4\left(\pi_t^w - \pi_{t-1}^w\right) + \chi_5\left[(\hat{y}_{t+1} - \hat{y}_t) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_{t+1} - \hat{a}_t)\right]\\
& +\chi_6\left[(\hat{y}_t - \hat{y}_{t-1}) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_t - \hat{a}_{t-1})\right] + \chi_7\left[(\hat{y}_{t-1} - \hat{y}_{t-2}) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_{t-1} - \hat{a}_{t-2})\right]
\end{aligned} \tag{D.19}$$

where

$$\begin{aligned}
\chi_1 &= \frac{1+\theta^p}{\theta^p}\frac{1}{\kappa^p}\\
\chi_2 &= -\beta\frac{1+\theta^w}{\theta^w}\frac{1}{\kappa^p\kappa^w}
\end{aligned}$$

$$
\begin{aligned}
\chi_3 &= \frac{1+\theta^w}{\theta^w}\frac{1}{\kappa^w} \\
\chi_4 &= \frac{1+\theta^w}{\theta^w}\frac{1}{\kappa^p\kappa^w} \\
\chi_5 &= -\frac{\beta}{\kappa^p\kappa^w} \\
\chi_6 &= \left(\frac{1}{\kappa^p\kappa^w} + \frac{\beta}{\kappa^p\kappa^w} + \frac{1}{\kappa^p} + \frac{1}{\kappa^w}\right) \\
\chi_7 &= -\frac{1}{\kappa^p\kappa^w}.
\end{aligned}
$$

Another way of writing the optimal targeting rule is

$$
\begin{aligned}
0 \quad = \quad & \left\{\frac{1+\theta^p}{\theta^p}\pi_t + \left[(\hat{y}_t - \hat{y}_{t-1}) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_t - \hat{a}_{t-1})\right]\right\} \\
& + \frac{1}{\kappa^w}(1+\beta+\kappa^p)\left\{\frac{1+\theta^w}{\theta^w}\pi_t^w + \left[(\hat{y}_t - \hat{y}_{t-1}) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_t - \hat{a}_{t-1})\right]\right\} \\
& - \frac{\beta}{\kappa^w}\left\{\frac{1+\theta^w}{\theta^w}\pi_{t+1}^w + \left[(\hat{y}_{t+1} - \hat{y}_t) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_{t+1} - \hat{a}_t)\right]\right\} \\
& - \frac{1}{\kappa^w}\left\{\frac{1+\theta^w}{\theta^w}\pi_{t-1}^w + \left[(\hat{y}_{t-1} - \hat{y}_{t-2}) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_{t-1} - \hat{a}_{t-2})\right]\right\}
\end{aligned} \tag{D.20}
$$

which boils down to the targeting rule in the standard NK model with flexible wages for $\frac{1}{\kappa^w} = 0$.

E Additional Results for Section 5

The lack of robustness of the optimal targeting rules may depend on our modelling choices. We investigate two avenues to explore the sensitivity of this result: (i) wage indexation in the sticky wage model, and (ii) the preferences assigned to the policymaker.

The estimation results in Table 2 suggest that the empirical fit of the sticky wage model improves if we allow for indexation of wages to past inflation. With full wage indexation, the focus of optimal monetary policy in the sticky wage model shifts from smoothing wage inflation to smoothing the difference between wage inflation and lagged price inflation, i.e., $\pi_t^w - \pi_{t-1}$. This change in focus of the optimal policy is also reflected in the optimal targeting rule derived for the sticky wage model with $\iota^\omega = 1$.

Figure 1 plots selected impulse responses to a markup shock when the sticky wage model features full wage indexation and we repeat the previous exercise of comparing the outcomes in the search and matching model and the sticky wage model (now with $\iota^\omega = 1$) under the optimal targeting rules derived in the two models, respectively. Under full wage indexation, the optimal monetary policy in the sticky wage model refrains from stabilizing wage inflation; to reduce welfare-costly dispersion in the nominal wage, the central bank smooths the term $\pi_t^w - \pi_{t-1}$. Under the markup shock, the decline in the real wage is still engineered by raising inflation in the impact period. Yet, the rise in price inflation this period pushes up nominal wages in the subsequent period through indexation which in turn offsets most of the decline in the real wage. To compensate for this effect, price inflation rises by more in the impact period under the optimal policy in the model with indexation than absent indexation. Turning to the optimal targeting rule derived in the search and matching model, this rule with its focus on reducing price inflation induces even bigger welfare losses (measured as CEV) in the sticky wage model with full indexation than in the model without indexation (now 1.9728 instead of 1.3033), confirming the lack of robustness of the optimal targeting rules.

For the search and matching model, wage indexation in the sticky wage model only impacts the responses under the optimal targeting rule from the sticky wage model relative to the previous discussion; the differences between the two targeting rules are mostly quantitative in nature. Given the modified focus of the new targeting rule from the sticky wage model, wage inflation is not stabilized as forcefully as in Figure 4. Yet, since nominal wages in period t move to offset past inflation, the downward adjustment in the real wage demands even larger movements in inflation than under the no-indexation targeting rule. Thus, the overall welfare loss in the search and matching model (measured as CEV) rises (now 0.1680 instead of 0.1133 for $\iota^{\omega} = 0$).

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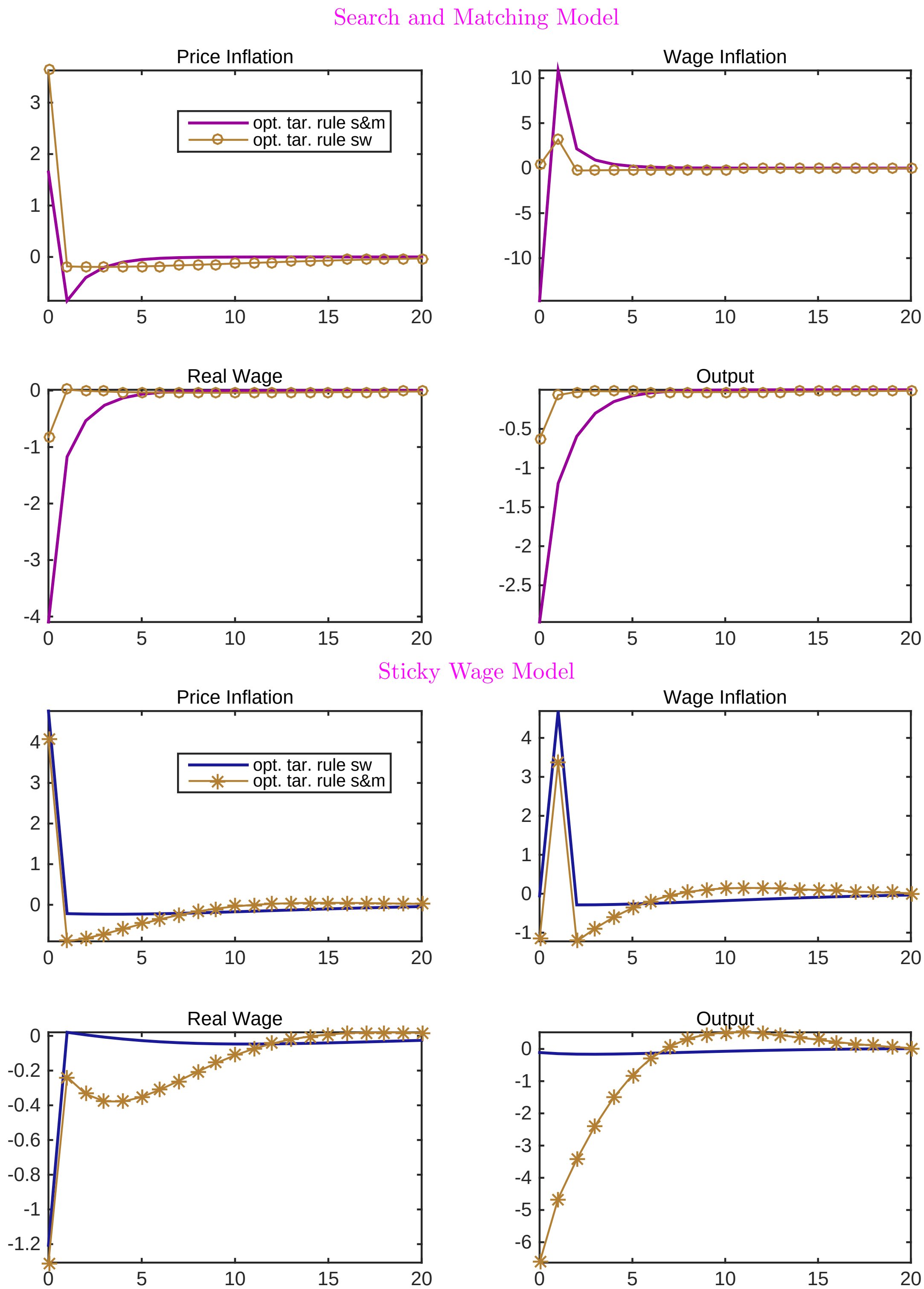
Figure 1: Targeting rules with wage indexation in the sticky wage model: price markup shock



Note: Figure 1 compares the performance of optimal targeting rules for both the search and matching model and the sticky wage model in response to a *price markup shock* when the sticky wage model features wage indexation.