Real Keynesian Models and Sticky Prices

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Abstract

In this paper we present a generalized sticky price model which allows, depending on the parameterization, for demand shocks to maintain strong expansionary effects even in the presence of perfectly flexible prices. The model is constructed to incorporate the standard three-equation New Keynesian model as a special case. We refer to the parameterizations where demand shocks have expansionary effects regardless of the degree of price stickiness as Real Keynesian parameterizations. We use the model to show how the effects of monetary policy– for the same degree of price stickiness– differ depending whether the model parameters are within the Real Keynesian subset or not. In particular, we show that in the Real Keynesian subset, the effect of a monetary policy that tries to counter demand shocks creates the opposite tradeoff between inflation and output variability than under more traditional parameterizations. Moreover, we show that under the Real Keynesian parameterization neo-Fisherian effects emerge even though the equilibrium remains unique. We then estimate our extended sticky price model on U.S. data to see whether estimated parameters tend to fall within the Real Keynesian subset or whether they are more in line with the parameterization generally assumed in the New Keynesian literature. In passage, we use the model to justify a new SVAR procedure that offers a simple presentation of the data features which help identify the key parameters of the model. The main finding from our multiple estimations, and many robustness checks is that the data point to model parameters that fall within the Real Keynesian subset as opposed to a New Keynesian subset. We discuss both *(i)* how a Real Keynesian parametrization offers an explanation to puzzles associated with joint behavior of inflation and employment during the zero lower bound period and during the Great Moderation period, *(ii)* how it potentially changes the challenge faced by monetary policy if authorities want to achieve price stability and favor employment stability.

Key Words: Monetary Policy, Business Cycle; **JEL Class.:** E3, E32, E24

Introduction

Monetary policy is often presented as an effective tool, and sometimes even an optimal tool, to counter the effects of demand shocks on the economy without bearing a cost in terms of increased inflation variability. The main reasoning behind such a view is linked to a property of many commonly used macroeconomic models; a view according to which the effects of demand shocks are mainly transmitted to the real economy through the presence of nominal sticky prices. When the degree of price stickiness tends to zero, the expansionary effects of demand shocks on the economy also tend to disappear. Hence, if monetary authorities can reproduce the flexible price outcome, which generally involves stabilizing inflation, it implies that economic activity will become more stable. In this sense, a monetary policy that can implement the economy's flexible price equilibrium can often counter the expanionary effects of demand shocks and maintain inflation stability.[1]

The starting point of this paper is to show how two simple modifications of commonly used sticky nominal price models – when included simultaneously– expand the mechanism by which demand shocks affect the economic activity and, as a result, alter the challenges faced by monetary policy in terms of favoring inflation and employment stability.[2] In particular, by changing the specification of borrowing costs faced by households and firms, we extend the canonical three-equation New Keynesian model along two dimensions. In particular, our modifications allow for both a discounted Euler Equation formulation [3] and for a cost channel to monetary policy.[4]

With these two simple extensions, we show that the model's parameter space inherits an interesting dichotomy. In one region, as the frequency of price adjustment goes to infinity,

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[1] Let us emphasize here that we are focusing on the positive effects of monetary policy not the normative effects. It is possible that a monetary policy that both stabilizes inflation and insulates the economy against demand shocks may not be an optimal policy.

[2] This paper was initially motivated by the following question. We know that the effects of demand shocks can be transmitted to the real economy through mechanisms associated with sticky nominal prices or through other real mechanism. The question is then: if one believes that sticky prices are a feature of modern economies, is it important for our understanding of the positive effects of monetary policy to know whether the effects of demand shocks are only transmitted through sticky prices or whether they also work through other real mechanisms? The answer provided in the paper points to the second alternative.

[3] See for example Del Negro, Giannoni, and Patterson [2012], Gabaix [2016], McKay, Nakamura, and Steinsson [2016a], [2016b] and Farhi and Werning [2017].

[4] See for example Christiano, Eichenbaum, and Evans [2005], Ravenna and Walsh [2006] and Chowdhury, Hoffmann, and Schabert [2006]

the expansionary effects of demand shocks on economic activity disappear (and possibly reverse to become contractionary), while in the other region they survive regardless of the degree of nominal price rigidities. The former parametrization includes the standard New Keynesian model as a special case, while we refer to the later parameterizations as a Real Keynesian parameterization since the expansionary effects of positive demand shocks do not rely solely on sticky nominal prices. We then examine the positive effects of monetary policy in our extended environment. For any given degree of price stickiness, we show that the qualitative effects of monetary policy change depending on whether one is a Real Keynesian parameterization or not. In the more standard parameterization (which includes the canonical New Keynesian configuration), a monetary authority faced with demand shocks can aim to stabilize inflation and this will also help stabilize economic activity. However, under a Real Keynesian parametrization, the situation is more challenging. If monetary authorities aim at stabilizing inflation it will lead to an increased effect of demand shocks on activity. Moreover we show that under a Real Keynesian parameterization Neo-Fisherian effects emerge in that persistent increases in interest rates are likely to raise inflation instead of lowering it (see Cochrane [2013] for a discussion. of Neo-Fisherian effects) even though such a monetary contraction leads to a decrease in output. It is worth noting that in our environment, Neo-Fisherian effects emerge even if the economy admits a unique stationary solution.

Our framework suggests that knowing whether the economy is in a Real Keynesian configuration or not is important for understanding the implications of different monetary policies.[5] Hence, the second part of the paper is devoted to empirically exploring whether the post-war U.S. data points towards parameters that are more in line with a standard New Keynesian model or if instead they are more supportive of a Real Keynesian configuration. In particular, we begin by estimating our extended sticky price model on different samples of post-war data, for example, the pre-Volker disinflation period, the post-Volker disinflation period and the more recent Zero Lower Bound (ZLB) period. In all the case we explore, the estimates repeatedly point towards a Real Keynesian configuration. We then aim to

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[5] Here we are again making a statement about the positive effects of monetary policy not the normative effects.

isolate the data patterns which contribute to this result. To do this, we present a new Structural Vector Autoregression (SVAR) approach that highlights the impulse responses that a structural model needs to explain. There are two features of the data isolated using our SVAR approach that favor a Real Keynesian interpretation. First, our SVAR procedure isolates monetary shocks that are much more persistent than generally found. We observe that the identified monetary contractions are associated with a recessions, but they also lead to increased inflation at all horizons. Second, we find that demand shocks have less effect on inflation during the ZLB period than in previous periods. As we shall show, both these patterns are hard to explain in the conventional setup but are fully consistent with a Real Keynesian configuration.

Given that our estimates of the effects of monetary shocks play an important role in distinguishing between a Real Keynesian configuration and a more standard New Keynesian configuration, we spend considerable effort clarifying why our results differ from other segments of the SVAR literature which find that monetary contractions are generally very short lived and usually lead to a fall in inflation. One potentially important element we isolate to explain the difference is the treatment of the Volker dis-inflation period. The issue being whether the pre- and post-Volker disinflation periods should be treated as one integrated monetary regime with the disinflation period itself being interpreted as a shock; or if instead it is best to treat these samples separately as different monetary regimes and interpret the Volker dis-inflation period as a change in regime. While our preferred specifications are not very sensitive to such a distinction, the only alternative specifications we find that support the more conventional story[6] relies on treating the Volker disinflation as a shock instead as a change in regime; which we believe is highly questionable. We also discuss why it is essential to isolate the effects of persistent monetary shocks if one wants to distinguish between the two different types of parameter configurations we focus upon. In particular, we clarify why isolating the effects of very temporary monetary shocks – as for example is found in the literature exploiting Federal Reserve announcement windows[7]– is not very helpful for answering the question we explore.

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[6] We refer here to monetary shocks that lead to both a decrease in economic activity and in inflation.

[7]See for example Rigobon and Sack [2004], Nakamura and Steinsson [2013] and Gertler and Karadi [2015].

In the final section of the paper we discuss how thinking of the economy as being in a Real Keynesian configuration offers new insights on recent monetary periods. For example, we highlight that the framework offers an explanation to why inflation has been so stable during the ZLB period. We also discuss how such a configuration accounts for the Great Moderation period. In particular, this framework does not support the notion that the reduction in inflation during the 1983-2007 period was due mainly to an effective stabilization by monetary authorities of real activity. Instead, it points to a change in the speed of price adjustment, likely due to a lower target inflation rate, as the main reason for the observed more stable inflation. In fact, we show that real activity – as measured by changes in the variability of employment rates or unemployment rates – essentially did not change over the period even as inflation became much more stable. Overall, we argue that – even in an economy without cost-push shocks– viewing monetary policy as being conducted in an economy characterized by a Real Keynesian configuration suggests that monetary policy can be used effectively to stabilize inflation, but that it can't simultaneously be used to insulate the economy from demand shocks. Other tools are needed if one wants to pursue both goals.

The remaining sections of the paper are structured as follow. In Section 1 we present our extension of a standard three-equation sticky nominal price model which includes the benchmark New Keynesian model as a special case. We show how the parameter space of this extended model can be dichotomized in a region where the expansionary effects of demand disturbances vanish (or become negative) as price stickiness goes to zero and one region where this is not the case. It is here that we introduce the notion of a Real Keynesian parametrization. In Section 2 we first derive an irrelevance result in showing that the effects of demand and cost-push shocks are qualitatively identical whether the economy is within the Real Keynesian or non-Real Keynesian parameterization. The more surprising result is that despite this irrelevance result, we find that the tradeoffs associated with monetary policy depend on the whether one is or not in a Real Keynesian configuration. In Section 4 we explore the model empirically to see whether the data is more favorable to a Real Keynesian configuration or not. In passage, we present a new SVAR methodology that highlights a set of data features which needs to be explained. In Section 5 we discuss the relationship between our findings and other findings in the literature with respect to the

effects of monetary shocks. In Section 6 we use the Real Keynesian structure to highlight how it offers an alternative framework for understanding recent monetary episodes as well as to discuss some of its implication for policy choices.

1 Extending the Canonical Sticky Price Model

The goal of this section is to extend the canonical three-equation New Keynesian model, as presented in Galí [2015] and Woodford [2004]. First we want to allow for a formulation of the log-linearized household's Euler equation in which the coefficient on expected consumption can be smaller than one. Various micro-foundations for this so-called "discounted Euler equation" have been proposed by Del Negro, Giannoni, and Patterson [2012], Gabaix [2016], McKay, Nakamura, and Steinsson [2016a], [2016b] and Farhi and Werning [2017], as a way to address the "forward guidance puzzle" discovered by Del Negro, Giannoni, and Patterson [2012]. Second, we want to allow for a cost channel to monetary policy, by having the interest rate entering in the definition of the marginal cost of production. Such a cost channel has been also extensively studied in the literature. It was mentioned by Farmer [1984], then modeled by Blinder [1987], Fuerst [1992], Christiano and Eichenbaum [1992], Barth and Ramey [2002]) and discussed in the framework of the New Keynesian model by Christiano, Eichenbaum, and Evans [2005], Ravenna and Walsh [2006] and Chowdhury, Hoffmann, and Schabert [2006].

Although each of these extensions has been analyzed extensively, the implication of allowing them to appear simultaneously has not – to our knowledge– been explored. In the model below, we provide a framework which allows both modifications to arise in a simple manner. It is clear that there are many alternative ways of getting these elements into a model, and the precise way they are motivated could be very important for normative analysis. However, since we will be focusing on the positive implications of our model, the precise micro-foundations that give rise to these extensions are not very crucial for our purposes.

1.1 Firms

Firms behave similarly to that in the benchmark New Keynesian setup (see Galí's [2015] textbook). Since the elements are very well known, we only outline the main steps here

and leave details to Appendix A. There is a final good sector with constant returns to scale that uses a continuum of intermediate goods as inputs. Each of intermediate goods is a produced by a monopolist which has access to a CRS technology that uses a basic input. The intermediate good sector faces nominal rigidities which take the form of Calvo adjustment. The monopolistic firms take the cost of the basic input and the aggregate price index as given. When focusing on the log-linearized behavior around the steady state, we get inflation being determined as

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \ mc_t + \mu_t, \tag{1}$$

where mc_t is the real marginal cost, and variables are expressed as log deviations from the zero-inflation steady state. The parameter κ captures the effects of price adjustment, with κ going to infinity as price adjustment is allowed to become arbitrarily more frequent. Cost-push shocks μ are assumed to enter the model through shocks to the markup. The determination of the marginal cost for the intermediate good producers is determined by the price of the basic input. The basic input is produced by a set of competitive firms that employ labor and final good in fixed proportions to produce. These firms must borrow to buy the final good they use in production at the beginning of the period. This gives rise to a (real) marginal cost for intermediate firms of the form (again in log deviations from steady state)

$$mc_t = a_1\Big(w_t - p_t - \theta_t\Big) + a_2\Big(i_t - E_t[\pi_{t+1}]\Big),$$

where θ_t is a productivity index, p_t is the price of the final good, i_t is the nominal interest rate and π_t is the rate of inflation. For now, we will assume that productivity is constant as to focus on demand shocks and set θ_t to zero.

1.2   Households

There is a measure one of identical households indexed by i. Each of them chooses a consumption stream and labor supply to maximize discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{t-1}(U(C_{it}) - \nu(L_{it}))$, where ζ is a discount shifter. Changes in ζ will act as demand shocks. There are two modifications we make relative to standard formulation. First, we require that households borrow the amount $D^M = P_t C_{it}$ in the morning of each period to finance their consumption pur-

chases and only receive their labor income in the afternoon, when they can also decide to borrow or lend D_{it}^A for intertemporal smoothing. Second, we assume that each household faces an upward sloping supply of funds schedule when borrowing, that is, the cost of borrowing may increase as an individual wants to borrow more. These two assumptions will give rise to Euler equation of a more general form. Indeed, a household will face the following decision problem

$$\max \sum_{t=0}^{\infty} \beta^t \zeta_{t-1} E_0 \big[U(C_{it}) - \nu(L_{it}) \big]$$

$$s.t. \quad D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H) D_{it}^A + (1 + i_{it-1}^H) P_t C_{it},$$
$$1 + i_{it}^H = (1 + i_t) \left(1 + \rho \left(\frac{D_{it+1}}{P_t} \right) \right),$$

where i_{it}^H is the interest rate faced by the household, i_t is the policy rate, $D_{it} = D_{it}^M + D_{it}^A$ is amount of real debt outstanding held by the agent at the beginning of a period (which in equilibrium will be C_{it}) and Ω_{it} are the profits received from the firms. $\rho'(\cdot) > 0$ will capture the default premium perceived by the agent. In Appendix A, we present a model with asymmetric information (some agents can commit to repay their debt, some cannot) where the pooling equilibrium is characterized by such a perceived default premium even though there is no default in equilibrium.

The first order conditions (evaluated at the symmetric equilibrium in which $D_{it} = P_t C_{it}$ $\forall i$) associated with this problem are:

$$U'(C_t) = \beta \frac{\zeta_t}{\zeta_{t-1}} E_t \left[U'(C_{t+1})(1 + i_t)\big(1 + \rho(C_t) + C_t \rho'(C_t)\big) \frac{P_t}{P_{t+1}} \right],$$
$$\frac{\nu'(l_t)}{U'(C_t)} = \frac{W_t}{P_t}.$$

Accordingly, this gives rise to an Euler equation in linearized form which can be written as (omitting constant terms)

$$\ell_t = \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t,$$

using the fact that in equilibrium $c_t = \ell_t$ and where $d_t = \log \left(\frac{\zeta_t}{\zeta_{t-1}} \right)$ is a discount factor shock. We show in Appendix A that $0 < \alpha_\ell < 1$. This "discount factor" in the Euler Equation is created by the asymmetry of information on the debt market. Because of this asymmetry, agents face an interest rate that is increasing in the amount of debt they issue.

The (log linear) labor supply equation for the household is standard. Evaluated at equilibrium $D_t = 0$ and $c_t = \ell_t$), it is given by

$$\ell_t = a_3(w_t - p_t).$$

1.3 Resulting Linearized Equilibrium Conditions

The equilibrium conditions determining employment L_t and inflation Π_t are given by the following two equations (omitting constant terms), where we have used the fact that $C_t = Y_t = L_t$ and where $\ell_t = \log L_t$ and $\pi_t = \log \Pi_t$:

$$\ell_t = \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t, \tag{EE}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_\ell \ell_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC}$$

where "EE" stands for Euler Equation and "PC" for Phillips Curve. To close the model we need to specify the conduct of monetary policy, which we will return to later. We refer to this model as the "extended model". Note that the above two equations embed the standard New Keynesian model as a special case, when $\alpha_\ell = 1$ and $\gamma_r = 0$. For ease of exposition, we will assume that d_t and μ_t follow AR(1) processes with auto-regressive coefficients ρ_d and ρ_μ.

1.4 Deriving a Real Keynesian Condition for Parameters

The object of this section is to contrast how the above model behaves as the economy converges to one without nominal rigidities. As we will prove, the model behavior will depend on on whether $\gamma_r(1 - \alpha_\ell)$ is large or small relative to $\gamma_\ell \alpha_r$. Full flexibility of prices corresponds to the case in which the parameter κ tends to infinity. First result concerns the New Keneysian model.

Proposition 1. *In the New Keynesian model (i.e., when $\alpha_\ell = 1$ and $\gamma_r = 0$), the effects of demand shocks on employment go to zero as prices become fully flexible.*[8]

This is a well known result, as discussed recently in Cochrane [2013]. Note that it holds for any value of α_ℓ (i.e., for un-discounted or discounted Euler equations, as long as $\gamma_r = 0$). We now explore the case in which we allow for the cost channel (i.e., $\gamma_r \neq 0$).

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[8]This proof of this result as well as all the following ones are gathered in Appendix B.

Proposition 2. *In the extended model, when prices become fully flexible, demand shocks always maintain a positive effect on employment if and only if*

$$(1 - \alpha_\ell)\gamma_r > \gamma_\ell \alpha_r. \tag{RK}$$

We will refer to this condition as the RK (for "Real Keynesian") condition, as we define a Real Keynesian model as follows.

Definition 1. *A Real Keynesian model is defined as a parameterization in which demand shocks maintain a positive effect on employment even when prices are flexible.*

It is interesting to note that the Real Keynesian condition will not hold if either the Euler equation is not discounted (i.e., $\alpha_\ell = 1$) or there is no cost channel (i.e., $\gamma_r = 0$). The combination of the two extra features that we have added to the canonical model are needed for Real Keynesian properties to emerge. The RK condition is indeed a condition on the relative size of the marginal cost elasticities to the wage and the real interest rate, as it can be seen by rewriting the Real Keynesian condition as $\frac{\gamma_r}{\gamma_\ell} > \frac{\alpha_r}{1 - \alpha_\ell}$. Both these forces ($\gamma_\ell$ and γ_r) may be small near the steady state but their ratio could be either big or small.

In the next section, we return to the case in which prices are sticky (κ is finite and fixed), and study the implications of being in a Real Keynesian configuration for the effect of shocks and for stabilization trade-offs.

2 Implications of Being in a Real Keynesian Model When Prices are Sticky

We pursue three objectives in this section. First, we look at differences in the response of the economy to shocks (demand and cost-push) in the New Keynesian and Real Keynesian configurations. . Second, we look at monetary policies that aim at stabilizing the economy in response to demand shocks, and derive the potential trade-offs between employment and inflation stabilization in New Keynesian and Real Keynesian models. Note that we restrict ur attention here to the positive effect of monetary policy rules, and do not address any normative questions. Third, we study the behavior of the economy for a fixed nominal interest rate (e.g., at the Zero Lower Bound) in the New Keynesian and Real Keynesian

configurations. Before we start, we discuss of the class of monetary policy rules that we consider.

2.1 Monetary Policy Rules

With sticky prices, allocation properties are greatly dependent on the specific nominal interest rule one considers. The main characteristic we want the policy rule to have is for it ensures determinacy of inflation in the sense that the model inherits a unique stationary equilibrium outcome. One possibility is to choose a policy rule of the form $i_t = \phi_\pi \pi_t + \phi_\ell \ell_t$. However, the difficulty with such a policy rule is that the values of ϕ_π and ϕ_ℓ that ensure determinacy change as we move between a New Keynesian and Real Keynesian parameterization. For ease of presentation, it is much simpler to work with a rule of the form $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ with $\phi_\ell > 0$, [9] as this guarantees determinacy under all parameter configurations [10] and, within this rule, it remains easy to talk about policies that are more or less aggressive towards stabilizing employment through the choice of ϕ_ℓ. In Appendix E, we discuss how there is no great loss of generality in moving from a policy rule of the type $i_t = E_t[\Pi_{t+1}] + \phi_\ell \ell_t$ to one of the form $i_t = \phi_\pi \pi_t + \phi_\ell \ell_t$, as long as we focus on policies that induce determinacy.

2.2 Responses to Shocks

Now we derive the qualitative properties of the economy responses to shocks in the New Keynesian and Real Keynesian parametrization. From now on, we assume that the shocks are autoregressive processes of order 1. Proposition 3 looks at the effects of demand shocks.

Proposition 3. *Assuming monetary policy is given by $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ with $\phi_\ell > 0$, then a positive demand shock increases employment, inflation and interest rates (real and nominal) under both RK and non-RK parameterizations.*

Proposition 3 indicates that the qualitative effects of demand shocks, for a finite κ and a large class of monetary policies, will be similar regardless of whether the economy is within

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[9]Note that this policy will not guarantee finite inflation when κ goes to infinity (i.e., full price flexibility) if $\gamma_2 \neq 0$. This is not a problem here as we keep κ finite

[10]See Proposition B.1 in Appendix B.

a New Keynesian or Real Keynesian parametrization. Hence knowing that demand shocks tend to increase employment, inflation and interest rate will tell us nothing about the relevant parameterizations.

Proposition 4 gives the same irrelevance result for cost-push shocks, so the response to cost-push shocks is not discriminating the Real Keynesian parametrization from the New Keynesian one.

Proposition 4. *Assuming monetary policy is given by $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ with $\phi_\ell > 0$, then a positive cost-push shock increases inflation and the nominal interest rate, while leaving employment and the real interest rate unchanged under both RK and non-RK parameterizations.*

2.3 Policy Trade-off with Demand Shocks

When only demand shocks hit the economy, what happens as we change ϕ_ℓ, that is, how does the variance of inflation and employment change as we adopt a policy that is more or less aggressive in terms of reacting to the demand shock?[11] The surprising result is that the existence of a trade-off between inflation and output variability depends on whether under quasi flexible prices demand shocks have positive effects or not (i.e., whether the Real Keynesian condition holds or not).

To see this, let us derive the relationship between inflation variance σ_π^2 and employment variance σ_ℓ^2 as we change ϕ_ℓ for a given variance of the demand shocks σ_d^2. This relation is given by :

$$\sigma_\pi^2 = \left(\frac{1}{\alpha_r}\right)^2 \left(\frac{\kappa}{1-\beta}\right)^2 \left(\frac{\gamma_r}{\alpha_\ell}\sigma_d + [\gamma_r(1 - \rho_d\alpha_\ell) - \gamma_\ell\alpha_r]\sigma_\ell\right)^2. \tag{2}$$

We therefore have the proposition:

Proposition 5. *Assuming that monetary policy rule is given by $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ with $\phi_\ell > 0$, then a more aggressive policy (larger ϕ_ℓ) always leads to a higher variance of inflation and a lower variance of employment if and only if the economy is in a Real Keynesian configuration.*

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[11]In Appendix C, we study this trade-off with cost-push shocks.

In a New Keynesian parametrization ($\gamma_r = 0$ and $\alpha_l = 1$), monetary policy will never face a trade-off: if it is aggressive at stabilizing employment or inflation, it manages to also stabilize the other variable. In contrast, under a Real Keynesian parametrization, there is always a tradeoff. If monetary policy tries to aggressively stabilize either inflation of employment, it destabilizes the other.

To understand how this result arises it is helpful to look at the issue graphically. For presentation purposes we can assume that demand shocks as $i.i.d.$. First we can look in the (r_t, ℓ_t) plane (where $r_t = i_t - E_t[\pi_{t+1}]$) and plot the Euler equation and the policy rule in this space, with the slope of the latter governed by ϕ_ℓ. This is what is shown on the top graph of panel (a) in Figures 1, 2 and 3. Expected variables are zero because of the $i.i.d.$ assumption for the shock. In this plane, the nature of the Phillips curve and the parametrization (Real Keynesian of not) does not matter. On this graph, the grey zone around the Euler equation line corresponds to all the possible locations of the Euler equation, as implied by the demand shocks d_t. Where the parametrization matters (New or Real Keynesian) is for the Phillips Curve in the (π_t, ℓ_t) plane. In the New Keynesian case, as depicted on Figure 1, this Phillips curve is invariant with respect to policy. In the extended model (Figures 2 and 3), we can define a pseudo Phillips curve as the Phillips curve in which we have substituted the nominal interest rate by its expression in the policy rule. This pseudo Phillips curve is given by (3), and is not invariant to the policy parameter ϕ_ℓ:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(\gamma_\ell + \gamma_r \phi_\ell)\ell_t. \tag{3}$$

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

What is the effect of a change in policy? Consider increasing ϕ_ℓ. As shown on top of panel (b) in Figures 1 to 3, this will mitigate the effects of demand shocks on activity. In the New Keynesian case, this is always good in terms of stabilizing both activity and inflation as the Phillips curve does not change, as shown on the bottom of panel (b) However, in the more

general case, as we render policy more aggressive (greater ϕ_ℓ), we also steepen the Phillips curve, as seen in Figure 2 and 3 on bottom of panel (b). If γ_r is very small (as in Figure 2), this will still lead to a stabilization of both activity and inflation. However, as γ_r gets big relative to $\gamma_\ell \alpha_r$ (as in Figure 3), things will eventually switch. Eventually there will appear a negative tradeoff between employment and inflation variances. When does the switch arise? Exactly for the same condition under which demand shocks have positive effect on activity under flexible prices. If the parameterization is Real Keynesian, then monetary policy faces an unpleasant trade-off where insulating employment variability to demand shocks comes at the cost of increasing inflation variability.

2.4 Implications of the Real Keynesian Parametrization for the Economy at Zero Lower Bound

The previous result of an unpleasant trade-off between employment and inflation stability in the Real Keynesian configuration could be seen as a theoretical curiosity, that would be difficult to test empirically. To illustrate how the implications of a change in policy can be very different depending whether the economy is in a Real Keynesian parametrization or in a New Keynesian one, it happens that looking at the ZLB period is quite instructive. Let us contrast how the economy would behave if we change policy from an activist policy of the form $i = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ with $\phi_\ell > 0$ to a policy of fixed interest rates $i_t = 0$. [12] To ease the exposition, we assume that there are only demand shocks. It is well known that with a New Keynesian parametrization, the equilibrium for the economy becomes indeterminate under fixed interest rate rule, so it is difficult to predict exactly how inflation may change. Proposition 6 shows that equilibrium properties are different in a Real Keynesian parametrization.

Proposition 6. *Assuming that the economy is in a Real Keynesian parameterization and there are only demand shocks, then moving from the policy $i = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ with $\phi_\ell > 0$ to the policy $i_t = 0$, the equilibrium remains determinate with the variance of employment σ_ℓ^2 increasing but the variance of inflation σ_π^2 decreasing.*

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[12]i_t represents here deviation form the steady state, so that this fixed nominal interest rate regime needs not to be strictly the Zero Lower Bound, but should be interpreted as any policy that maintains the nominal interest rate fixed.

Interestingly, this proposition can be interpreted as suggesting that hitting the zero lower bound under an RK parametrization could result in an increase the variance of employment but a decrease in the variance of inflation. It it is helpful to contrast this result with the case where the economy exhibits a discounted Euler equation $(0 < \alpha_\ell < 1)$ but no cost channel $(\gamma_r = 0)$. Such a parametrization has been extensively studied in the "forward guidance puzzle" literature (Del Negro, Giannoni, and Patterson [2012], Gabaix [2016], McKay, Nakamura, and Steinsson [2016a] and Farhi and Werning [2017]). The nice feature of this parametrization is that the equilibrium can remain determinate under an interest rate peg. Accordingly, in this case we can unambiguously compute the variances of employment and inflation. As stated in Proposition 7, in such a case, switching to an interest peg is accompanied with an increase in both the variance of employment and inflation.

Proposition 7. *Consider an economy with $\alpha_\ell < 1$ and $\gamma_r = 0$. If α_ℓ is sufficient small to maintain determinacy as policy moves from $i = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ to $i_t = 0$, then the variance of both employment σ_ℓ^2 and inflation σ_π^2 will increase.*

3 Structural Estimation

3.1 The Estimated Equations

Now that we have highlighted the differences of a model in the Real Keynesian parametrization compared to a New Keynesian one, the goal of this section is to look at data through the lens of our simple extended sticky price model, where we don't *a priori* take any stance on whether parameters are in Real Keynesian subset or not. Our objective is to see whether a Real Keynesian parameterization may offer a better fit of the data than more standard New Keynesian parameterizations. The initial model we want to estimate includes the following two equations

$$
\begin{aligned}
\ell_t &= \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t, \\
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_\ell \ell_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t,
\end{aligned}
$$

where d_t and μ_t are assumed to be independent AR(1) processes. Two issues immediately arise. First we need to specify a class of monetary policies. We begin by disregarding the

ZLB period, and choose the class of policies rules

$$i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t. \tag{Policy}$$

This class of policy rules is attractive as it minimizes difficulties associated with indeterminacy while simultaneously being very flexible as it allows monetary policy to react to the state space of the system. In appendix E, we show that for any monetary rule that reacts to current endogenous variables and that guarantees determinacy of equilibrium, equilibrium allocations can be replicated with a our class of policy rules. Note that in this policy rule, ν_t will represent monetary shocks, that we also assume to be AR(1).

A second issue that arises is whether we should think of cost-push shocks (μ_t) as only affecting the Phillips curve or whether they should potentially also directly affect households demand. While the framework presented in the previous section implies that cost-push shocks should only affect the Phillips curve, it is not difficult to think of extensions in which cost shocks may also have a direct impact on demand. For example, in a richer model with incomplete markets, such shocks may favor precautionary savings which would lead then to enter the household's Euler equation. Even though we will not derive such a case formally here, we choose to allow in our estimation that μ_t shocks enter both the Phillips Curve and the Euler Equation, and we will let the data decide if such a mechanism may be at play.[13] Accordingly, the model on which we base our estimations is given by the following three equation system

$$\ell_t = \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + \alpha_\mu \mu_t + d_t, \tag{EE}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_\ell \ell_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC}$$

$$i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t. \tag{Policy}$$

This is a quite simple linear system of three equations in three unknowns, ℓ, π and i. As such, this system has low dimension and is entirely forward looking, and is unlikely to capture well the rich dynamics of the economy. We will therefore later explore extensions which allow for endogenous dynamics and more shocks. An attractive feature of starting from such a simple

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[13]This extension allows for the following possibility. A cost-push shock μ_t could create inflation, without much change in i, thereby decreasing the real interest rate while simultaneously leading to a fall in employment. If μ_t is not allowed to directly affect the Euler Equations, such a pattern could not arise.

model is that all the mechanisms at play can be understood easily. The drawback is that it may be an over-simplification. We believe that it is a useful starting point as it allows us to ask whether the simple narrative of a striped down New Keynesian model offers a better interpretation of the data than what could be offered by a Real Keynesian parametrization–which is a parameterization not generally considered in the literature.

3.2   Estimation, Identification and Sample Period

We will begin by estimating the above model by maximum likelihood on post-war U.S. data excluding the ZLB period. However, as written the model is slightly under-identified. [14]   First, it will not be able to separately identify κ, γ_ℓ and γ_r. Instead we can only get estimates of $\kappa\gamma_\ell$ and $\kappa\gamma_r$. Without loss of generality, we therefore normalize $\kappa = 1$ when estimating over one sample period. Later, when we estimate over different samples, we will be able to estimate changes in κ over time if we assume that γ_ℓ and γ_r do not vary. After this normalization, the model still has two parameters more than what can be identified from the data. As commonly done in the empirical macroeconomic literature, we do not estimate β and α_r. For β we set it to .99, which is in line with large parts of the literature. Our results are not sensitive to changing β around this level. As for α_r (which is largely determined by the inverse of the agents risk aversion parameter), there is considerable debate regarding its value. In our baseline estimation, we set this value to .33, which is at the average or at the high end of the range of most micro-level estimates. Because there is controversy over the value of such parameter, we explore the sensitivity of our results with allowing α_r to vary between .1 and 1.

Our initial data sample is for quarterly U.S. data over the period `1953Q3-2007Q1` . We stop the sample before Great Recession period where the interest rate was constrained by the ZLB for much of the time. We will later include that period, but then estimate only the Euler equation and the Phillips curve for ZLB period. Our measure of interest rates is the Federal Funds rate.[15] For inflation, we use either the GDP deflator or the CPI growth rate. For the employment rate, as we want a series that is as close to stationary as possible, we use

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[14]We are able to solve analytically the model and report in appendix G the explicit mapping between reduced form parameters and structural parameters.

[15]See Appendix D for data definition and sources.

either the negative of the unemployment rate or the linearly detrended employment rate.[16].
As a robustness check, we will also estimate the model on a smaller sample that focuses on
the Post Volker dis-inflation period, `1983Q4-2007Q1` . This period has the advantage that
it may be less subject to the possibility of indeterminacy, as discussed in Clarida, Galí, and
Gertler [2000].

In the text, we only present results obtained with (minus) unemployment as a measure
of employment, the GDP deflator growth as a measure of inflation, and with $\alpha_r = .33$, which
we refer to as the baseline estimation. In Appendix H, we present results from estimating
eight variants of the model (two different periods, two measures of inflation , two measures
of employment); each with five different values of α_r (.1, .3, .5, .75, 1). This makes for forty
estimations.

3.3 Results

Table 1 presents the baseline estimation of our forward looking sticky price model. In this
estimation none of the free parameters is constrained in terms of sign or size. The first aspect
to note from the table is that the signs of the estimates are the expected ones. Monetary
policy is observed to increase interest rates in response to demand shocks ($\phi_d > 0$) and to
decrease it in response to cost-push shocks ($\phi_\mu < 0$). The "discount factor" in the Euler
equation α_ℓ is estimated to be .65, which is between 0 and 1. This estimate may appear
low, but it is unclear to us how it should be compared with the literature. It should also
be noted that this value is quite sensitive to the choice of α_r. If α_r is set to .1 instead, the
estimate of α_ℓ becomes over .8. Let us immediately note that results are surprisingly robust
across our 40 variations (see Appendix H for more details).

[Table 1 about here.]

The key element we want to explore from this table is whether the estimated parameter
configurations suggest that the data are in the New Keynesian or the Real Keynesian param-
eters configuration (or in between). Recall that the model in the the Real Keynesian region
if the condition $(1 - \alpha_\ell)\gamma_r - \gamma_\ell\alpha_r < 0$ holds. We actually find that this condition is met

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[16] This linear trend is a simple way of removing some of the effects on the employment rate that are due
to increased female participation over our period.

for all cases we explored that gave interpretable results. To give a sense of our uncertainty regarding this inference, in Figure 4 we plot the distribution of the quantity $(1 - \alpha_\ell)\gamma_r - \gamma_\ell\alpha_r$ as implied by the parameter given in Table 1, given sample uncertainty. As can be seen in the figure, most of the implied distribution (79%) is in the Real Keynesian configuration.

[Figure 4 about here.]

Estimation Using Pooled Sub-Periods :   The results presented in Table 1 are implicitly imposing that the period from 1954 to 2007 is treated as one with constant monetary policy (i.e., no change in ϕ_d and ϕ_μ) and no change in the processes of shocks. However, it is generally agreed that the conduct of monetary policy changed quite drastically over the time span when Paul Volcker was the Chairman of the Federal Reserve. For this reason, it may be more reasonable to estimate our model allowing for different monetary policies over different periods. In such a case, we could estimate each period separately, which allows all parameters to vary across periods, or we could pool the samples allowing some parameters to change over periods and some to stay the same. In this subsection, we explore results from such a pooling exercise. Here our idea is to estimate the model over three sub-periods, allowing for the monetary policy to change over periods and for the exogenous shock process to change. We also allow the degree of price rigidity κ to change between the pre-Volcker dis-inflation period, the post-Volcker dis-inflation and the ZLB period. Other parameters are assumed fixed. Such an exercice gives us new estimates of parameters for which we can check the Real Keynesian condition. This is attractive as the estimates will incorporate information obtained by the change in monetary regimes. In particular, using such change in monetary policy regime should help us evaluate the Real Keynesian condition as we know that changes in policy will have different effects depending on whether we are in an Real Keynesian or New Keynesian parameters subset.

We now perform a pooling estimation over three sub-periods; the pre-Volcker dis-inflation period `1954Q3-1979Q1`; the post-Volcker dis-inflation period `1983Q4-2007Q1` and the zero lower bound period `2009Q1-2016Q3`.[17] For the two first sub-periods, we continue to assume

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[17]Results from pooling just the first two sub-periods or the two last sub-periods give similar results in terms of finding support for a Real Keynesian parametrization. Moreover, results are not very sensitive with

that monetary policy is of the form $i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t$, where ϕ_d and ϕ_μ allowed to vary between periods, while over the ZLB period, the monetary regime period is taken to be a fixed interest rate rule.[18] In this last regime, because of the fixed interest rate rule, indeterminacy of equilibrium could arise. However, in contrast to a standard New Keynesian parameterization, this will not always happen in our extended model. In particular, if one is in a Real Keynesian configuration, we have shown previously that the equilibrium stays determinate under a fixed interest rule. Hence, we proceed to estimate the model as if the equilibrium is determinate even in the fixed interest rate regime, and then we verify whether the implied parameters are consistent with determinacy.

Table 2 provides results from estimating our model by pooling the three sub-samples, allowing for monetary policy and exogenous processes to changes across regimes. Interestingly, our estimate of α_ℓ is increased in this case to .83 even when α_r is set at .33. A few other interesting elements shall be stressed. First, we estimate that monetary policy reacted more to demand shocks in the post-1983 period versus the pre-1979 period, which is in line with the standard narratives over the period. Also, we see that the degree of price rigidities κ decreased substantially from the first sub-period to the two last ones, which is consistent with many estimates of the Phillips curve over the period. One issue that arises is that the parameter estimate for γ_ℓ is found to be negative, although very small in absolute value and not significantly different from zero. There are two way to interpret this. One possibility is that γ_ℓ should be treated as being essentially zero, in which case the Real Keynesian condition $(1 - \alpha_\ell)\gamma_r > \gamma_\ell \alpha_r$ will be satisfied. The other possibility is to accept that γ_ℓ is indeed negative (a possibility discussed in Appendix F), in which case the Real Keynesian condition becomes $(1 - \alpha_\ell)\gamma_r > |\gamma_\ell|\alpha_r$, which is again satisfied. Therefore, regardless of the above choice, parameters are in the Real Keynesian zone for the three sub-periods. Finally, given the estimated parameters, determinacy of equilibrium is maintained even in the fixed interest rate regime.

[Table 2 about here.]

respect to changing the exact start and end dates of each of these periods. The estimation procedure is a two-step estimation that will be explained in the next section.

[18]In each case we are assuming that agents behave as if the regime will stay constant forever.

We can also use the parameters in Table 2 to interpret aspects of the Great Moderation as well as aspect of the ZLB period. For example, the estimates suggest that the Great Moderation period was mainly a nominal phenomena, with the fall in inflation volatility being due primarily to a fall in κ (i.e., an increase in price stickiness). Although not often discussed, the actual change in the variance of employment between periods was extremely minor, as shown in Table 3. This suggests that inflation stability did not come mainly as a result of a more activity stabilizing monetary policy. Instead, what we observed over during the Great Moderation was mainly demand driven non-inflationary business cycles.[19] With respect to the ZLB sub-period, the estimates in Table 2 suggest that inflation should have become more stable in the ZLB period in comparison to the preceding period, as monetary policy was no longer able to react to demand shocks. Recall that in a Real Keynesian regime, a less agressive monetary policy in terms of a lower ϕ_d should lead to lower inflation variability, which is indeed what we observe. Interestingly, in a Real Keynesian parameter configuration, seeing determinate and stable inflation during a ZLB period is not only easy to explain, it is actually what is predicted by the model.

3.4 Allowing for Habit Persistence

The model we have studied so far has no internal dynamics. We now assume that utility shows habit persistence in consumption, so that preferences can be written $u(c_t - hc_{t-1}) - v(\ell_t)$. As is common in the literature, we assume that the habit term is external to the household. Under this assumption, the model allocations are the solution of the three following equations for all $t > 0$:

$$\ell_t = \alpha_\ell E_t[\ell_{t+1}] + \alpha_{\ell_{-1}}\ell_{t-1} - \alpha_r(i_t - E_t[\pi_{t+1}]) + \alpha_\mu \mu_t + d_t, \tag{EE'}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_\ell \ell_t + \gamma_{\ell_{-1}}\ell_{t-1} + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC'}$$

$$i_t = E_t[\pi_{t+1}] + \phi_{\ell_{-1}}\ell_{t-1} + \phi_d d_t + \phi_\mu \mu_t + \nu_t. \tag{Policy'}$$

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[19] The parameters estimate suggest that if monetary authorities keep the real interest rate close to constant, then demand shock will drive employment fluctuations with inflation being stable.

In this new set of equations, we see that the Euler equation inherits a lagged term in ℓ_{t-1}, the Phillips Curve also inherits such a term as labor supply– and therefore wages– is affected by the habit term. Finally, we extend the monetary policy rule to also allow monetary authorities to react to the term ℓ_{t-1} which is inline we our previous assumption that we want to allow monetary authorities to react to the state space of the system (see Appendix E). In this extended model, the condition under which the economy is in a Real Keynesian configuration is given by

$$(1 - \alpha_\ell - \alpha_{\ell_{-1}})\gamma_r > |\gamma_\ell|\alpha_r. \tag{RK'}$$

Table 4 shows estimated parameters, which have again expected signs and magnitudes. Once again, the Real Keynesian condition (RK') is satisfied.

[Table 4 about here.]

4 How to Interpret the Results: a SVAR Approach

Results in Tables 1, 2 and 4 suggest that the economy may be best characterized as corresponding to a Real Keynesian configuration (which recall is a configuration not usually even allowed for in most estimations of sticky price models). As we have noted, this result is robust to many changes in terms of data and sample periods. However, a common problem with structural estimation is that the results often seem as coming from a black-box, with little indication of what data features are driving the results. Our goal now is to make our estimation results more transparent and especially we will aim to clarify the following two claims.

Claim 1. *When looking within a sample period, our Real Keynesian results are driven largely by implied effects of monetary shocks.*

Claim 2. *When pooling periods, our Real Keynesian results are further helped by changed behavior of inflation in response to demand shocks in the pre- versus post-ZLB period.*

We will support these two claims by exploiting evidence from an identification property of the VAR representation for the class of model we are considering, and which most often holds for DSGE models that are estimated by Maximum Likelihood or Bayesian methods.

21

Indeed, one can obtain structural impulse responses for the structural shocks of this classe of models simply by exploiting the assumption that shocks are independent processes. As the SVAR approach we want to use has many potential applications, we start by discussing it in some general terms before going to our precise application.

4.1 Using Shocks Independence to Derive a Structural VAR Representation

The first goal of this section is to highlight how certain assumptions implied by a large class of structural model can be exploited to derive SVAR representation of the data and therefore the associated structural impulse responses. In particular, we want to discuss how structural impulse response can be obtained before imposing any cross-equations restrictions and can be often obtained by simply exploiting the assumption that the driving forces are independent processes.

To set ideas,[20] suppose we have a structural model that has a solution of the form:

$$X_t = AX_{t-1} + B\Theta_t, \tag{4}$$

$$\Theta_t = R\Theta_{t-1} + \varepsilon_t, \tag{5}$$

where X_t and Θ_t are stationary processes. For example, X_t could be the vector $\{\pi_t, i_t, \ell_t\}$ as in our case, and the elements of ε would then be the innovations in demand, cost-push and monetary policy. The first common restrictions that such a structural model imposes are that R be diagonal and that the variance-co-variance matrix of ε be diagonal and normalized to identity. In other words many structural models assume that the driving forces of the model are independent processes. A second type of restrictions such a model often imposes is that certain elements of A may be zero. For example, in our purely forward model, the matrix A is assumed to be zero. In our habit persistence model, only the third column of A is non zero. Finally, a structural model also generally imposes that the elements in A and B are a function of both a set of underlying structural parameters and of the elements in R. These restrictions are generally referred to as cross equation restrictions. The imposition of cross equation

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[20]See Appendix I for a detailed exposition.

restrictions is often the most contentious part of a structural model.[21] The idea behind the SVAR literature is to try to derive structural impulses response implied by a theoretical framework without imposing (or being precise about) all of a model implications, which may in particular apply to controversial ones such as implied by cross equation restrictions. This is the avenue we want to pursue here.

To our knowledge, what has been rarely recognize (or at least little discussed in the SVAR literature), is that if one thinks that data is generated by a structural model which takes the form given by Equations (4) and (5) where Θ_t is composed of a set of independent processes, then this is generally enough to identify the models impulse structural responses.[22] In Appendix I we discuss this in more detail. Obviously, imposing all the models implications will generally give more efficient estimates of the structural impulse responses if the implications are true, but simply exploiting the diagonal structure of R is generally sufficient. More generally, the structural response implied by a structural model of the form (4) and (5) can most often be obtained by exploiting the zero restrictions implied by the model on the matrices R and A, without needing to impose cross equation restrictions. This is what we will exploit to highlight the data features that are are driving our findings of a Real Keynesian configuration. One challenge with this approach is that once the structural impulse responses are estimated, one will still need to use judgement to give them the right labels. This may or may not be difficult depending on the setting. For example, in our case, a demand shock should lead to an increase in activity, interest rates and inflation. If there is only one shock among the identified structural impulse responses that has such property then it can naturally be labeled the demand shock. If there is no shock among the identified ones that have that property, it implies that something is at odds with the model. Finally, if more than one shock has the same qualitative features, then this method does not allow us to distinguish them.

More generally, when one is thinking within the framework of a model of the type given by (4) and (5), one can use the following two-step approach to estimate the model parameters. First, one can estimate (by Maximum Likelihood for example) the matrices A, R and B,

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[21]It is worth noting the rather uncommon restriction in a structural model is that it imposes zeros in the matrix B. However, many SVAR approaches exploit restrictions on B.

[22] This assumes that X_t has at least two elements.

using only the zero restrictions implied by the model. Then given the estimates of these matrices, one can recover structural parameters using a minimum distance estimator to best match the estimated matrices A, R and B. If the model is just identified, there is no loss of efficiency in using such a two step procedure of the model. Even if the model is over-identified, one may still want to proceed using such a two-step procedure since, by allowing one to see structural impulse responses before imposing the cross-equation restrictions, one can get a good sense of what properties of the data are being used to identify parameters.[23]

4.2   First SVAR Results with the Baseline Model

We start by estimating by Maximum Likelihood the following model, in which Θ is a latent variable:

$$\underbrace{\begin{pmatrix} \pi_t \\ i_t \\ \ell_t \end{pmatrix}}_{X_t} = A \begin{pmatrix} \pi_{t-1} \\ i_{t-1} \\ \ell_{t-1} \end{pmatrix} + B \underbrace{\begin{pmatrix} \Theta_{1t} \\ \Theta_{2t} \\ \Theta_{3t} \end{pmatrix}}_{\Theta_t},$$

$$\begin{pmatrix} \Theta_{1t} \\ \Theta_{2t} \\ \Theta_{3t} \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{pmatrix}}_{R} \begin{pmatrix} \Theta_{1t-1} \\ \Theta_{2t-1} \\ \Theta_{3t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix}}_{\varepsilon_t}.$$

In Figure 5 we report the structural impulse responses implied by our initial simple model when we estimate it over the longer sample 1954Q3-2007Q1. These impulse responses are obtained by the estimation of matrices B and R. As we inspect the data through the lens of the fully forward model, matrix A is constrained to be zero in the estimation. For the same reason, the matrix R is assumed to be diagonal (and that the shocks are uncorrelated). No cross equation restrictions are being imposed at this stage. In Figure 6, we show the impulse responses to the three structural shocks as obtained from the Maximum Likelihood estimation of the model presented in the previous section, where all the theoretical cross-restrictions are imposed. Because the model parameters were just identified, the impulse responses are the same to those obtained using our SVAR strategy. In other words, Figures 5 and 6 are identical. This needs not to be exactly the case if the model were over-identified, as is the case when we estimate it over the three sub-samples.

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[23]In the estimation of Table 2, we used such a two step procedure.

[Figure 5 about here.]

[Figure 6 about here.]

In Figure 7 we derive the structural impulse responses the same way for each of our sample treated separately; pre-Volcker dis-inflation, post-Volcker dis-inflation and ZLB. For the ZLB we only estimate a bivariate process as the nominal interest rate i is essentially fixed. Later we will document the robustness of the results with respect to allowing more shocks and for allowing endogenous dynamics.

[Figure 7 about here.]

The first aspect to note from Figures 5 and 7 is the similarities between the impulse response patterns across periods. In all case, the impulse response to ε_1 appears to un-ambiguously be picking up effects of a demand shocks as all three variables respond positively to this shock, and the other columns do not have this property. The more controversial aspect is the naming of the monetary shock and the cost-push shock. As it can be seen on Figure 6, the model implies that a monetary shock that increases interest rates should lead to a fall in output, while the model does not put any constraints on the effects of the markup shocks. Given this, the only coherent interpretation is that the impulse responses to ε_2 correspond to the effects of the cost-push shock while the response to ε_3 corresponds to the effects of the monetary shock. The controversial aspect with this labeling is that the implied monetary shocks are *(i)* very persistent and *(ii)* lead to a rise in inflation. Once view in such a way, it becomes much clearer why the estimates of structural parameters presented in the previous section tended to favor parameters in the Real Keynesian subset, in order to produce the impulse responses to $(\varepsilon_d, \varepsilon_\mu, \varepsilon_\nu)$ of Figure 6 that fit the responses to $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ of Figure 5. In particular, an increase in inflation following a monetary shock ε_ν is a pattern that cannot be explained by a New Keynesian parameterization. However, it is precisely the pattern expected in a Real Keynesian configuration. An additional reason to believe that ε_3 is picking up a monetary shock is that, during the lower bound period, when there are no more monetary shocks to identify, the impulse response that disappears is that which we associated with a monetary shock, as shown in Figure 7.

25

What other feature of these impulse response also tend to favor a Real Keynesian interpretation? If we look at the impulse response to demand shocks (ε_1) across regimes, we can see that the response of inflation to a demand shock in the ZLB period has less (normalized) impact on inflation than in the previous adjoining period. If we were in a standard New Keynesian setup, the response of inflation to a demand shock that has the same effect on activity should lead to the same effect on inflation regardless of the monetary regime. In contrast, this is not the case in a Real Keynesian parameterization. The impact on inflation of a demand shock should be less, given that the inflationary pressure of a rise in interest rates is absent.[24]

The impulse responses in Figures 5 and 7 allow for a clear understanding of why our estimation favor a Real Keynesian parametrization. However, given the simplicity of the model three questions arise. Firstly, are these results robust to allowing the model to have endogenous propagation? Secondly, are these results robust to allowing the model to have more shocks? Thirdly, how do we reconcile the monetary shock impulse responses we identify with what is found more generally in the literature? We now address each of these questions in turn.

4.3 Adding Habit Persistence to Obtain More Endogenous Dynamics

Our baseline model has a fully forward structure which implies that it does not involve any endogenous dynamics. We have chosen it as our baseline because it builds off the standard three-equation New Keynesian model and focuses on two core extensions that we were needing, namely discounted Euler equation and cost channel. However, it may be that in pursuit of simplicity we have biased our exploration by omitting other important mechanisms. We could at this point write a much more complicated model with several sources of accumulation and adjustment costs and take it to the data. This is not how we choose to proceed because the range of such models is almost infinite. Instead, we choose to proceed by small steps to understand what results appear robust and which may be more fragile. To this end, we begin by exploring the very simple extension of our baseline model

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[24] This effect should be sufficiently less so that the variance of inflation is reduced if the shock process for demand has not changed, as we have shown in proposition 6

in which we include habit formation in agents' utility, as we have done in Subsection 3.4. In this framework, the matrix A in the SVAR representation has now on non-zero column (the third one). Figure 8 reports the structural impulse response associated with applying this SVAR methodology to our long sample `1954Q3-2007Q1`, while Figure 9 presents the structural impulse response functions for our three sub-samples.

[Figure 8 about here.]

[Figure 9 about here.]

There are three elements we want to emphasize on these figures. First, with the exception of adding a small amount of richer dynamics, the impulse response in Figure 8 and 9 are qualitatively very similar to those from Figures 5 and 7 and they support the same labelling of shocks. Second, we again see that the identified monetary shock (ε_3) leads to an increase in inflation, which is not a possible outcome with a New Keynesian parametrization, while it is consistent with a Real Keynesian parametrization. Finally, we see that the inflation barely responds to a demand shock in the ZLB sub-period, which is again consistent with a Real Keynesian interpretation. These observations suggest that the forward model, although extremely simple, may be providing a good summary account of the data.

4.4   Allowing for Richer Internal Dynamics

Allowing for habit persistence in consumption was an easy way of introducing internal dynamics into our baseline model. This extension allowed us to explore the robustness of our initial results to the addition of this simple internal propagation mechanism. We could continue to offer explicit generalization of our baseline model along these lines and verify whether estimation suggests Real Keynesian type parameterization. However, at this point it should become clear that the main issue of whether the data likely favor a Real Keynesian parameterization or an New Keynesian parameterization depends primarily on the robustness of the properties of the structural impulse responses implied by models with independent driving processes. If it is true that, from SVAR results that exploit the independence of the driving forces, we continue to find monetary contraction lead to increased inflation, and that the ZLB period is associated with demand shocks that have less effect on inflation, this places

into question typical New Keynesian parameterizations and favor an interpretation in the direction of our Real Keynesian parametrization. Accordingly, in this section we present a set of estimate of structural impulse responses which uses our SVAR strategy. In particular, we want to think of the data as being driven by a model of the form given by Equations (4) and (5), which we repeat here.[25]

$$X_t = AX_{t-1} + B\Theta_t,$$
$$\Theta_t = R\Theta_{t-1} + \varepsilon_t.$$

where R is always assumed to be a diagonal matrix, while the matrix A may or may not contain some zero restrictions. To begin with, we maintain X_t to be the vector $\{i_t, \pi_t, \ell_t\}$. Later, we will allow X_t to be of higher dimension to allow for more shocks and further internal propagation.

In Figures 8 and 9, we reported structural impulse response for the case where the matrix A in (4) was only allowed to have non zero elements in its third column (the column associated with ℓ_{t-1}). In Figures 10, we allow for more flexibility in the matrix A. Figure 10 is based on using our longer sample `1954Q3-2007Q1`. In the first panel of Figure 10 , we allow for the A matrix to have non zero elements in both the columns associated with ℓ_{t-1} and with the column associated with π_{t-1}. Such an extension could be derived by introducing elements developed in the "Hybrid New Phillips curve" literature where inflation determination is not fully forward looking but may also include a backward looking element, as introduced by Galí and Gertler [1999] and discussed in Rudd and Whelan [2005] and Galí, Gertler, and Lopez-Salido [2005]. In the second panel of the figure, we report impulse responses when we do not place any constraints on the matrix A. Such a formulation could be justified by combining a Hybrid New Phillips Curve specification, habit persistence and sluggish nominal interest rates rule. As we noted above, we are not being explicit about the details of the model that would justify such a specification as we instead want to focus on properties of the implied structural impulse responses that are consistent with a class of such models.

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[25] We also explored robustness of to the reduced form of the model to be

$$X_t = AX_{t-1} + B_1\Theta_t + B_2\Theta_{t-1},$$
$$\Theta_t = R\Theta_{t-1} + \varepsilon_t.$$

[Figure 10 about here.]

When looking at results presented in Figure 10, we see that the additional flexibility in A changes little the properties of the structural impulse responses relative to Figure 8. The exception is that the response of inflation to demand is slightly negative on impact in such extensions. The key element for us is that the response to the identified monetary shock (ε_3) is not much changed with the shock still leading to increased inflation.

4.5 Allowing for More shocks

In our baseline model we are only allowing for three shocks. However, if there are other important shocks driving the system, our reported structural impulse response could be severely biased. To explore this possibility, we examine here the robustness of the structural impulse responses to allowing for more shocks. In particular, we report results when we allow for an explicit oil price shock or a technology shock, as both these are prominent in the literature.[26]

To explore the potential bias due to the omission of oil prices, we extend our SVAR specification to include the real price of oil as the fourth variable. We report results for both our long sample `1954Q1-2007Q1` as well as the Post-Volcker dis-inflation period `1983Q1-2007Q1`. Moreover, we report results corresponding to a fully forward version of the model ($A = 0$) as well as the case with habit persistence (the column of A associated with ℓ_{t-1} is non-zero). The results are presented in Figures 11 and 12. Figure 11 reports that the impulse responses of oil prices to the four shocks. As can be seen in this figure, the shock labelled ε_4 is the main shock that affects oil prices. In fact, in the long sample, it is the shock that explains almost all the variance of the oil price. Accordingly we label this shock the oil price shock. The other shocks maintain their same ordering with ε_1 being the demand shock, ε_2 being the markup shock and ε_3 being the monetary shock. In the post-Volcker sample, we see that the demand shock has also had a tendency to increase the price of oil.

[Figure 11 about here.]

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[26]We have also explore the possibility of shocks to the natural rate of employment. Results are consistent with the one of the three-shock model.

Figure 12 reports the response of π, i and ℓ to each of the four shocks. From these figures, in both the full sample and the post-1983 sample, we see that the responses we have been associating to demand, markup and monetary policy are essentially unchanged with the addition of the oil price in the system. In Figures 13 and 14, we do a similar exercise by adding the growth in GDP in our system and interpreting the fourth shock as a TFP shock. Again, we find that the addition of such a shock changes very little the responses we find for the other shocks.

[Figure 12 about here.]

[Figure 13 about here.]

[Figure 14 about here.]

4.6 Does Our SVAR Methodology Ever Produce a Deflationary Effect of a Monetary Contraction ?

In all the results we have shown up to now using our SVAR methodology (which is solely based on imposing zero restrictions on R and possibly A in the reduced form representation of the model solution), we have found that the inferred effect of a monetary contraction is to cause a fall in employment while simultaneously increasing inflation. This result is obviously at odds with segments the SVAR literature which find that monetary contraction, at least eventually, leads to gradual falls in inflation, as illustrated for example in Christiano, Eichenbaum, and Evans [1999] and [2005]. Does our methodology sometimes deliver the more conventional response, and if so under what conditions does it arise? While we certainly cannot claim to have covered all possibilities, we have found our methodology to only occasionally deliver such a response, and this has arises only in cases we consider quite fragile. To see this, consider extended the reduced form representation to

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + B\Theta_t, \tag{6}$$

$$\Theta_t = R\Theta_{t-1} + \epsilon_t. \tag{7}$$

In this specification, we are allowing further complexity in the internal propagation by introducing the term $A_2 X_{t-2}$. To set a baseline for case with the extended dynamics, we begin

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by reporting the structural impulse responses for our three-equation specification when we allow for habit to take on a richer dimension, that is, we have instantaneous utility of consumption written as $U(C_t - h_1 C_{t-1} - h_2 C_{t-2})$. In this case, only the third columns in A_1 and A_2 are not zero. In Figure 15 we report the implied structural impulse responses for this case for both our longer sample and our post-Volcker dis-inflation sample. What can be seen from these figures is again the robustness of our previous results regarding the effect of our identified monetary shocks. The very short run behavior of inflation in the post-Volcker sample following a demand shock becomes a bit erratic but besides this, results are qualitatively very similar to result based on the fully forward representation or representations with one lag of X. We also report on Figure 15 the responses when we allow the matrix A to be unrestricted, a configuration that we label "Habit persistence, gradual adjustment of i and Hybrid New Phillips curve". For our post-Volcker sample, results are once more robust (except for creating some very short term erratic behavior). In contrast, for the full sample, we see that the effect of an identified monetary shock is now very different from before along two dimensions: the increase in the interest rate following a monetary shock is now very temporary (it was very persistent before), and now we see such monetary contraction being associated with a gradual drop in inflation (after a short increase near impact). Interestingly this response is very similar to the type of responses reported in monetary VARs as Christiano, Eichenbaum, and Evans [2005]. The problem we see with this response is that it is not robust to a change in sample.[27] In particular, as shown, it is not apparent in the post-Volcker sample.[28]. In order to get this more traditional result, we need to firmly believe that it is best to treat the whole period as representing one monetary regime, as it is only in this case that it arises. For us, this seems problematic. It is quite clear there was a change in regime over the 1979-1983 period, so for meaningful structural impulse response functions, preference should be given to results that are robust to excluding a period of regime change. What we believe is happening in this segment of the monetary VAR literature is that with the greater flexibility when allowing for more lagged effects of i_t, the dominant monetary shocks isolated this way using the longer sample is the ones associated with the period around 1979-1983.

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[27]It should be noticed that the results also relies heavily on allowing the interest rate to enter with more than one lag.

[28]We have checked that it is not apparent in the pre-Volcker sample.

While this may be great for for understanding the effect of monetary policy rule change, we believe that it may be quite uninformative about the effect of a monetary shock to a stable rule.

[Figure 15 about here.]

4.7 Summary of Structural Estimation and SVAR results

Using a simple SVAR methodology, which mainly exploits the independence of the exogenous driving forces, we have repeatedly identified monetary shocks that are is very persistent, with the property that a contraction leads simultaneously to decreased employment and increased inflation. Moreover, we have found that the effects of demand shocks on inflation decreases under the ZLB. Both these results are easy to rationalize with a Real Keynesian parametrization but difficult to explain with a New Keynesian parameterization. This explains why, when we estimate model parameters structurally, we find that they favor a Real Keynesian configuration over a New Keynesian configuration. In fact, we believe that if one accepts our SVAR results, then accepting Real Keynesian as the plausible parametrization is almost immediate. This is why the robustness of the SVAR results is key to knowing whether a Real Keynesian or a New Keynesian parametrization may be preferred.

After searching in many directions, we have found our SVAR patterns results to be robust with one notable exception which arises when the VAR exploits primarily variations induced in the period 1979-1983 to identify monetary shocks. As it is general accepted that there was change in monetary policy over the period 1979-83, therefore the 1979-83 period is unlikely to be a good candidate for identifying the effects of monetary shock, as monetary shock should represent random departures within a stable regime. A period like 1979-1983 is likely better for thinking about how one may implement a reform, how it becomes credible and how the economy adapts to a change in regime; it is not a period that tell us how the economy reacts to a monetary shock within a stable regime.

5 The Identified Monetary shocks

The SVAR approach based on shock restrictions we exploited to identify shocks, and in particular monetary shocks, may be viewed as being quite coarse. It may be thought that one can gain much more insight by using identification techniques that focus in a more precisely way on monetary shocks. For example, it may be thought that using identification strategies that exploit observed changes in policy rates around FOMC meetings provide better information. Such more precisely identified shocks may not represent very important monetary shocks in terms of variance decompositions, but since they are better identified, they could be more informative about whether we may be in a Real Keynesian or an New Keynesian parameterization.

In this section we want to briefly discuss why the findings from more precise – for example event study based– identification schemes, while useful for many questions, are in practice quite un-informative for the question we pose in this paper. In order to clarify this issue, let us focus on the case of our model where there are internal dynamics as induced by the presence of habit.[29] The aspect we want to highlight is the different information content of impulse responses associated with more persistent versus more temporary monetary shocks.[30] The issue here involves only the response of inflation. For employment, the implied qualitative response does not depend on whether the economy is in a RK parameterization or not, nor does it depend on the persistent of shocks: a monetary tightening is always predicted in our setup to lead to a fall in employment. For inflation the potential responses are more subtle. Suppose for example we observe the response to a rather temporary monetary contraction which eventually leads to a fall in inflation. Given that this is the type of pattern often found in the literature, it would be helpful if it were informative for us. However, such a pattern is consistent with either a Real Keynesian or a non-Real Keynesian parameterization as long as there is no significant fall in inflation precisely on impact. Either parameterizations is consistent with an initial period where inflation either stays close to zero or an initial period

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[29]We choose to focus on this case as the case with no internal dynamics is very special since it implies that impact effects and longer run effects are the same.

[30]As already explained, a monetary shock is interpreted as a shock ν_t in a policy rule of the form $i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \phi_\nu \nu_t$, which is a monetary rule that guarantees determinacy for all parameterizations. The shock process for ν_t is assumed to be an AR(1) process with auto-covariance ρ_ν. Hence when talking about a more or less persistent shock, we are referring to cases where ρ_ν is higher or lower.

where inflation actually rises (the so called price puzzle) for a short while. For this reason, such responses to temporary monetary shocks – even if they are well identified– are not helpful for the question at hand as they don't allows us to discriminate between the two parameterizations. In contrast, the response to persistent monetary shocks can be more informative. In fact, we have the following proposition:

Proposition 8. *Under a Real Keynesian parameterization,*[31] *if a monetary shock becomes sufficiently persistent, then inflation will not fall at any horizon. In contrast, in a non-Real Keynesian parameterization, following a sufficiently persistent monetary contraction inflation will always eventually fall at some horizon.*

This result indicates that a non-Real Keynesian parameterization is potentially consistent with a initial period of the price puzzle, but if the monetary shock is sufficiently persistent, then inflation should always eventually decrease.[32]. In contrast, if we can observe a monetary contraction which is sufficiently persistent and is not associated with a decrease in inflation, this would refute a non-Real Keynesian parameterization in favor of a Real Keynesian parameterization. Recall that this is precisely the type of pattern which we found using our SVAR technique based on exploiting shock restrictions: we recuperated from the data very persistent monetary shocks with no evidence of an eventual fall in inflation. This pattern is only consistent with the Real Keynesian parameterization.

6   Some Extensions and Narratives

6.1   Endogenous Price Rigidity κ

Up to now we have taken the measure of price rigidity κ as exoegnously fixed. However, it may be reasonable to think of κ as potentially being endogenous with its value depending positively on the variability of inflation and on the steady state inflation level. For example, it is interesting to briefly consider the case where κ takes the form of a function $\kappa(\sigma_\pi, \bar{\pi})$ which would be increasing in both arguments. [33] If this were the case, then for a given

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[31] We are focusing here on the case where the model contains internal dynamics as induced by habit.

[32] In fact, it can be further shown that in a non-Real Keynesian parameterization, then length of the price puzzle is decreasing in the persistent of monetary shocks, while it is increasing in the Real Keynesian case.

[33] Such a function $\kappa(\cdot)$ could be for example derived in a rational inattention setup. See Sims [2010] for a detailed exposition.

policy rule, it is easy to verify that higher steady state inflation gives rise to more variable inflation, regardless of the parameterization. However, the feedback and amplification from a choice of policy rule to the variability of inflation will depend on the parameterization.

In the strict New Keynesian case ($\gamma_r = 0$, $\alpha_\ell = 1$), or more generally when $\gamma_r < \gamma_\ell \alpha_r$, when κ is endogenous, choosing a more aggressive monetary policy in terms of employment stability will likely lead a virtuous cycle in terms of inflation stability. To be more precise, more stabilization in terms of higher ϕ_ℓ – in a policy rule of the form $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ – directly stabilizes inflation, which can lead to lower κ and even more stabilization. This is the beauty of aggressive monetary policy in a New Keynesian parametrization, that makes monetary policy such a effective tool to stabilize demand shocks.

In Real Keynesian case, the effect of employment stabilization is likely to creats a vicious cycles in terms of its effect on inflation. In this case, if monetary policy aims at stabilizing employment, this increases inflation variability, which increases κ which in turn increases inflation variability further. Hence, in an Real Keynesian parametrization, monetary policy is much less attractive to use to stabilize employment. It may actually be the case that with an endogenous κ, it becomes impossible to find a ϕ_ℓ that fully stabilizes the employment even when there are only demand shocks.[34]

6.2   Interpreting the Great Moderation

One of the strengths of the standard New Keynesian model is that it provides a compelling narrative to the Great Moderation period. According to this narrative, by adopting a more anti-inflationary stance in the post-Volker period, the economy benefited from both greater inflation stability and greater employment stability as a reduction in inflation variability is predicted to go hand-in-hand with greater employment stability if demand shocks are the dominant driving force. This contrasts with the implication of a Real Keynesian parameterization which suggests that such a tradeoff is reversed, since it predicts that greater inflation stability should be associated with less employment stability. As we already mentioned it,

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[34] Note that if monetary policy hasn't been very aggressive in the past (ϕ_ℓ low), and has aimed at low inflation, then κ would tend to be low even in a Real Keynesian regime. In such a situation, a surprise reaction to a demand shock could have a huge stabilizing role, even if a systematic policy may not stabilize. This is similar in narrative to that found in Lucas [1972], but the mechanism is quite different.

it is important to recognize that variance of employment (or unemployment) did not actually decline during the pre versus post Volker period, as shown in Table 3. The greater moderation period was in fact mainly associated with a fall in inflation volatility, with quasi constant employment volatility.

How could a Real Keynesian perspective explain the Great Moderation? Given our estimates, we interpret that the main change induced in the post-Volcker period to be a fall in κ. If one is ready to think of κ as potentially endogenous, a fall in κ is a natural outcome when a policy is targeted towards inducing a lower average inflation rate. The reduction in κ by itself is able to explain why we observed more stable inflation in the Post Volker period, without this implying that the variability of employment needs to increase. Hence, if one only had this policy change in our sample, it would be difficult to differentiate the two possibilities when one takes into account the change in κ. However, the presence of the later lower bound period turns out to be helpful in this respect, as we discuss in the following sub-section. Under the Real Keynesian interpretation, one should observe lower inflation variability and higher output variability when monetary policy is more constrained, as implied by the lower bound, while such a pattern is hard to explain with an New Keynesian parameterization.

6.3 Zero Lower bound

The zero lower bound period has been characterized by three features: important changes in employment, a low variance in inflation and inflation only slightly below target. As previously noted, the Real Keynesian configuration gives a simple explanation of these joint features if the ZLB is interpreted as an induced fixed interest rate period. First, in such a configuration, inflation stays determinate. Moreover, in such a case, the effect of a fixed interest rate is to stabilize inflation, since interest rate changes are no longer contributing to inflation variability. Hitting the zero lower bound also implies greater changes in employment as the employment stabilizing role of monetary is unopperative.

To get a greater sense of how a zero lower bound affects inflation and employment in a Real Keynesien configuration, it is useful to consider once again the case of the fully forward model with only *i.i.d.* demand shocks, so that all expected variables are zero. In

this configuration, the policy rule is then given by

$$r_t = i_t = \max\left\{0, \phi_\ell \ell_t\right\},$$

while the Euler equation and the Phillips curve are given by the two following equations:

$$\ell_t = -\alpha_r r_t + d_t,$$
$$\pi_t = \kappa(\gamma_\ell \ell_t + \gamma_r r_t).$$

In the top row of Figure 16, we plot the Euler equation and the policy rule in the (r_t, ℓ_t) plane space. The light gray Euler equation corresponds to the initial situation, and the dark gray one to a situation with a negative demand shock. The New Keynesian (panel (a)) and Real Keynesian (panel (b)) are no different in that space. But implications for inflation are quite different, as shown in the bottom row of Figure 16. The main aspects we want to highlight is that a large demand shock in the Real Keynesian configuration will lead inflation to be slightly below its long run target, but well above the level that would have been predicted using pre-lower bound observations given the size of the shock. In the Real Keynesian configuration, the Phillips curve flattens out precisely when the economy is at the ZLB, which happens with adverse demand shocks. For the same drop in ℓ, a negative demand shock causes a smaller drop in π in the Real Keynesian configuration (right column), as compared to the New Keynesian configuration) if the ZLB binds.

[Figure 16 about here.]

6.4 Non-Linear Model

It is beyond the scope of this paper to examine a non-linear version of our sticky price model. However, it is worth noting that a Real Keynesian configuration could be a local phenomena, applicable only near the steady state. For example, γ_ℓ may be very close to zero (or equal to zero) when one is near the steady state of the system, which leads the Real Keynesian condition to be satisfied. When the economy deviates far from the steady state, it may be that γ_ℓ increases causing the parametrization to switch from Real Keynesian to New Keynesian. For example, it could be that γ_ℓ is a function of ℓ_t, in which case the Phillips curve is best seen as a non-linear function. If γ_ℓ were to be represented as a quadratic

function of ℓ_t –as to represent that the effect of market tightness on wages may be more operative when far from the steady state– then the Phillips curve could take a form like:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(\gamma_\ell \ell_t^3 + \gamma_r(i - E_t[\pi_{t+1}])).$$

Now suppose that monetary policy was of the form $i_t = E_t[\pi_{t+1}] + \phi_d d_t$, and for complete simplicity, assume that the demand shock d_t is an iid process and the only shock in the economy. In such a case, inflation will be given by:

$$\pi_t = \kappa\big(\gamma_\ell(1 - \alpha_r \phi_d)^3 d_t^3 + \gamma_r \phi_d d_t\big).$$

In this case, whether increasing ϕ_ℓ stabilizes or destabilizes inflation depends on the distribution of shocks. If the distribution has a large variance, then activist policy (in terms of higher ϕ_ℓ) may help stabilize inflation while if the shocks are not too large, it could destabilize inflation. Alternatively, in such a framework, one may want to choose a monetary policy that reacts very differently to small versus large shocks.

7 Conclusion

This paper has explored an extended sticky price model which allows for the canonical three-equation New Keynesian model as a special case, but more importantly allows for a parametrization where the expansionary effects of demand shocks do not disappear as nominal rigidities vanish. We referred to the later as Real Keynesian parameterizations. We began by documenting why the effect of monetary policy could be very different depending on whether the economy is best characterized by a Real Keynesian parametrization or not. In particular, we emphasized how a monetary policy aimed at stabilizing employment faces a much less attractive tradeoff under a Real Keynesian parameterization than under parameterizations usually assumed in the New Keynesian literature. Moreover, as an example, we discussed how a Real Keynesian parametrization may help explain why inflation can be very stable at the ZLB period while employment remains very variable. Given these differences, a large part of the paper is devoted to showing that post-war US data on employment, inflation and interest rates may be more easily explained by a Real Keynesian parametrization than with a more conventional parametrization. If this is true, it suggests a need to rethink the

role of monetary policy in economic stabilization since stabilizing inflation and countering demand shocks may be more conflictual in terms of policy than usually recognized. The Real Keynesian parameterization suggests that two tools are required to approach such goals.

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Appendix

A Micro-Foundations for a Real Keynesian Model

The two extensions that we add to the standard New Keynesian model are that *(i)* the real interest rate enters in the marginal cost of firms and *(ii)* the household Euler equation is discounted. We provide below the micro-foundations for both extensions

A.1 Firms and the derivation of the augmented New Phillips curve

The introduction of the real interest rate in the marginal cost of firms is not new (Christiano, Eichenbaum, and Evans [2005], Ravenna and Walsh [2006]). The twist we introduce here allow for arbitrary elasticities of the marginal cost with respect to respectively the real wage and the real interest rate. Let's do the derivation of the marginal cost, that can be done looking at the static optimal choice of inputs.

A.1.1 Production

Each monopolist produces a differentiated good using basic input as the only factor of production, and according to a one to one technology. The marginal cost of production will therefore be the price of that basic input. It is assumed that the basic input is produced by a representative firm that behaves competitively. The representative firm produces basic input Q_t with labor L_t and the final good M_t according to the following Leontief technology

$$Q_t = \min(a\Theta_t L_t, bM_t).$$

As in the main text, we assume that Θ_t is constant and normalized to one. The optimal production plan implies $Q_t = aL_t = bM_t$, so that the optimal input demands are $L_t = \frac{Q_t}{a}$ and $M_t = \frac{Q_t}{b}$. Denote by $\mathcal{C}(Q_t) = W_t L_t + \Phi_t M_t$ the total cost of production, where the exact expression of Φ_t will be derived later. Using the optimal input demands, we obtain

$$\mathcal{C}(Q_t) = \left(\frac{W_t}{a} + \frac{\Phi_t}{b}\right) Q_t,$$

so that marginal cost is

$$\mathcal{C}'(Q_t) = \frac{W_t}{a} + \frac{\Phi_t}{b}.$$

Log-linearizing the above expression gives the following expression of the real marginal cost, where the variables are now in logs and where constant have been omitted:

$$mc_t = \left(\frac{\frac{W}{a}}{\frac{W}{a} + \frac{\Phi}{b}}\right)(w_t - p_t) + \left(\frac{\frac{\Phi}{b}}{\frac{W}{a} + \frac{\Phi}{b}}\right)(\phi_t - p_t).$$

A.1.2 Derivation of the cost Φ_t

The unit price of the final good that enters the production of basic input is P_t. We assume that in the morning of each period, the basic input representative firm must borrow D^B_{t+1} at the risk-free

nominal interest rate i_t to pay for the input M_t. In the afternoon, it produces, sells its production, pays wages, repays the debt contracted the previous period D_t^B and distributes all the profits Ω_t^B as dividends. Those profits will be zero in equilibrium. The period t budget constraint of the firm is therefore:

$$D_{t+1}^B + \widetilde{P}_t Q_t = W_t L_t + (1 + i_{t-1}) D_t^B + P_t M_t,$$

with $D_{t+1}^B = P_t M_t$. Period t profit writes

$$\Omega_t^B = \widetilde{P}_t Q_t - W_t L_t - (1 + i_{t-1}) P_{t-1} M_{t-1},$$

where $\widetilde{P}_t$ the price of the basic input. Assuming that the firm maximises the expected discounted sum of profits real profits Ω_t^B / P_t with discount factor β, and using $Q_t = a L_t = b M_t$, we obtain the first order condition

$$\widetilde{P}_t = \left(\frac{W_t}{P_t} + \frac{\beta}{b} E_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} \right] \right) P_t.$$

Therefore, the real marginal cost of the basic input firm will be given by

$$MC_t = \frac{W_t}{P_t} + \frac{\beta}{b} E_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} \right].$$

The price of the basic input $\widetilde{P}_t$ is equal to the nominal marginal cost of the basic input firm, and is also the marginal cost of the intermediate input firm, which is the relevant one for pricing decisions. In logs, the real marginal cost will write (omitting constants):

$$mc_t = \underbrace{\left(\frac{\frac{1}{a} \frac{W}{P}}{\frac{1}{a} \frac{W}{P} + \frac{\beta}{b} \frac{1+i}{1+\pi}} \right)}_{\widehat{\gamma}_\ell} (w_t - p_t) + \underbrace{\left(\frac{\frac{\beta}{b} \frac{1+i}{1+\pi}}{\frac{1}{a} \frac{W}{P} + \frac{\beta}{b} \frac{1+i}{1+\pi}} \right)}_{\gamma_r} (i_t - E_t[\pi_{t+1}]).$$

A.1.3 Pricing

As in the standard New Keynesian model, intermediate firms play a Calvo lottery to draw price setting opportunities. Except for the use of the basic input, the modelling is very standard. The optimal household labor supply, that we will derive later, will give us

$$\frac{\nu'(L_t)}{U'(C_t)} = \frac{W_t}{P_t},$$

which write in logs, using $C_t = a L_t$ and omitting constants

$$w_t - p_t = \left(\frac{L \nu''(L)}{\nu'(L)} - \frac{C U''(C)}{U'(C)} \right) \ell_t.$$

As $C_t = Y_t = a L_t$, the marginal cost does not depend on the scale of production and is the same for all the intermediate input firms. It is written as

$$mc_t = \underbrace{\widetilde{\gamma}_\ell \left(\frac{L \nu''(L)}{\nu'(L)} - \frac{C U''(C)}{U'(C)} \right)}_{\gamma_\ell} \ell_t + \gamma_r \left(i_t - E_t[\pi_{t+1}] \right).$$

The rest of the model is standard, and we obtain the New Phillips curve

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \, mc_t + \mu_t.$$

Once using the expression of the real marginal cost, we have

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \left(\gamma_\ell \ell_t + \gamma_r \left(i_t - E_t[\pi_{t+1}] \right) \right) + \mu_t.$$

This give us equation (PC) in the main text.

A.2 Households and the derivation of the discounted Euler equation

The derivation of the discounted Euler equation relies on two set of assumptions. First, because of asymmetry of information and lack of commitment, individual households will face an upward sloping supply of funds when borrowing. To maintain tractability, we will consider a equilibrium in which agents never default, so that the income and wealth distribution will have a unique mass point. For exposition simplicity, we will derive the main features of the equilibrium in a two-period model, and explain why the extension to an infinite horizon is trivial. Second, we will assume a particular timing of income and expenditure flows. Those two assumptions will allow us to derive a discounted Euler equation.

A.2.1 A simple two-period model with asymmetric information and lack of commitment

It is a deterministic mode with two periods. There are two types of households and a zero profit risk neutral representative bank that has access to an unlimited supply of funds at cost $\overline{R}$. Households receive no endowment in the first period, and ω in the second period. The consumption good is the numéraire.

Some households (superscript c) have access to commitment and always repay their debt, while the other type (superscript nc) cannot commit to repay. Type is not observable. Because of this, risk neutral banks will want to charge a risk premium on their loans. More specifically, they propose to the households a schedule $R(d)$ that is increasing in the level of debt d.

Preferences over consumption are given by $u(c_1) + \beta u(c_2)$. Households also bear an additively separable utility cost of defaulting $\psi(d)$ which is an increasing and convex function of the amount of defaulted debt.

When households borrow (and they will always do under regularity conditions on preferences u), they will consume (c_1, c_2) and their debt is $d = c_1$. Committed type households maximise their utility under the budget constraint $c_2 = \omega - R(c_1)c_1$. and the optimal choice for c_1 satisfies

$$u'(c_1^c) = \beta \left(R(c_1^c) + R'(c_1^c)c_1^c \right) u'(\omega - R(c_1^c)). \tag{A.1}$$

The non-committed type household optimally decide whether they will default (superscript d) or not (superscript nd) in period 2, and this choice can made in period 1 because there is no uncertainty in this example. If they repay (non default), then they behave as the committed type, so that

$$c_1^{nc,nd} = c_1^c.$$

If they default, then they will borrow in period 1 so that to equalise marginal utility of consumption with marginal psychological cost of default. The optimal choice will then satisfy

$$u'(c_1^{nc,d}) = \psi'(c_1^{nc,d}), \tag{A.2}$$

while $c_2^{nc,d} = \omega$.

The optimal decision to default or not depends on the direction of the following inequality:

$$\underbrace{u(c_1^c) + \beta u\left(\omega - R(c_1^c)c_1^c\right)}_{\text{if no default}} \gtrless \underbrace{u(c_1^{nc-d}) + \beta u(\omega) - \psi(c_1^{nc,d})}_{\text{if default}}.$$

For given $u(\cdot)$, β and ω, there is always a psychological cost function $\psi(\cdot)$ such that household of the non-committed type choose to imitate the committed ones. In this case, we have a pooling equilibrium in which all households behave the same and in which there are no defaults. From the banks zero profit, we should have $R(c_1^c) = \overline{R}$, as there is no default. This condition is the only restriction put on the $R(\cdot)$ schedule, so that any off-equilibrium belief $R'(\cdot) > 0$ is consistent with a no default pooling equilibrium.

Extension to an infinite horizon model : If we assume that past actions (default or not) are not observable, then the logic of the two-period model still holds in a standard infinite horizon model. With asymmetric information on the household type (access or not to commitment), one can sustain an equilibrium with no default with the following properties: (i) households always make the same consumption and saving choices (no observed heterogeneity), (ii) there is no risk premium on the interest rate in equilibrium and (iii) households consistently face an upward sloping interest schedule $R(b)$. The interest of this modelling is the absence of observed heterogeneity, that allows for a simple solving of the model.

A.2.2 Household problem with upward sloping interest schedule.

There is a measure one of identical households indexed by i. Each of them chooses a consumption stream and labor supply to maximizes discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{t-1}(U(C_{it}) - \nu(L_{it}))$, where ζ is a discount shifter.

We split the period into a morning and an afternoon. There is no difference in information between morning and afternoon. In the morning, household i must order and pay consumption expenditures $P_t C_{it}$ and cannot use previous savings to do so. They must borrow $D_{it+1}^M = P_t C_{it}$ units of money (say dollars) at a nominal interest rate i_{it}^H that, for the reasons sketched above, will depend on the total borrowing of the household in period t (hence the subscript i). In the afternoon, household i can borrow D_{it+1}^A for intertemporal smoothing motives, receives labor income $W_t L_{it}$ and profits from intermediate firms Ω_{it} and must repay principal and interest on the total debt inherited from the previous period $(1 + i_{it-1}^H)(D_{it}^M + D_{it}^A)$. The morning budget constraint is therefore given by:

$$D_{it+1}^M = P_t C_{it},$$

and the afternoon one by:

$$D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H)(D_{it}^M + D_{it}^A).$$

Putting together, we obtain the following budget constraint of period t:

$$D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H)D_{it}^A + (1 + i_{it-1}^H)P_{t-1}C_{it-1}.$$

As there are no information between and afternoon, the interest rate i_{it}^H faced by household i is a function of the total real net debt subscribed in period t. We write it as a premium over the risk free nominal rate:

$$c1 + i_{it}^H = (1 + i_t)\left(1 + \rho\left(\frac{D_{it+1}^M + D_{it+1}^A}{P_t}\right)\right) = (1 + i_t)\left(1 + \rho\left(C_{it} + \frac{D_{it+1}^A}{P_t}\right)\right),$$

with $\rho > 0$, $\rho' > 0$ and $\rho'' > 0$.

The decision problem of household i is therefore given

$$\max \sum_{t=0}^{\infty} \beta^t \zeta_{t-1} E_0 \big[U(C_{it}) - \nu(L_{it}) \big]$$

$$s.t. \quad D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H) D_{it}^A + (1 + i_{it-1}^H) P_t C_{it},$$

$$1 + i_{it}^H = (1 + i_t) \left(1 + \rho \left(C_{it} + \frac{D_{it+1}^A}{P_t} \right) \right).$$

The first order conditions (evaluated at the symmetric equilibrium in which $D_{it+1}^A = 0 \ \forall i$) associated with this problem are:

$$U'(C_t) = \beta \frac{\zeta_t}{\zeta_{t-1}} E_t \left[U'(C_{t+1})(1 + i_t)\big(1 + \rho(C_t) + C_t \rho'(C_t)\big) \frac{P_t}{P_{t+1}} \right],$$

$$\frac{\nu'(l_t)}{U'(C_t)} = \frac{W_t}{P_t}.$$

Assuming that utility is CRRA ($U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$), the Euler equation can be log-linearized to obtain (omitting constants and with $L_t = C_t$) :

$$\ell_t = \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t,$$

where ℓ and c are the logs of L and C and with $\alpha_\ell = \frac{\sigma}{\sigma + \varepsilon_\rho} \in]0,1[$, $\alpha_r = \frac{1}{\sigma + \varepsilon_\rho} > 0$, $\varepsilon_\rho = \frac{C(2\rho' + C\rho'')}{\rho + C\rho'} > 0$ and $d_t = -\frac{1}{\sigma + \varepsilon_\rho}(\log \zeta_t - \log \zeta_{t-1})$. This give us equation (EE) in the main text.

B   Proofs

Proof of Proposition 1

As κ goes to infinity, the Phillips curve implies[35] that $\ell_t = 0$ (recall that ℓ is the deviation of employment from its steady state level) when $\gamma_r = 0$. Hence, employment is not affected by demand shocks when $\gamma_r = 0$.

Proof of Proposition 2

Step 1 :   This step consists in showing that under the RK condition, demand shocks maintain an expansionary effect of employment even as κ goes to infinity. As κ goes to infinity, the Philips curve implies that

$$i - E_t[\pi_{t+1}] = \frac{\gamma_\ell}{\gamma_r} \ell_t.$$

Replacing this is the Euler equation implies

$$\ell_t = \frac{\alpha_\ell}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} E_t[\ell_{t+1}] + \frac{1}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} d_t.$$

Since under the RK condition, $\frac{\alpha_\ell}{1 + \alpha_r \frac{\gamma_\ell}{\gamma_r}}$ is between 0 and 1, this has a unique stationary equilibrium where current and all future expected demand disturbances have a positive effect on employment.

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[35] This assumes that monetary policy is conducted in such away that inflation stays bounded.

Step 2 : This step consists in showing that if the RK condition is not satisfied then, as κ goes to infinity, demand shocks do not maintain (always) a positive effect on employment.

There are two cases to consider. In the first case, the equilibrium is determinate when the RK condition is not satisfied. This arises when $\alpha_r \gamma_\ell > (1 + \alpha_\ell)\gamma_r$. The second case is when the equilibrium is indeterminate, which happens either when $\gamma_r < \alpha_r \gamma_\ell < (1 + \alpha_\ell)\gamma_r$, or when $\gamma_r > \alpha_r \gamma_\ell > (1 - \alpha_\ell)\gamma_r$.

First case : When $\alpha_r \gamma_\ell > (1 + \alpha_\ell)\gamma_r$, the equilibrium is determinate since $-1 < \frac{\alpha_\ell}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} < 0$ and $\frac{1}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} < 0$, and the solution is given by

$$\ell_t = \frac{1}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} E_t \left[\sum_{i=0}^{\infty} (\frac{\alpha_\ell}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}})^i d_{t+i} \right].$$

Therefore the effects of current demand disturbances in negative, and the effects of future demand disturbances oscilates between positive and negative.

Second case : When $\gamma_r < \alpha_r \gamma_\ell < (1 + \alpha_\ell)\gamma_r$ or when $\gamma_r > \alpha_r \gamma_\ell > (1 - \alpha_\ell)\gamma_r$.), the equilibrium is indeterminate and admits a continuum of solutions. We consider the full set of equilibrium of the form $\ell_t = \sum_{i=0}^{\infty} \psi_i d_{t-i}$ under the assumption that d_t follows an AR(1) process of the form $d_t = \rho_d d_{t-1} + \epsilon_t$. Since all we need to show is that demand shock do not always have an expansionary under this configuration, this will be sufficient. The Euler equations is

$$\ell_t = \frac{\alpha_\ell}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} E_t[\ell_{t+1}] + \frac{1}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} d_t.$$

Then the set of solutions can be written as

$$\ell_t = \frac{1}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} \sum_{i=0}^{\infty} \theta_i d_{t-i},$$

with $\theta_0 = \frac{1 + \frac{\alpha_\ell}{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}} \theta_1}{1 - \frac{1}{1 + \alpha_r \frac{\gamma_\ell}{\gamma_r}} \rho_d}$ and $\theta_i = (\frac{1 - \alpha_r \frac{\gamma_\ell}{\gamma_r}}{\alpha_l})^{i-1} \theta_1$ for $i > 0$. Since $\frac{\alpha_\ell}{1 + \alpha_r \frac{\gamma_\ell}{\gamma_r}}$ is greater than one in absolute value, θ_1 can take on an arbitrary value (this captures the indeterminacy). When $\gamma_r < \alpha_r \gamma_\ell < (1 + \alpha_\ell)\gamma_r$, there is no choice of θ_1 that makes demand shocks only have positive effects (either demand always has negative effects on employment, or it oscillates between positive and negative effects). When $\gamma_r > \alpha_r \gamma_\ell > (1 - \alpha_\ell)\gamma_r$, it is possible to construct an equilibrium where demand shocks have only positive effects if ρ_d is sufficiently small, but not if ρ_d is sufficiently close to one.

Proof of Propositions 3 and 4: Before proving Proposition 3 and 4, we first prove Lemma B.1.

Lemma B.1. *If monetary policy is given by $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$ with $\phi_\ell > 0$, then there is a unique stationary equilibrium.*

Under the monetary rule $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$, the Euler equation becomes

$$\ell_t = \frac{\alpha_\ell}{1 + \alpha_r \phi_\ell} E_t[\ell_{t+1}] + \frac{1}{1 + \alpha_r \phi_\ell} d_t.$$

Since $\frac{\alpha_\ell}{1+\alpha_r\phi_\ell} < 1$, this has the unique stationary solution

$$\ell_t = \frac{1}{1+\alpha_r\phi_\ell} \sum_{i=0}^{\infty} E_t\left[\frac{\alpha_\ell}{1+\alpha_r\phi}\right]^i d_{t+i}. \tag{B.3}$$

The Philips curve becomes

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(\gamma_\ell + \gamma_r\phi_\ell)\ell_t + \mu_t,$$

which also has a unique stationary solution of the form

$$\pi_t = \kappa(\gamma_\ell + \gamma_r\phi_\ell)\sum_{i=0}^{\infty} \beta^i E_t[\ell_{t+i}] + \sum_{i=0}^{\infty} \beta^i E_t[\mu_{t+i}]. \tag{B.4}$$

Note that although this monetary policy can maintain determinacy for any value of κ, under many parameter configurations it will imply that the variance of inflation will go off to infinity as κ goes to infinity. Hence, this policy is not always admissible given how the limit is taken when κ goes to infinity in Proposition 2.

Proposition 3 and 4 following directly from Lemma B.1. In particular, equation (B.3) implies that a demand shock always has a positive effect of employment, regardless of the parameterization, while a cost push shock has no effect on employment. In contrast, equation (B.4) implies that a demand shock and a cost push shock increase inflation regardless of the parameterization. The implications for nominal and real interest rates then follow directly from the policy rule.

Proof of Proposition 5 : Equation 2 implies a positive (negative) tradeoff between σ_π and σ_ℓ if $\gamma_r(1 - \rho\alpha_\ell) - \gamma_\ell\alpha_r > 0 \ (< 0)$. This tradeoff will always be positive (regardless of the value of ρ_d) if and only if $\gamma_r(1 - \alpha_\ell) - \gamma_\ell\alpha_r > 0$, which is the RK condition.

Proof of Proposition 6: We are considering the following system of equations

$$\begin{aligned}
\ell_t &= \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t, \\
\pi_t &= \beta E_t[\pi_{t+1}] + \gamma_\ell\ell_t + \gamma_r(i_t - E_t[\pi_{t+1}]), \\
i_t &= \phi_\pi E_t[\pi_{t+1}] + \phi_\ell\ell_t, \\
d_t &= \rho_d d_{t-1} + \varepsilon_{dt}.
\end{aligned}$$

For this system we want to compare its implication for the variance of inflation and employment as we move from $\phi_\pi = 1$ and $\phi_\ell > 0$ to the situation where $\phi_\pi = 0$ and $\phi_\ell = 0$. First, it is easy to verify that, under a Real Keynesian parameterization, this system maintains determinacy for all ϕ_{pi} and ϕ_ℓ such that $0 \leq \phi_\pi \leq 1 \ 0 \leq \phi_\ell$ and therefore the solution can be written in the form $\ell_t = \Omega_\ell d_t$ and $\pi_t = \Omega_\pi d_t$ where

$$\Omega_\ell = \frac{1 - \beta\rho_d + \gamma_r(1 - \phi_\pi)}{(1 - \beta\rho_d)(1 + \alpha_r\phi_\ell - \alpha_\ell\rho_d) + (\phi_\ell - 1)\rho_d[\alpha_r\gamma_\ell - \gamma_r(1 - \alpha_\ell\rho_d)]},$$

$$\Omega_\pi = \frac{\gamma_\ell + \gamma_r\phi_\ell)}{(1 - \beta\rho_d)(1 + \alpha_r\phi_\ell - \alpha_\ell\rho_d) + (\phi_\pi - 1)\rho_d[\alpha_r\gamma_\ell - \gamma_r(1 - \alpha_\ell\rho_d)]}.$$

This implies that $\frac{\partial\Omega_\ell}{\partial\phi_\pi} < 0$ and $\frac{\partial\Omega_\ell}{\partial\phi_\ell} < 0$, hence the variance of employment necessarily increases as we move from $\phi_\pi = 1$ and $\phi_\ell > 0$ to the situation where $\phi_\pi = 0$ and $\phi_\ell = 0$.

Moreover, we have that $\frac{\partial\Omega_\pi}{\partial\phi_\pi} > 0$ and $\frac{\partial\Omega_\pi}{\partial\phi_\ell} > 0$ when the parameterization is Real Keynesian (and $\phi_\pi \leq 1$). Hence the variance of inflation necessarily decreases as we move from $\phi_\pi = 1$ and $\phi_\ell > 0$ to the situation where $\phi_\pi = 0$ and $\phi_\ell = 0$.

Proof of Proposition 7:   The proof of this proposition builds on that for Proposition 6. In the case where $\gamma_r = 0$ and α_ℓ is sufficiently small to maintain determinacy, the solution is still of the form $\ell_t = \Omega_\ell d_t$ and $\pi_t = \Omega_\pi d_t$ with the same expressions for $\Omega_p i$ and Ω_ℓ. Hence we still that $\frac{\partial \Omega_\ell}{\partial \phi_\pi} < 0$ and $\frac{\partial \Omega_\ell}{\partial \phi_\ell} < 0$, hence the variance of employment continues to increases as we move from $\phi_\pi = 1$ and $\phi_\ell > 0$ to the situation where $\phi_\pi = 0$ and $\phi_\ell = 0$. However, we now have that $\frac{\partial \Omega_\pi}{\partial \phi_\pi} < 0$ and $\frac{\partial \Omega_\pi}{\partial \phi_\ell} < 0$. Hence the variance of inflation now increases as we move from $\phi_\pi = 1$ and $\phi_\ell > 0$ to the situation where $\phi_\pi = 0$ and $\phi_\ell = 0$.

Proof of Proposition 8:   We are now considering the dynamics induced by the monetary innovation ε_ν with following system of equations which includes external habit

$$\ell_t - h\ell_{t-1} = \alpha_\ell E_t[\ell_{t+1} - h\ell_t] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t,$$
$$\pi_t = \beta E_t[\pi_{t+1}] + \gamma_\ell \ell_t + \gamma_r(i_t - E_t[\pi_{t+1}]),$$
$$i_t = E_t[\pi_{t+1}] + \phi_\nu \nu_t,$$
$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu t}.$$

In the case with external consumption habit, the RK condition becomes

$$(1 - h)(1 - \alpha_\ell)\gamma_r > \gamma_\ell \alpha_r.$$

Note that this condition is identical to the previous derived RK condition when $h = 0$.

Given the policy rule, it is again easy to verify that the system admit a unique stationary solution as long as $0 \le \alpha_\ell < 1$ which is of the form $\ell_t = \Omega_{\ell\ell}\ell_{t-1} + \Omega_{\ell\nu}\nu_t$ and $\pi_t = \Omega_{\pi\ell}\ell_{t-1} + \Omega_{\pi\nu}\nu_t$ with

$$\Omega_{\ell\ell} = h,$$
$$\Omega_{\ell\nu} = -\frac{\alpha_r \phi_\nu}{1 - \alpha_\ell \rho_\nu},$$
$$\Omega_{\pi\ell} = \frac{\gamma_\ell h}{1 - \beta h},$$
$$\Omega_{\pi\nu} = \frac{1}{1 - \beta\rho_\nu}\left(\gamma_r \phi_\nu - \frac{\alpha_r \phi_\nu}{(1 - \alpha_\ell \rho_\nu)(1 - \beta h)}\right).$$

Since $\Omega_{\ell\ell}$ and $\Omega_{\ell\nu}$ are positive, it follows that contractionary monetary shocks, as captured by increases in ν_t always have a negative effects of output regardless of whether we are in an RK parameterization or not. The more complicated case is the relationship between π_t and all ν_{t-i}. When $h < \rho$ this relationship can be expressed as

$$\pi_t = \Omega_{\pi\nu}\nu_t + \sum_{i=1}^{\infty} \frac{\rho_\nu^i \phi_\nu}{1 - \beta\rho_\nu}\left(\gamma_r - \frac{\alpha_r \gamma_\ell}{(1 - \alpha_\ell \rho_\nu)(1 - h)}\Phi(i, \rho_\nu)\right)\nu_{t-i},$$

where

$$\Phi(i, \rho_\nu) = \frac{(1 - h)[(1 - \frac{h}{\rho_\nu}) + (1 - \beta\rho_\nu)(1 - (\frac{h}{\rho_\nu})^i)h]}{(1 - \frac{h}{\rho_\nu})(1 - \beta)},$$

with $\frac{\partial \Phi}{\partial i} > 0$ and $\Phi(i, 1) \le 0$.

Therefore, as $\rho \mapsto 1$, all the coefficients on inflation will be positive under a Real Keynesian parametrization. However, if we are not in a Real Keynesian parametrization, the effect of a monetary shock will eventually be negative since $\gamma_r - \frac{\alpha_r \gamma_\ell}{(1 - \alpha_\ell)(1 - h)}\Phi(i, 1)$ is negative for sufficiently large i.

50

C   Policy Trade-off with Cost-Push Shocks

Here we study the stabilization of cost-push shocks, and assume that the policy authorities can recognize a cost shock, so that the policy rule for setting the interest rate is now given by

$$i_t = E_t[\pi_{t+1}] + \phi_\mu \mu_t.$$

The type of policy trade-off that arises as the authorities change ϕ_μ is described in the next proposition. We are assuming that the cost shocks follow a first order autoregressive process.

Proposition C.1. *If the monetary policy rule is given by $i_t = E_t[\pi_{t+1}] + \phi_\mu \mu_t$ with $\phi_\mu > 0$, then increasing ϕ_μ always increases the variance of both inflation and employment when the model is in the Real Keynesian configuration. In contrast, when $\gamma_\ell \alpha_r > \gamma_r(1 - \rho_\mu \alpha_\ell)$ (which holds in the New Keynesian model), starting from $\phi_\mu = 0$, some increase in ϕ_μ will lead to a lower variance of π,*

Here again we get that the effects of changing policy has different implications if we are in the Real Keynesian case or in the New Keynesian one. In the Real Keynesian case, for stabilizing purposes, it is not desirable to react to cost shocks by increasing interest rates as this will destabilize both inflation and employment. In contrast, in the New Keynesian case, at least some increase in rates in response to cost shocks may be desirable as one can thereby stabilize both inflation even if it destabilizes employment.

Proof of C.1 We are now considering the dynamics induced by μ within the following system of equations

$$
\begin{aligned}
\ell_t &= \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]), \\
\pi_t &= \beta E_t[\pi_{t+1}] + \gamma_\ell \ell_t + \gamma_r(i_t - E_t[\pi_{t+1}]), \\
i_t &= E_t[\pi_{t+1}] + \phi_\mu \mu_t, \\
\mu_t &= \rho_\mu \mu_{t-1} + \varepsilon_{\mu t}.
\end{aligned}
$$

The solutions for ℓ_t and π_t are

$$
\begin{aligned}
\ell_t &= -\frac{\alpha_r \phi_\mu}{1 - \rho_\mu \alpha_\ell} \mu_t, \\
\pi_t &= \left[\left(\gamma_r - \frac{\gamma_\ell \alpha_r}{1 - \rho_\mu \alpha_r}\right)\phi_\mu + 1\right]\frac{\mu_t}{1 - \beta \rho_\mu}.
\end{aligned}
$$

From these two expression, one can see that if $\gamma_r - \frac{\gamma_\ell \alpha_r}{1 - \rho_\mu \alpha_r} > 0$, which is always the case under the RK condition, then an increase in ϕ_μ will lead to a greater variance in both π and ℓ_t. In contrast, if $\left(\gamma_r - \frac{\gamma_\ell \alpha_r}{1 - \rho_\mu \alpha_r}\right) < 0$, then an increase in ϕ_μ starting from $\phi_\mu = 0$ will lead to a decrease in the volatility of inflation and an increase in the volatility of employment.

D   Data Definition and Sources

- Inflation : Gross Domestic Product: Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`GDPDEF`). Sample is `1947Q1-2016Q3`.

- Inflation: Consumer Price Index for All Urban Consumers: All Items, Percent Change, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`CPIAUCSL`). Sample is `1947Q1-2016Q3`.

- Nominal interest rate: Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (`FEDFUNDS`). Sample is `1954Q3-2016Q3`.

- Unemployment: Civilian Unemployment Rate, Percent, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`FEDFUNDS`). Sample is `1948Q1-2016Q3`.

- Oil price: Spot Crude Oil Price: West Texas Intermediate (WTI), Dollars per Barrel, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (`WTISPLC`). Sample is `1946Q1-2016Q3`. The real oil price is then obtained by deflating by GDP price.

- U.S. Total GDP is obtained from the Bureau of Economic Analysis National Income and Product Accounts. Real quantities are computed as nominal quantities (Table 1.1.5) over prices (Table 1.1.4.). Sample is `1947Q1-2015Q2`.

- Non-Farm Business Hours is obtained from the Bureau of Labor Statistics. Sample is 1947Q1-2015Q2.

- U.S. Population: Total Population: All Ages including Armed Forces Overseas, obtained from the FRED database (`POP`) from 1952Q1 to 2015Q2. Quarters from 1947Q1 to 1952Q1 are obtained from linear interpolation of the annual series of National Population obtained from U.S. Census, where the levels have been adjusted so that the two series match in 1952Q1.

E   Equivalence of Different Forms of Policy Rules

In the main text, we make the case that the tradeoff between activity variance and inflation variation changes depending on whether $\gamma_r(1 - \alpha_\ell) > < \gamma_\ell \alpha_r$. To show this, it was easiest to work with a policy rule of the form $i_t = E_t[\pi_{t+1}] + \phi_d d_t$ (or $i_t = E_t[\pi_{t+1}] + \phi_\ell \ell_t$). The question is whether this was restrictive, Below are the elements of the proof that it is without loss of generality as long as the equilibirum is determinate.

Suppose we have a model (with no state variables) of the form

$$X_t = AE_t[X_{t+1}] + B\Big(i_t - GE_t[X_{t+1}]\Big) + Cd_t,$$

where $d_t = \rho_d d_{t_1} + \varepsilon_{dt}$ and where the eigenvalues of A are on the unit disc. Note that this model embeds our sticky price setup as a special case,[36] where $X_t = \{\pi_t, l_t\}$ and $GE_t[X_{t+1}]$ would simply be $E_t[\pi_{t+1}]$. Now suppose we consider a feedback policy rule of the form $i_t = DX_t$, where D is such that the solution is determinate.[37] Then the solution is

$$X_t = (I - (I - BD)^{-1}(A - BG)\rho_d)^{-1}(I - BD)^{-1}Cd_t = Fd_t.$$

Alternatively, we could suppose that the policy rule was of the form

$$i_t = GE_t[X_{t+1}] + F'd_t,$$

where if we choose to set F' according to $F' = (D - G\rho_d)F$, then it will give the same solution. In other words, in this case the model can be rewritten as

$$X_t = AE_t[X_{t+1}] + \Big(B(D - G\rho_d)(I - (I - BD)^{-1}(A - BG)\rho)^{-1}(I - BD)^{-1}C + C\Big)d_t,$$

---

[36] This does not cover the case where α_ℓ is exactly 1. We can easily generalize the following analysis for this case.

[37] That is, the eigenvalues of $(I - (I - BD)^{-1}(A - BG)\rho_d)$ are on the unit disc.

and it also has as its solution

$$X_t = Fd_t = (I - (I - BD)^{-1}(A - BG)\rho_d)^{-1}(I - BD)^{-1}Cd_t.$$

To see this, note that the solution is

$$X_t = (1 - A\rho_d)^{-1}(B(D - G\rho_d)(I - (I - BD)^{-1}(A - BG)\rho_d)^{-1}(I - BD)^{-1} + I)Cd_t.$$

Then it is easy to verify that

$$(1 - A\rho_d)^{-1}\Big(B(D - G\rho_d)(I - (I - BD)^{-1}(A - BG)\rho_d)^{-1}(I - BD)^{-1} + I\Big) =$$
$$\Big(I - (I - BD)^{-1}(A - BG)\rho_d\Big)^{-1}(I - BD)^{-1}.$$

Hence, as long as we are restricting attention to policy rules that induce determinacy, there is no loss of generality exploring properties of such a model under a policy rule of the form $i_t = DX_t$ or of the form $i_t = GE_t[X_{t+1}] + F'd_t$. In fact, the latter will cover the the full set of policy options offered by the former, but the reverse may mot be true.[38]

The same type of equivalence can be show for the case where the policy rule takes the form $i_t = GE_t[X_{t+1}] + F'X_t$ instead of $i_t = GE_t[X_{t+1}] + F'd_t$.

F   The Case of $\gamma_\ell < 0$

In the main, we have not considered the possibility that γ_ℓ may be negative, *i.e.* that the marginal cost could be in equilibrium a decreasing function of the production scale. This could happen if there are for example thick market externalities If $\gamma_\ell < 0$, then the system's response to increasing ϕ_ℓ is non monotonic on inflation. Nonetheless we have the following result.

Proposition F.2. *If $\gamma_\ell < 0$, then stabilizing the economy with respect to demand shocks will lead to increased inflation variability if $(1 - \rho_d\alpha_\ell)\gamma_r > -\gamma_\ell\alpha_r$*

So if we want to allow for case where γ_ℓ could be negative, the more general condition for unfavorable tradeoff between inflation and employment variability can be stated as

$$(1 - \alpha_\ell)\gamma_r > |\gamma_\ell|\alpha_r.$$

When monetary policy is of the form $i = E_{t+1}\Pi_t + \phi_\ell\ell_t$, it is easy to verify that demand shocks will again increase employment and inflation regardless of whether the economy is in a Real Keynesian regime or not.

---

[38]The mapping between the two policies is given by

$$F' = (D - G\rho_d)\Big(I - (I - BD)^{-1}(A - BG)\rho_d\Big)^{-1}(I - BD)^{-1}C.$$

For a given D, this gives an F'. In the other direction, given F' there are in general multiple solutions for D, since there are many feedback rules than can implement the same solutions as we are considering a one shock setup. Among this set, we would want to choose a D such that the eigenvalues of $(I - (I - BD)^{-1}(A - BG)\rho_d)$ are less than 1. This will still often leave many solutions. It may also be the case that there is no D that satisfies $F' = (D - G\rho_d)(I - (I - BD)^{-1}(A - BG)\rho_d)^{-1}(I - BD)^{-1}C$ and guarantees determinacy. For this reason, the rule of the type $i_t = GE_t[X_{t+1}] + F'd_t$ may be considered more general.

G   Explicit Mapping From Reduced Form to Parameters

We first solve for the structural model, and then inverse the mapping from structural parameters to the reduced form one. The model is given by

$$
\begin{aligned}
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_\ell \ell_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \sigma_\mu \mu_t, \\
i_t &= E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \sigma_\nu \nu_t, \\
\ell_t &= \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + \alpha_\mu \mu_t + \sigma_d d_t,
\end{aligned}
$$

with AR(1) exogenous shocks

$$
\begin{pmatrix} d_t \\ \mu_t \\ \nu_t \end{pmatrix} = \begin{pmatrix} \rho_d & 0 & 0 \\ 0 & \rho_\mu & 0 \\ 0 & 0 & \rho_\nu \end{pmatrix} \begin{pmatrix} d_{t-1} \\ \mu_{t-1} \\ \nu_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{dt} \\ \varepsilon_{\mu t} \\ \varepsilon_{\nu t,} \end{pmatrix},
$$

where the covariance matrix of the ε is identity. To solve the model, we guess a solution of the type

$$
\begin{pmatrix} \pi_t \\ i_t \\ \ell_t \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{23} & b_{33} \end{pmatrix} \begin{pmatrix} d_t \\ \mu_t \\ \nu_t \end{pmatrix},
$$

and solve for the undetermined coefficients b. The solution is given recursively by

$$
\begin{array}{lll}
b_{31} = \frac{\sigma_d - \alpha_r \phi_d}{1 - \alpha_\ell \rho_d}, & b_{32} = \frac{\alpha_\mu - \alpha_r \phi_\mu}{1 - \alpha_\ell \rho_\mu}, & b_{33} = \frac{-\alpha_r \sigma_\nu}{1 - \alpha_\ell \rho_\nu}, \\
b_{11} = \frac{\kappa(\gamma_\ell b_{31} + \gamma_r \phi_d)}{1 - \beta \rho_d}, & b_{12} = \frac{\sigma_\mu + \kappa(\gamma_\ell b_{32} + \gamma_r \phi_\mu)}{1 - \beta \rho_\mu}, & b_{13} = \frac{\kappa(\gamma_\ell b_{33} + \gamma_r \sigma_\nu)}{1 - \beta \rho_\nu}, \\
b_{21} = b_{11} \rho_d + \phi_d, & b_{22} = b_{12} \rho_\mu + \phi_\mu, & b_{23} = b_{13} \rho_\nu + \sigma_\nu.
\end{array}
\tag{G.5}
$$

We can invert the mapping (G.5) to derive the structural parameters. As the model has eleven parameters (not including the three persistence parameters ρ) and the reduced form nine (the coefficients of the B matrix, again excluding the three persistence parameters ρ), we reach just identification by imposing the value of β and α_r. The parameter κ is normalized to one. The other parameters are derived from the structural estimation as follows:

$$
\begin{array}{ll}
\phi_d = b_{21} - b_{11} \rho_d, & \phi_\mu = b_{22} - b_{12} \rho_\mu, \\
\sigma_\nu = b_{23} - b_{13} \rho_\nu, & \alpha_\ell = \frac{b_{33} + \alpha_r \sigma_\nu}{b_{33} \rho_\nu}, \\
\alpha_\mu = (1 - \alpha_\ell \rho_\mu) b_{32} + \alpha_r \phi_\mu, & \sigma_d = (1 - \alpha_\ell \rho_d) b_{31} + \alpha_r \phi_d, \\
\gamma_r = \left(\sigma_\nu - \frac{b_{33} \phi_d}{b_{31}}\right)^{-1} \left(\Gamma' - \frac{b_{33}}{b_{31}} \Gamma\right), & \gamma_\ell = \frac{\Gamma - \phi_d \gamma_r}{b_{31}}, \\
\sigma_\mu = b_{12}(1 - \beta \rho_\mu) - \kappa(b_{32} \gamma_\ell + \phi_\mu \gamma_r),
\end{array}
\tag{G.6}
$$

with $\Gamma = \frac{1}{\kappa}(1 - \beta \rho_d) b_{11}$ and $\Gamma' = \frac{b_{13}}{\kappa}(1 - \beta \rho_\nu)$.

H   Summary of the Results for the 40 Models

Here we present robustness analysis for our estimated extended model. The model is given by the three equations

$$
\ell_t = \alpha_\ell E_t[\ell_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + \alpha_\mu \mu_t + d_t,
\tag{EE}
$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_\ell \ell_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC}$$

$$i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t, \tag{Policy}$$

and the law of motions of the three shocks

$$
\begin{aligned}
d_t &= \rho_d d_{t-1} + \varepsilon_{dt}, &\text{(H.7)}\\
\mu_t &= \rho_\mu d_{t-1} + \varepsilon_{\mu t}, &\text{(H.8)}\\
\nu_t &= \rho_\nu d_{t-1} + \varepsilon_{\nu t}. &\text{(H.9)}
\end{aligned}
$$

We have shown in appendix G that the solution if this model takes the following form:

$$
\begin{pmatrix} \pi_t \\ i_t \\ \ell_t \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{23} & b_{33} \end{pmatrix} \begin{pmatrix} d_t \\ \mu_t \\ \nu_t \end{pmatrix}. \tag{H.10}
$$

The estimation is done with the two-step procedure we have described in the main text. We first estimate the system (H.7), (H.8), (H.9) and (H.10) by Maximum Likelihood, where d, μ and ν are latent variables. We then use the mapping (G.6) to derive the estimated parameters.

In the following tables, we present the results for two measures of inflation (using GDP deflator or CPI) and two measures of employment (*minus* unemployment or linearly detrended Non-Farm Business Hours per capita). The parameter α_r, that relates to the intertemporal elasticity of substitution of consumptio, is nit estimated, and we set it to five different values: .1, 1/3, .5, .75 and 1. In total, this amounts to estimating 40 models.

Results are displayed in Tables 5 to 12. In all the 40 cases, the parameters configuration is in the Real Keynesian zone. All parameters have expected signs except in two configurations. When $\alpha_r = 1$ for the full sample, GDP deflator and unemployment, we find a negative α_ℓ, which makes the model uninterpretable. For CPI inflation and hours over the full sample, we find $\alpha_\ell > 1$ except for $\alpha_r = 1$.

[Table 5 about here.]

[Table 6 about here.]

[Table 7 about here.]

[Table 8 about here.]

[Table 9 about here.]

[Table 10 about here.]

[Table 11 about here.]

[Table 12 about here.]

I Estimating the SVAR Representation of a Structural Model

I.1 Setup

Assume that the Data Generating Process is an economic model (typically a DSGE) of the type

$$\begin{aligned} X_t &= M_1 X_{t-1} + M_2 E_t[X_{t+1}] + M_3 \Theta_t, \\ \Theta_t &= R\Theta_{t-1} + \varepsilon_t. \end{aligned} \tag{I.11}$$

X is a $n \times 1$ vector of endogenous variables and Θ is a $n \times 1$ vector of structural shocks. Those shocks are assumed to be autoregressive of order one.[39] The structural innovations ε are normally distributed, with zero mean and their covariance matrix is identity. Some substitutions might be needed to eliminate some variables in order to obtain an expression with as many shocks as variables. Matrices M_1, M_2, M_2 are functions of the model deep parameters P, and R is the matrix of the shocks process parameters. Those matrices may contain some zero elements. It is assumed that all the variables in X are observable, while the shocks Θ are not.[40] Assuming determinacy of the model solution[41], the model solution can be written as

$$\begin{aligned} X_t &= A X_{t-1} + B\Theta_t, \\ \Theta_t &= R\Theta_{t-1} + \varepsilon_t. \end{aligned} \tag{I.12}$$

where A and B are functions of M_1, M_2, M_2 and R, and therefore of P and R, according to the mapping

$$(A, B) = \Phi(P, R).$$

Eliminating Θ, the model solution (I.12) can be written as the following Structural VAR(2) process:

$$X_t = \left(A + BRB^{-1}\right) X_{t-1} - BRB^{-1}A X_{t-1} + B\varepsilon_t. \tag{I.13}$$

Estimating a VAR(2) on the data, one can obtain the non structural VAR representation:

$$X_t = \Gamma_1 X_{t-1} + \Gamma_2 X_{t-1} + \nu_t. \tag{I.14}$$

where ν is a vector of innovations with covariance matrix Ω. The representation (I.14) is referred to as the non structural VAR. Matrices Γ_1, Γ_2 and Ω are functions of A, B and R according to the mapping

$$(\Gamma_1, \Gamma_2, \Omega) = \Psi(A, B, R).$$

I.2 Identification of Shocks and Parameters Estimation

Identification of the theoretical model (I.11) can be understood as involving two steps. The first one is the possibility of inverting the mapping Ψ in order to find the SVAR from the non structural VAR. This step is the one performed in the Structural VAR literature.

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[39]For expositional simplicity, we present only the case of a model with one lead and one lag and an order one process for shocks. The general idea of MS-VARs can be extended to higher order models.

[40] If the shocks are observable, then the identification of shocks problem is trivially solved.

[41]We haven't worked out the case of sunspots, but we think that the same logic than the one presented here goes through, as long as one keeps the same number of variables than the number of shocks.

If one can recover the SVAR representation –*i.e.* if the mapping Ψ is invertible, one can compute the theoretical impulse responses functions of the structural model, the variance decomposition, compute conditional correlations, etc... At this stage, the parameters R, that describe the exogenous shocks process, are identified, but it is not necessary to know the model deep parameters P. If the SVAR can be identified, one can then go a step further and identify the model parameters, provided that the mapping Φ is invertible.

The estimation of DSGEs, as exemplified by Smets-Wouters, can be understood as directly finding the mapping between non structural VAR $(\Gamma_1, \Gamma_2, \Omega)$ and the parameters (P, R), as $(\Gamma_1, \Gamma_2, \Omega)$ summarize all the information needed to compute the model likelihood.

As it requires less restrictions on the data, we think that it is of interest to highlight the hidden step in traditional DSGE estimation, which is the estimation of the Structural VAR (I.12) or (I.13).

I.3 Why is the SVAR identified?

Identifying the SVAR (I.13) means knowing matrices A, B and R. Absent of any restrictions, each matrix has n^2 elements, so that we have $3n^3$ unknown coefficients. The available information is the non structural VAR is given by $(\Gamma_1, \Gamma_2, \Omega)$. The system of equations that determines the elements of A, B and R is, using (I.13):

$$\begin{cases} \Gamma_1 &= A + BRB^{-1}, \\ \Gamma_2 &= -BRB^{-1}A, \\ \Omega &= BB'. \end{cases} \tag{I.15}$$

Because Ω is a symmetrical matrix, this system gives us $3n^2 - \frac{n(n-1)}{2}$ independent equations for $3n^2$ unknowns. This is the well-known problem of the identification of shocks in SVARs. If one adds some extra identifying assumptions (at least $\frac{n(n-1)}{2}$), then A, B and R can be identified.[42] Sims [1980] restriction than B is lower triangular brings exactly $\frac{n(n-1)}{2}$ restrictions, so that the VAR is just identified. But if one recalls that B is a complicated function of the M_1, M_2, M_2 and R, a lower triangular B matrix is quite a non generic configuration.

The Structural VAR approach we propose only uses the assumption that the shocks Θ are independent one from each other at all leads and lags – *i.e.* that R is a diagonal matrix. This assumption is an assumption about the theoretical Data Generating Process (the DSGE model), and is shared by the vast majority of the literature.[43] In plain words, the SVAR identifying assumption is that all the models shocks are independent AR(1) processes. This is enough to be able to identify the SVAR. In effect, R has now only n elements, so that we have $2n^2 + n$ unknowns and therefore $\frac{n(n-1)}{2}$ more restrictions than unknowns.[44] The SVAR is therefore generically identified.

The only disadvantage of this SVAR approach compared to the SVAR literature is that one cannot use OLS to estimate the SVAR. Under the form (I.12), the Θ are not observable, and the

---

[42]We follow here the literature by counting the number of parameters and equations to assess the possibility of identification. Of course, this is only a necessary condition for identification of the shocks.

[43]Exceptions are International Real Business Cycles models of the BKK type in which TFP are correlated with a lag between countries. Of course, this can also be accommodated in a SVAR, as long as enough zeros (at least $\frac{n(n-1)}{2}$) are imposed in the matrix R

[44]In the case $n = 1$, the SVAR is just identified. More precisely, a necessary condition for just identification is satisfied, but the case $n = 1$ is a well-known example of non identification although we have as many equations than parameters. Equation (I.12) writes $x_t = (a + r)x_{t-1} - arx_{t-2} + b\varepsilon_t$, and has to be compared to the non structural VAR (I.14), that is given by $x_t = \gamma_1 x_{t-1} + \gamma_2 x_{t-2} + \nu_t$. The endogenous and exogenous persistence parameters a and r enter symmetrically in the AR(2) representation, so that if $(a, r) = (\bar{a}, \frac{\gamma_2}{\bar{a}})$ is a solution, then $(a, r) = (\frac{\gamma_2}{\bar{a}}, \bar{a})$ is also a solution.

form (I.13) is non linear in the coefficients of A, B and R. One needs to use a maximum likelihood method, and we will do it by keeping the SVAR under the form

$$
\begin{aligned}
X_t &= AX_{t-1} + B\Theta_t, \\
\Theta_t &= R\Theta_{t-1} + \varepsilon_t.
\end{aligned}
\qquad (I.12)
$$

Such an estimation can be easily done, for example using the maximum likelihood model estimation routine of DYNARE. Note that under the null that model (I.11) is the Data Generating Process, the structural shocks will be recovered up to a sign normalization.

Figures

Figure 1: The New Keynesian Configuration
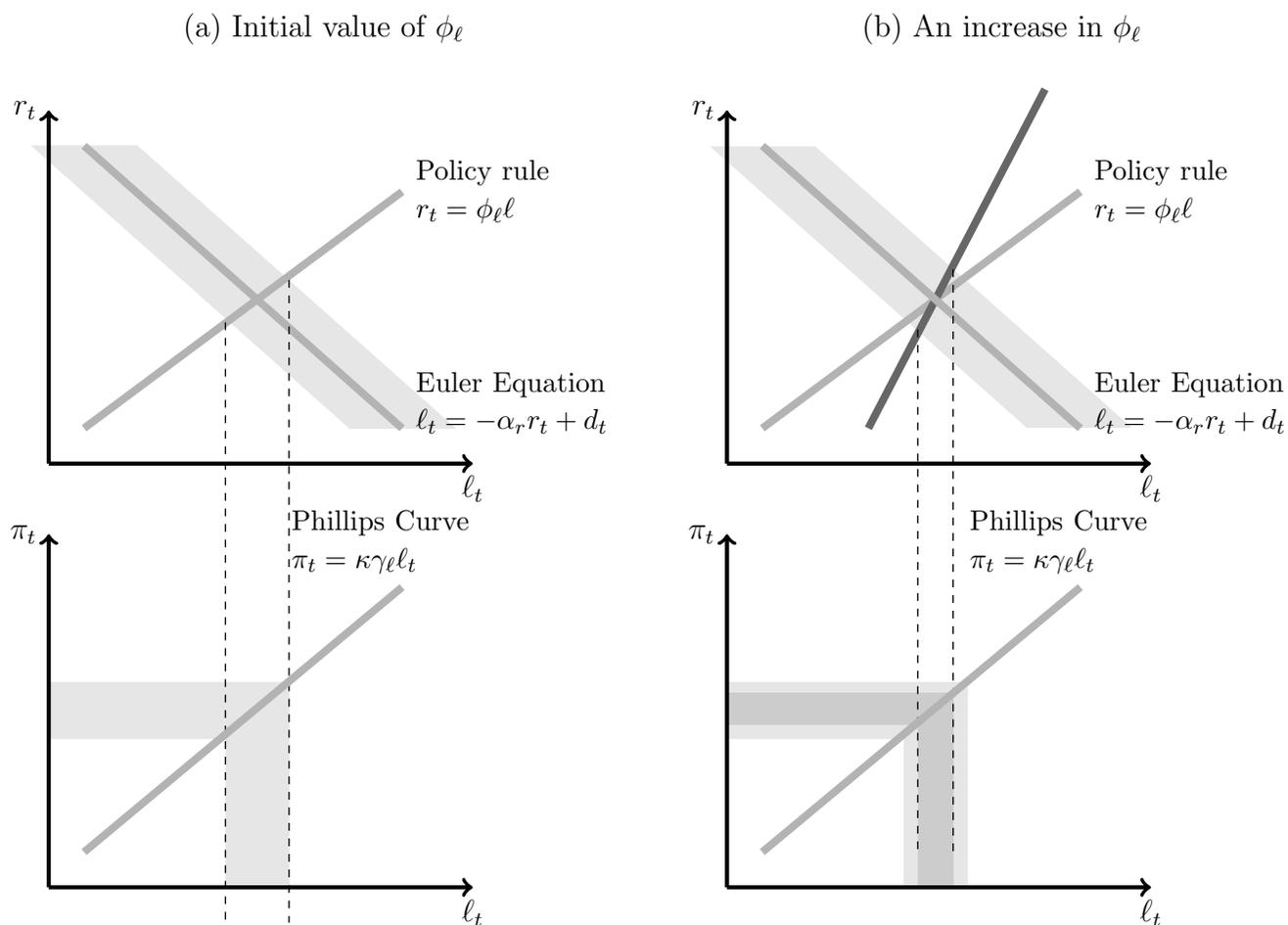
(a) Initial value of ϕ_ℓ (b) An increase in ϕ_ℓ

Policy rule
$r_t = \phi_\ell \ell$

Euler Equation
$\ell_t = -\alpha_r r_t + d_t$

Phillips Curve
$\pi_t = \kappa \gamma_\ell \ell_t$

Notes: The light gray zone corresponds to the range of fluctuations caused by the demand shocks for the initial value for ϕ_ℓ, while the dark gray zone corresponds to fluctuations with an increased ϕ_ℓ. Light grey lines correspond to initial ϕ_ℓ and dark grey ones to increased ϕ_ℓ.

Figure 2: The Extended Model in the non-Real Keynesian Configuration

(a) Initial value of ϕ_ℓ

(b) An increase in ϕ_ℓ

60

Figure 3: The Extended Model in the Real Keynesian Configuration

(a) Initial value of ϕ_ℓ

(b) An increase in ϕ_ℓ

Policy rule
$r_t = \phi_\ell \ell$

Euler Equation
$\ell_t = -\alpha_r r_t + d_t$

Pseudo Phillips Curve
$\pi_t = \kappa(\gamma_\ell + \gamma_r \phi_\ell)\ell_t$
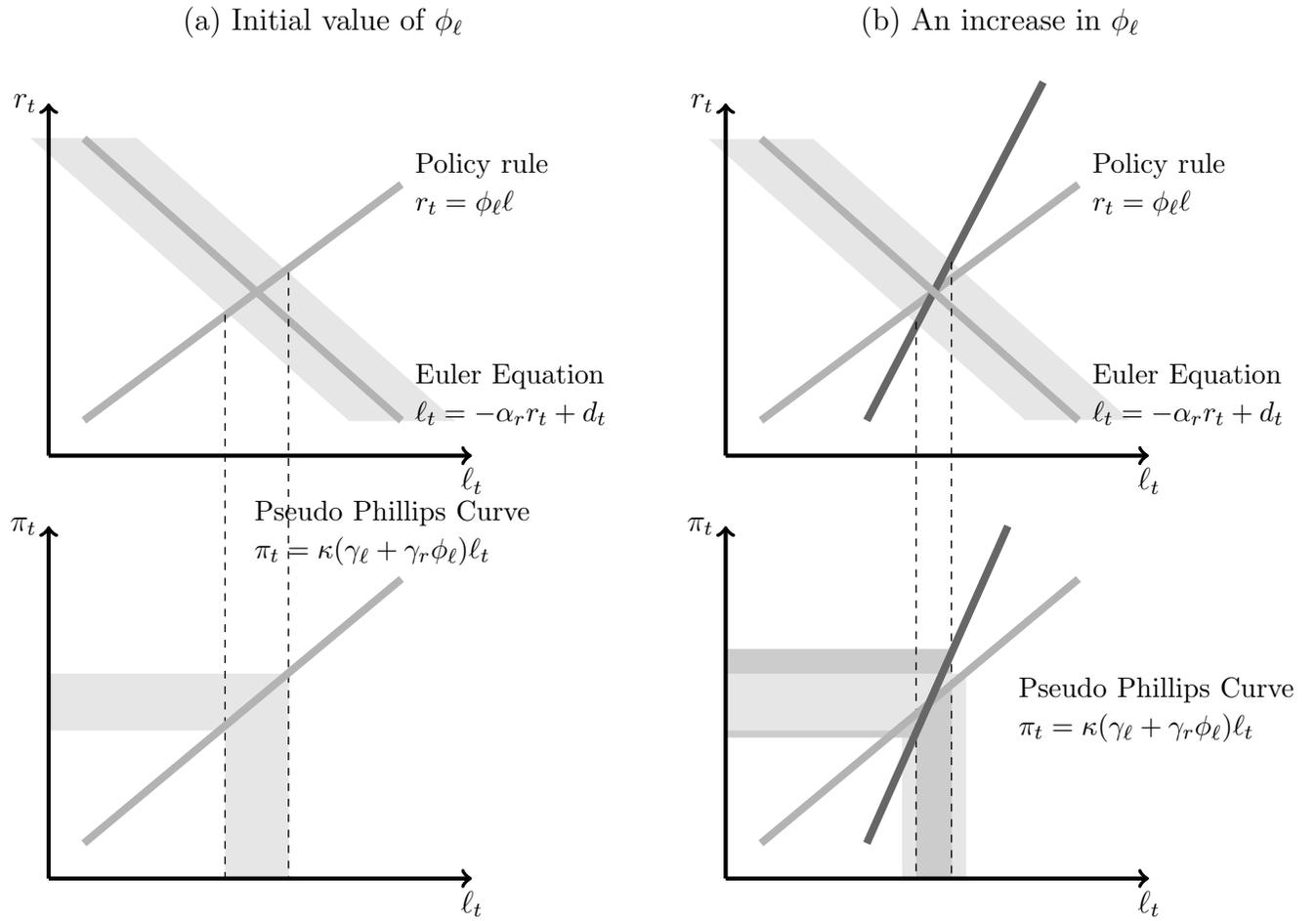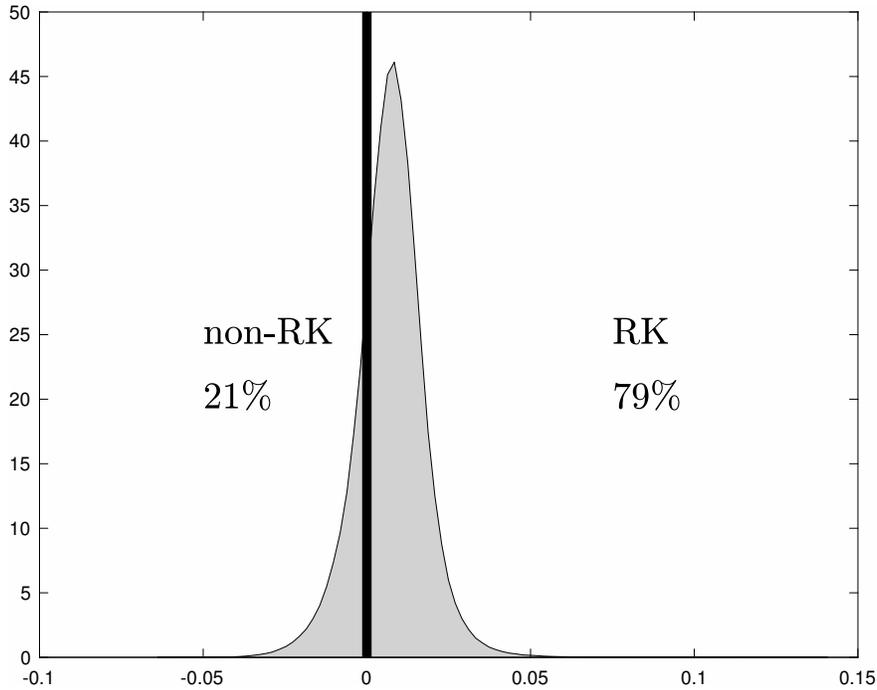
Notes: The light gray zone corresponds to the range of fluctuations caused by the demand shocks for the initial value for ϕ_ℓ, while the dark gray zone corresponds to fluctuations with an increased ϕ_ℓ. Light grey lines correspond to initial ϕ_ℓ and dark grey ones to increased ϕ_ℓ.

61

Figure 4: Distribution of $(1 - \alpha_\ell)\gamma_r - \gamma_\ell\alpha_r$

Notes: This Figure plots the distribution of the quantity $(1 - \alpha_\ell)\gamma_r - \gamma_\ell\alpha_r$ as implied by the parameters given in Table 1 (benchmark enstimation). The distribution is based on the one million draws of the parameters in a multivariate normal distribution with estimated mean and covariance matrix, and where the negative draws of α_ℓ have been trimmed off.

Figure 5: SVAR Impulse Responses for the Fully Forward Model, Long Sample
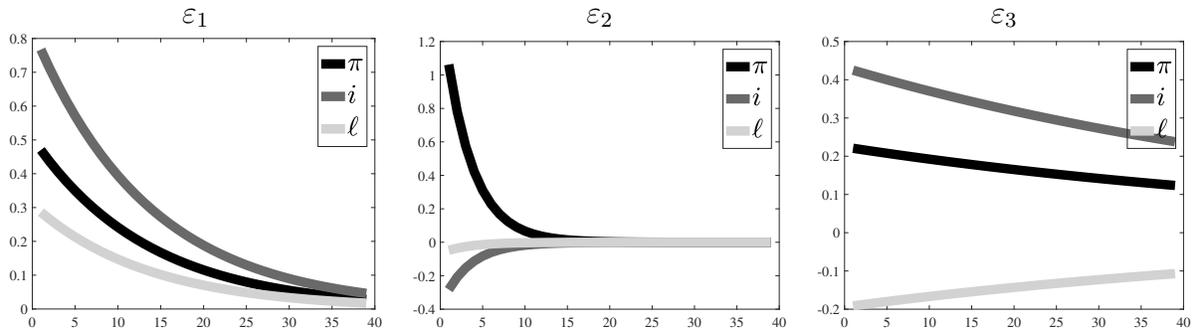
Notes: These impulse responses are obtained from the estimation of the SVAR (4) and (5) by Maximum Likelihood, over the sample 1954Q3–2007Q1. The matrix A is constrained to be zero.

Figure 6: Impulse Responses for the Fully Forward Model, Long Sample, ML Estimation of the Model
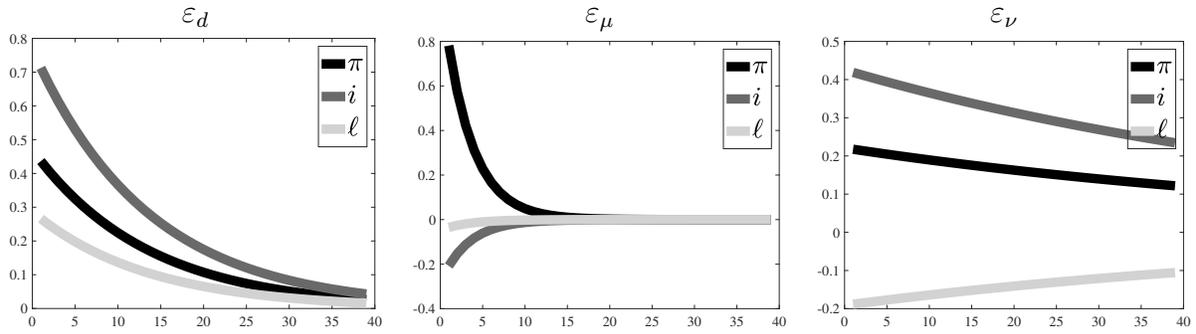
Notes: These impulse responses are obtained by solving the model (EE), (PC) and (Policy) with the parameter values obtained from the Maximum Likelihood estimation of (EE), (PC) and (Policy) over the sample 1954Q3–2007Q1.

Figure 7: SVAR Impulse Responses for the Fully Forward Model, Three Sub-Samples

First Sub-sample: `1954Q3–1979Q1`

ε_1

ε_2

ε_3

Second Sub-sample: `1983Q1–2007Q1`

ε_1

ε_2

ε_3

Third Sub-sample: `2009Q1–2015Q4`

ε_1

ε_2

Notes: These impulse responses are obtained from the estimation of the SVAR ([4](#)) and ([5](#)) by Maximum Likelihood, separately for the three sub-samples. The matrix A is constrained to be zero.

Figure 8: Full Sample,"Habit Persistence"
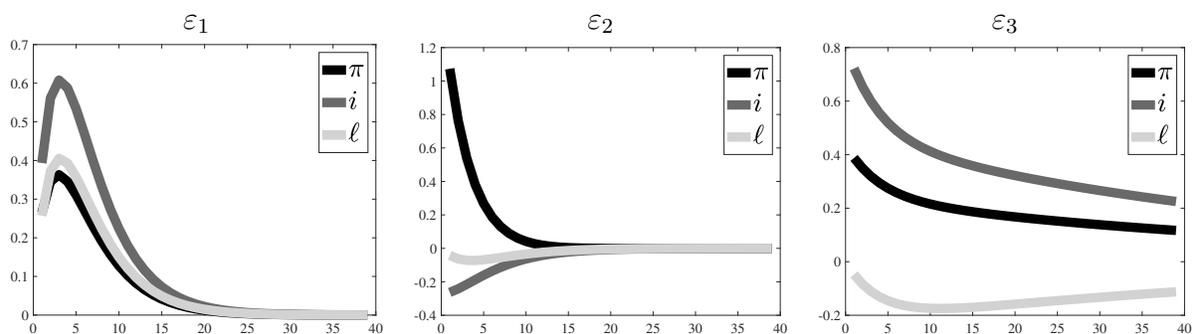
Figure 9: SVAR Impulse Responses for the Model with Habit Persistence, Three Sub-Samples

First Sub-sample: `1954Q3-1979Q1`

Second Sub-sample: `1983Q1-2007Q1`

Third Sub-sample: `2009Q1-2015Q4`

Notes: These impulse responses are obtained from the estimation of the SVAR (4) and (5) by Maximum Likelihood, separately for the three sub-samples. The matrix A is constrained to have the two first columns filled with zeros.

Figure 10: SVAR Impulse Responses When Allowing for More Propagation, Full sample

Habit persistence and "Hybrid New Phillips Curve"
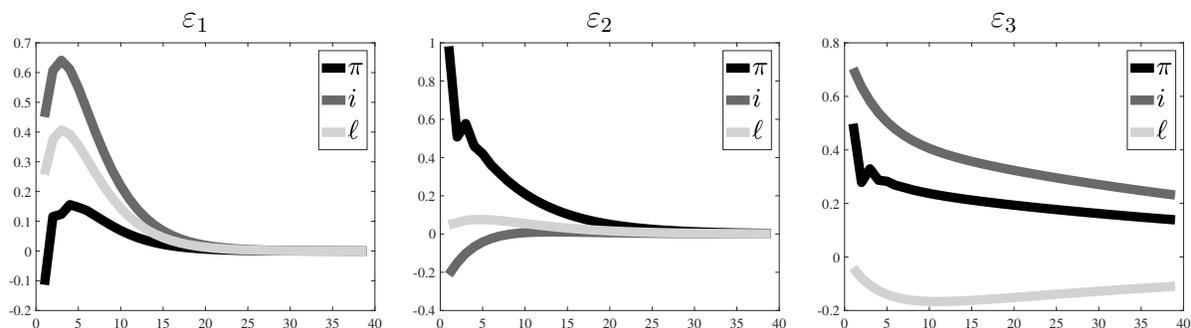
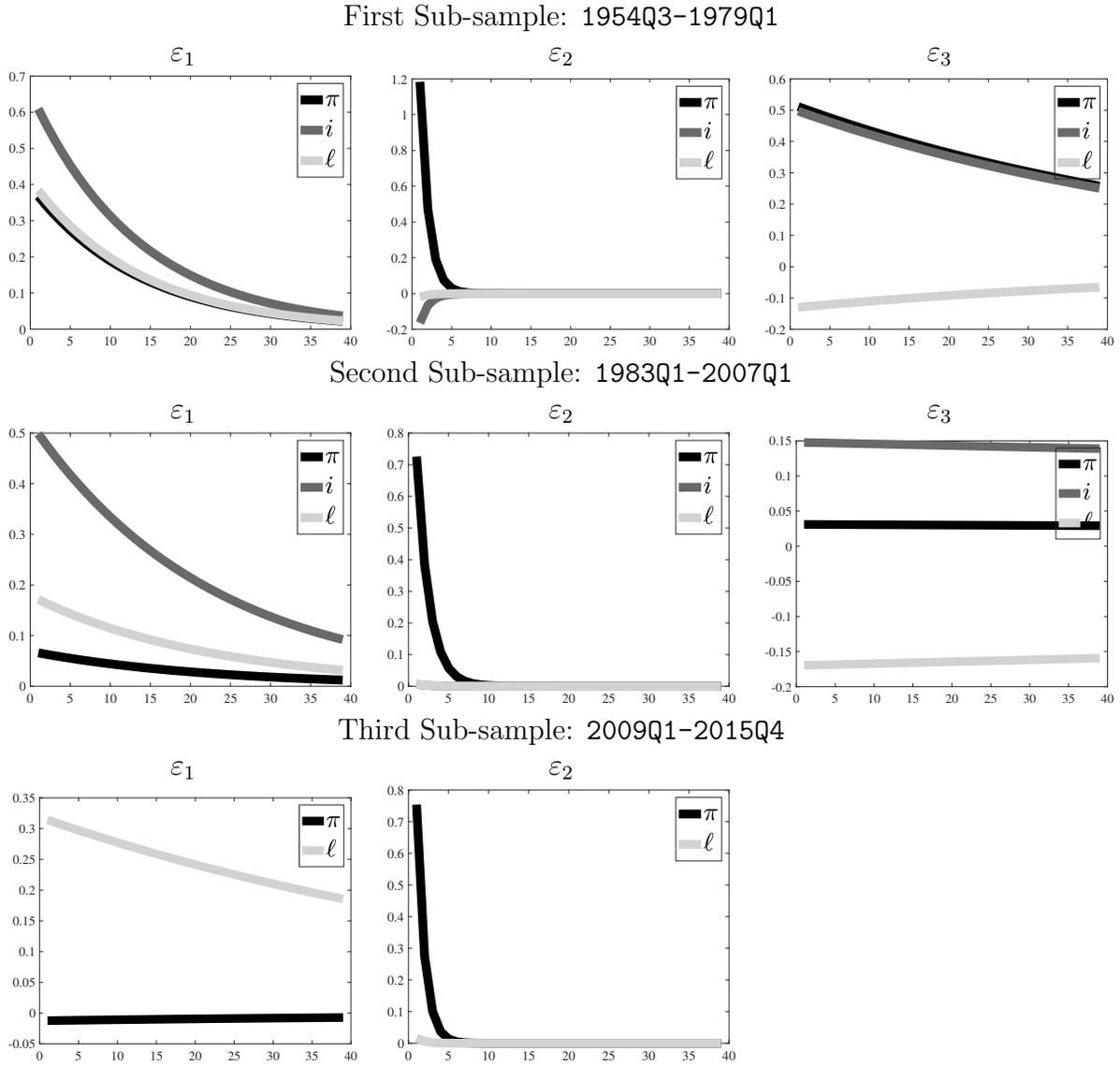Habit persistence, Gradual Adjustment of i and "Hybrid New Phillips Curve"

Notes: These impulse responses are obtained from the estimation of the SVAR (4) and (5) by Maximum Likelihood. In the first row, the matrix A is constrained to have the second column filled with zeros. In the second row, A is unrestricted.

Figure 11: SVAR Impulse Responses of the Price of Oil (p_o)
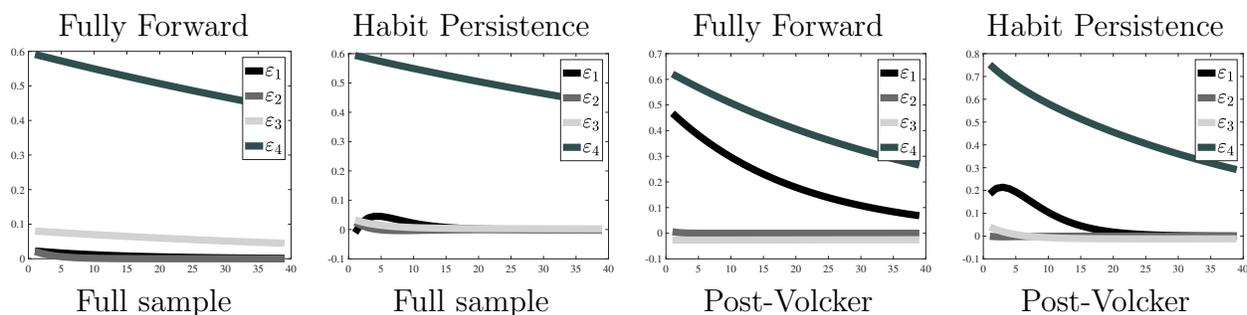
Notes: These impulse responses are obtained from the estimation of the SVAR (4) and (5) by Maximum Likelihood, adding the real oil price as the fourth variable. With habit persistence, the matrix A is constrained to have the first, second and fourth columns filled with zeros. In the fully forward case, A is constrained to be zero.

Figure 12: SVAR Impulse Responses with the Relative Price of Oil (p_o) as the Fourth Variable

Fully Forward Model, Full Sample

Habit Persistence Model, Full sample
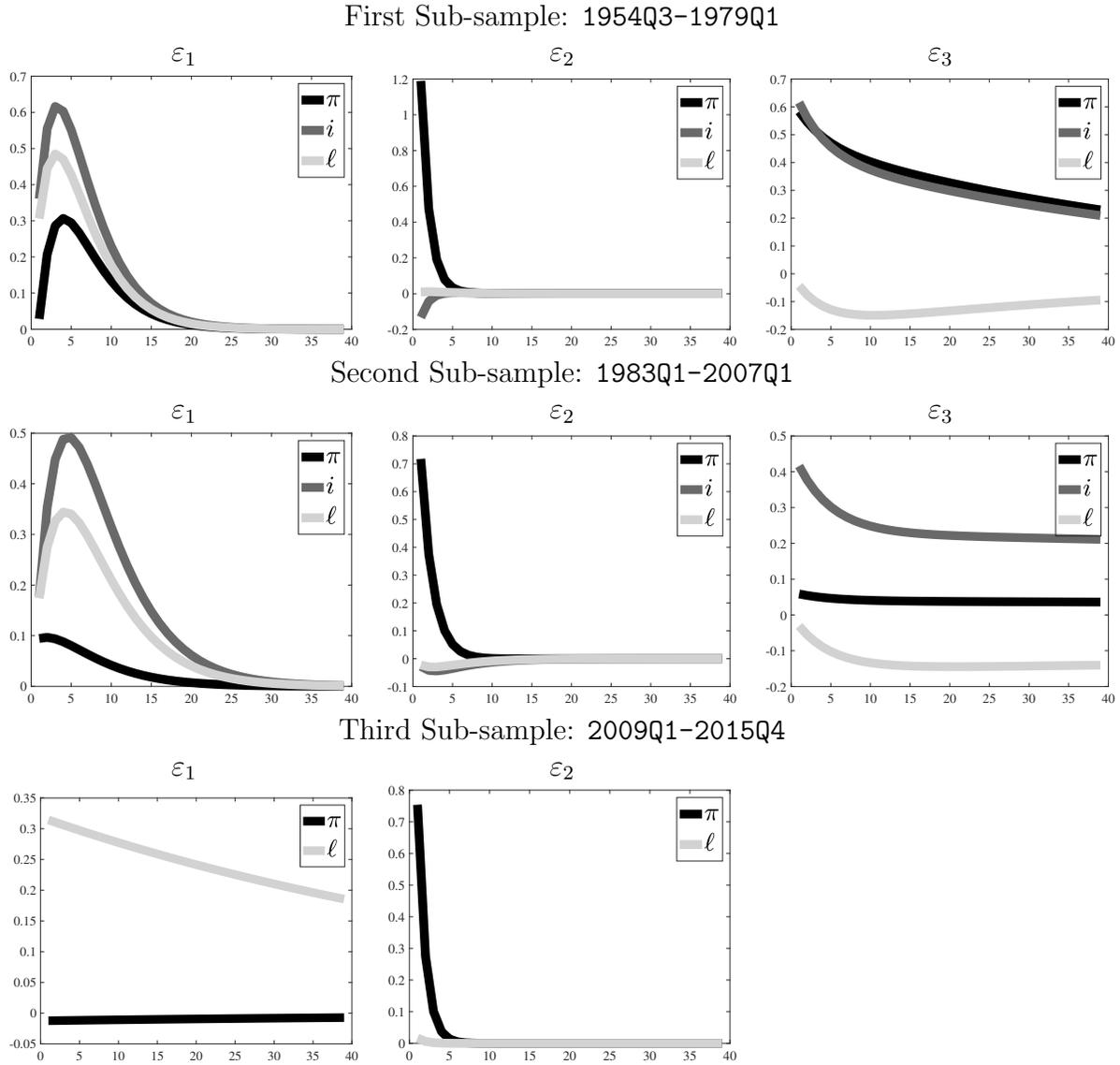

"Fully Forward", Post-Volcker

"Habit persistence", Post-Volcker

Notes: These impulse responses are obtained from the estimation of the SVAR (4) and (5) by Maximum Likelihood, adding the real oil price as the fourth variable. With habit persistence, the matrix A is constrained to have the first, second and fourth columns filled with zeros. In the fully forward case, A is constrained to be zero.

Figure 13: SVAR Impulse Responses of the Real growth (Δy)
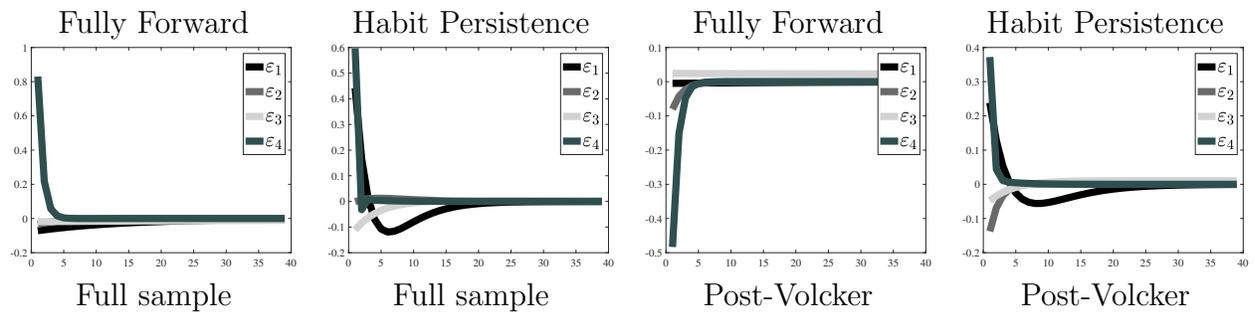
Notes: These impulse responses are obtained from the estimation of the SVAR (4) and (5) by Maximum Likelihood, adding the real GDP growth rate as the fourth variable. With habit persistence, the matrix A is constrained to have the first, second and fourth columns filled with zeros. In the fully forward case, A is constrained to be zero.

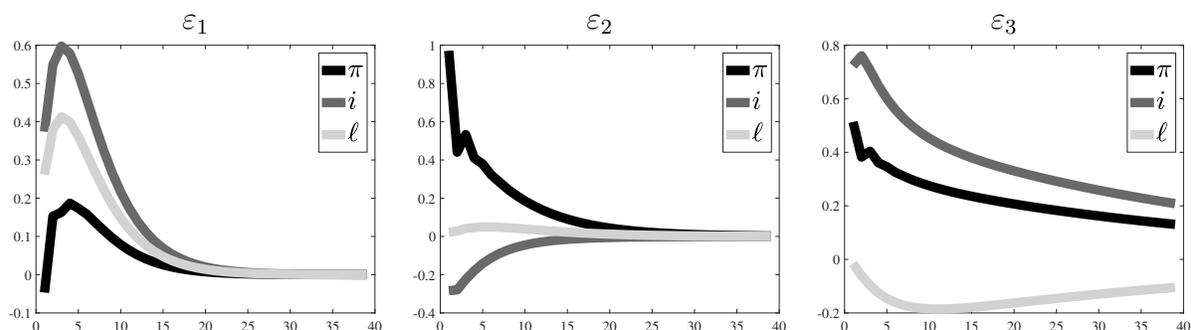Figure 14: SVAR Impulse Responses with the Real growth (Δy) as the Fourth Variable
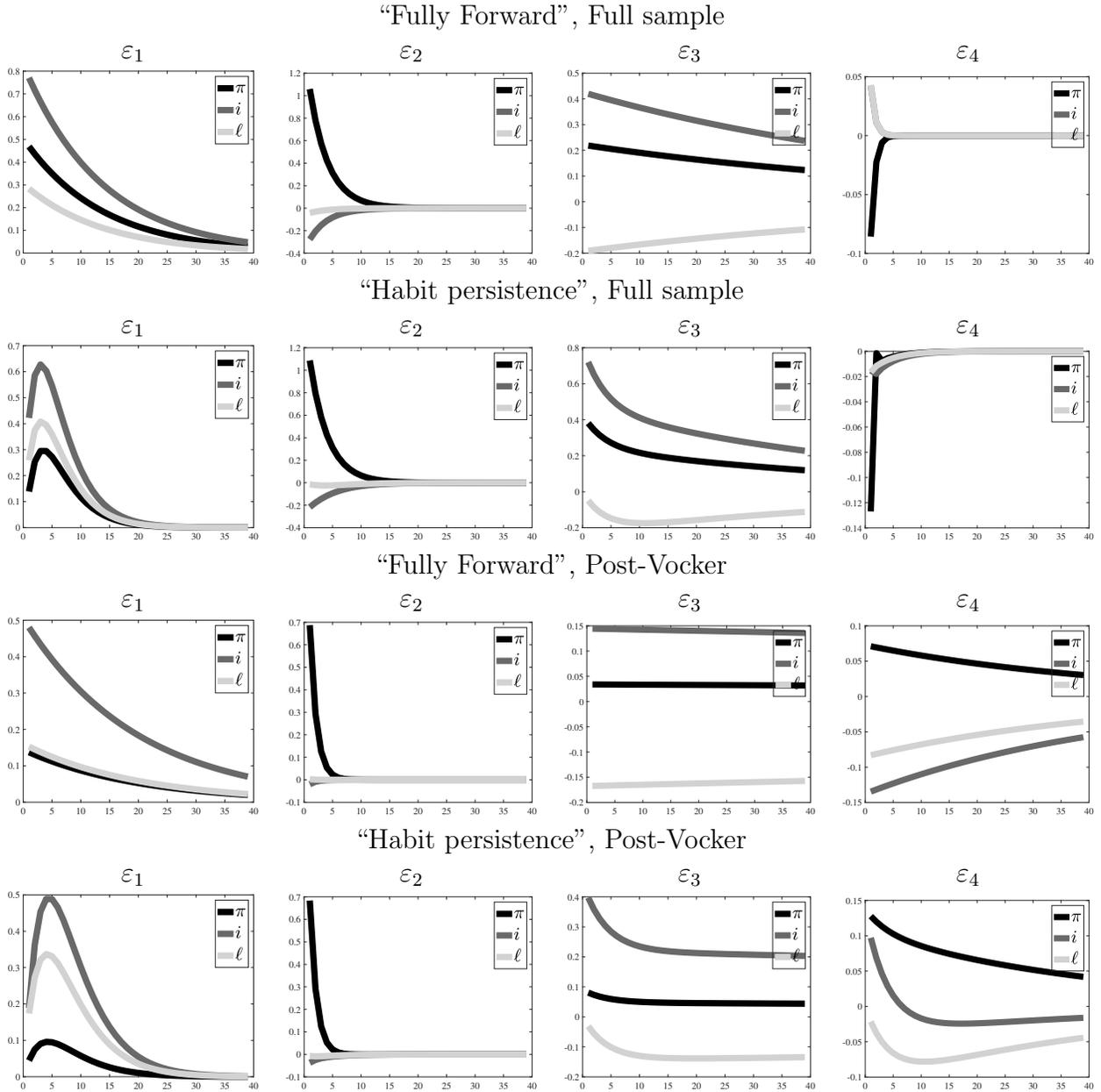
Notes: These impulse responses are obtained from the estimation of the SVAR (4) and (5) by Maximum Likelihood, adding the real GDP growth rate as the fourth variable. With habit persistence, the matrix A is constrained to have the first, second and fourth columns filled with zeros. In the fully forward case, A is constrained to be zero.

Figure 15: SVAR Impulse Responses with Order Two endogenous Dynamics

Habit Persistence, Full sample

Habit Persistence, Post-Vocker

Habit persistence, gradual adjustment of i and Hybrid New Phillips curve
Full sample

Habit persistence, gradual adjustment of i and Hybrid New Phillips curve
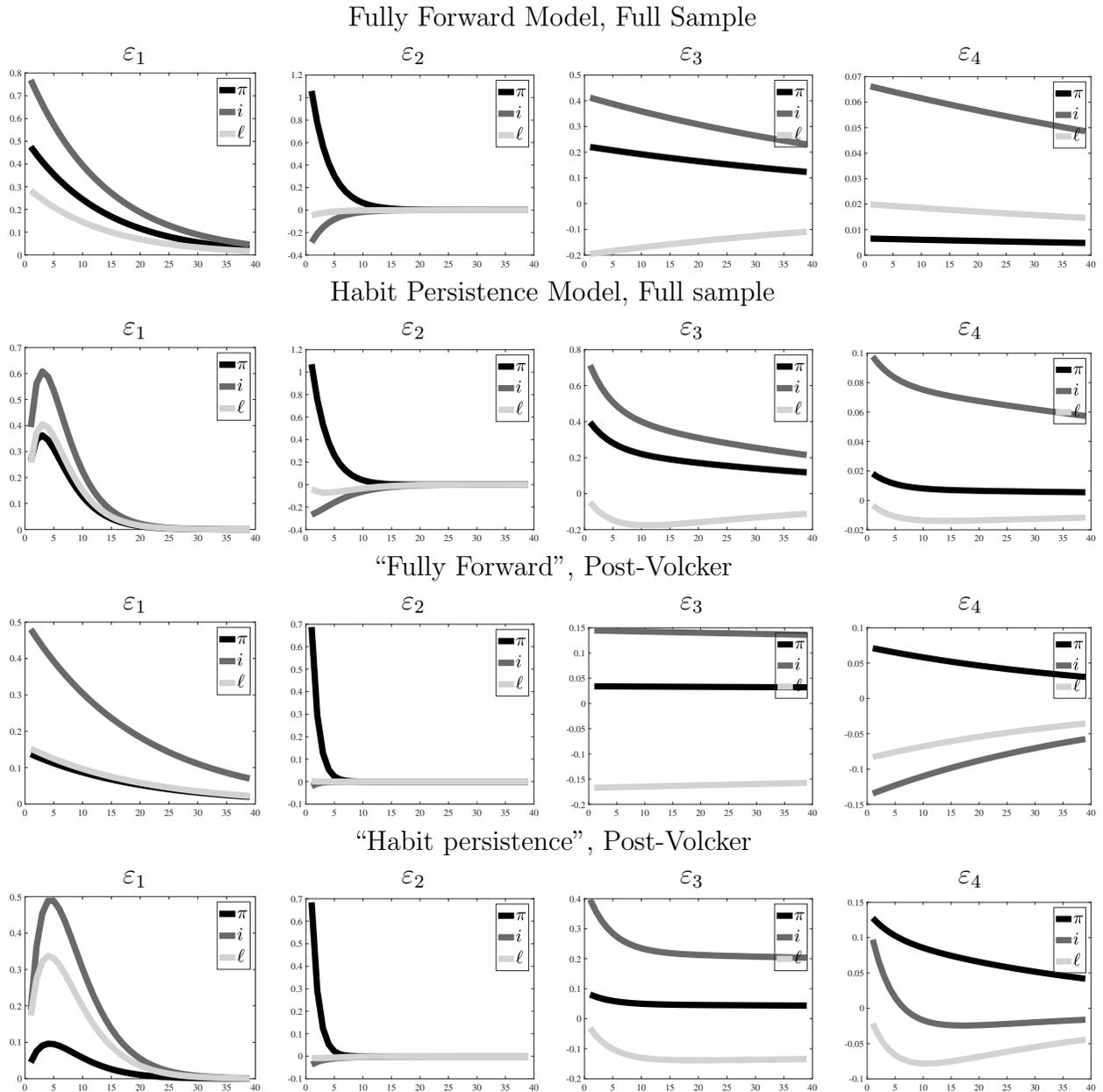Post-Vocker
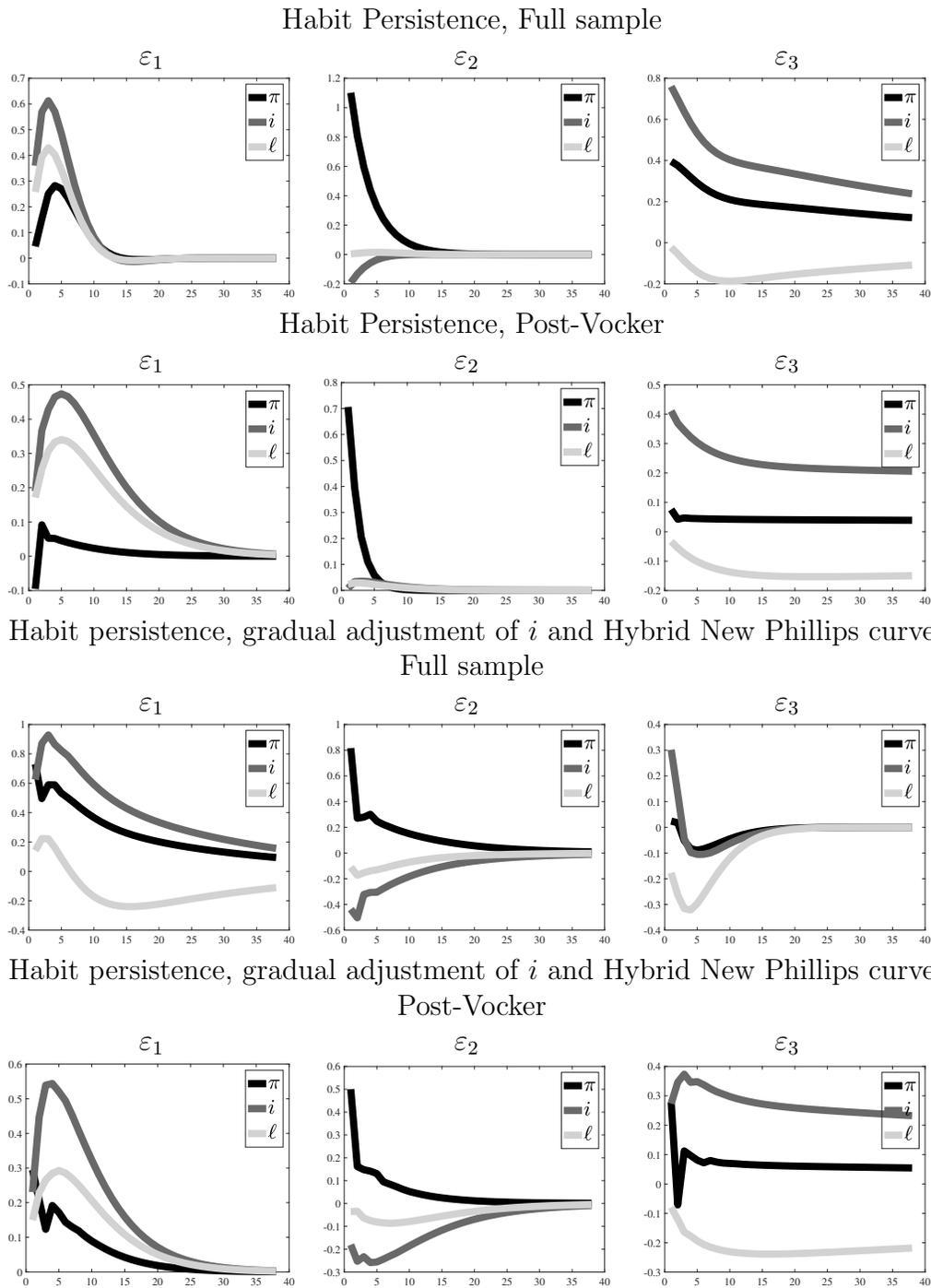
Notes: These impulse responses are obtained from the estimation of the SVAR (6) and (7) by Maximum Likelihood. With habit persistence, the matrix A is constrained to have the first and second columns filled with zeros. In the fully forward case, A is constrained to be zero.

Figure 16: Demand Shocks and Missing Inflation at the ZLB

(a) New Keynesian Configuration

(b) Real Keynesian Configuration
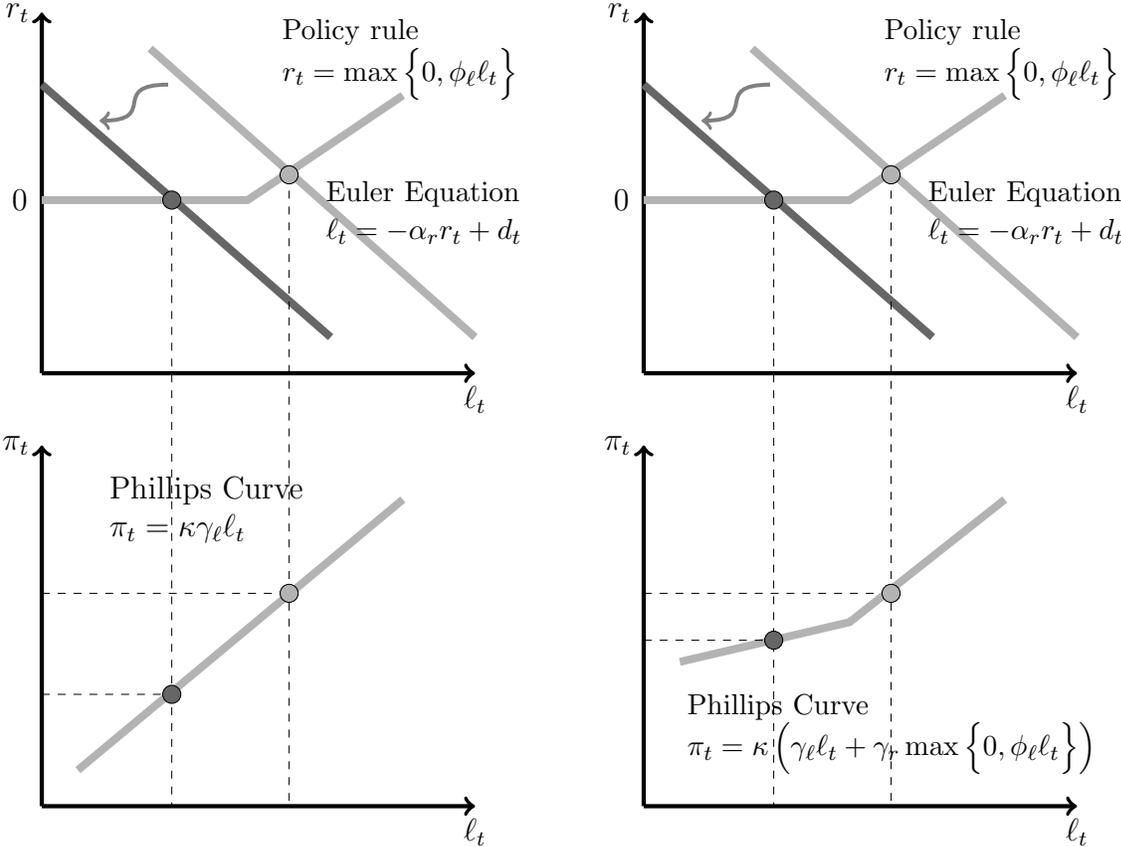
Notes: The light gray Euler equation corresponds to the initial situation, and the dark gray one to a situation with a negative demand. For the same drop in ℓ, a negative demand shock causes a smaller drop in π in the Real Keynesian configuration (right column), as compared to the New Keynesian configuration) if the ZLB binds.

Tables

Table 1: Estimated Parameters, Baseline Case

Not Estimated		Estimated	
α_r	0.33	ϕ_d	0.33 (.15)
β	0.99	ϕ_μ	-1.06 (.09)
κ	1.00	ρ_d	0.93 (.02)
Estimated		ρ_μ	0.74 (.05)
α_ℓ	0.65 (.21)	ρ_ν	0.98 (.01)
γ_ℓ	0.05 (.04)	σ_μ	0.37 (.06)
γ_r	0.07 (.04)	σ_d	0.22 (.08)
α_μ	-0.38 (.04)	σ_ν	0.21 (.08))

Notes: This table shows the estimated coefficients of equations (EE), (PC) and (Policy) over the sample 1954Q3-2007Q1 and using Maximum Likelihood. Standard deviations are shown between parenthesis.

Table 2: Joint Estimation Over The Three Sub-samples

Parameter	1954Q3-1979Q1	1983Q4-2007Q1	2009Q1-2016Q3
$\alpha_r^\star$	0.33	0.33	0.33
$\beta^\star$	0.98	0.98	0.98
α_ℓ	0.83	0.83	0.83
γ_ℓ	-0.04	-0.04	-0.04
γ_r	0.20	0.20	0.20
κ	1.00*	0.04	0.04
α_μ	-0.23	-0.12	-0.08
ϕ_d	0.25	0.45	–
ϕ_μ	-0.64	-0.38	–
ρ_d	0.93	0.96	0.99
ρ_μ	0.40	0.53	0.37
ρ_ν	0.98	0.99	–
σ_d	0.17	0.19	0.06
σ_μ	0.84	0.35	0.49
σ_ν	0.07	0.09	–

Notes: This table shows the estimated coefficients of equations (EE), (PC) and (Policy) pooled over the the subsamples, using two-step estimation method.

Table 3: Standard-Deviations of Employment ℓ, Inflation π and Nominal Interest rate i over the Three Sub-samples

	σ_ℓ	σ_π	σ_i
1954Q3–1979Q1 :	1.3%	2.6%	2.5%
1983Q4–2007Q1 :	1.3%	.9%	2.5%
2009Q1–2016Q3 :	1.7%	.8%	.1%

Notes: Employment is measured as minus unemployment, inflation is GDP deflator growth and nominal interest rate is the Effective Federal Funds Rate. All the variables are in percentage points, annualized for inflation and interest rate.

Table 4: Estimated Parameters, Model with Habit Persistence

Not Estimated		**Estimated**	
α_r	0.33	ϕ_d	0.08
β	0.99	ϕ_μ	-.40
κ	1.00	$\phi_{\ell_{-1}}$	1.13
Estimated		ρ_d	0.79
α_ℓ	-1.6	ρ_μ	0.53
$\alpha_{\ell_{-1}}$	2.18	ρ_ν	0.99
γ_ℓ	-0.008	σ_μ	0.36
γ_r	0.014	σ_d	0.64
$\gamma_{\ell_{-1}}$	0.019	σ_ν	0.36
α_μ	-0.2	–	–

Notes: This table shows the estimated coefficients of equations (EE'), (PC') and (Policy') over the sample 1954Q3–2007Q1, using two-step estimation method.

Table 5: GDP Deflator and *minus* Unemployment, Full Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	0.905	0.648	0.465	0.189	-0.086
γ_ℓ	0.049	0.049	0.049	0.049	0.049
γ_r	0.072	0.072	0.072	0.072	0.072
ϕ_d	0.331	0.331	0.331	0.331	0.331
ϕ_μ	-1.064	-1.064	-1.064	-1.064	-1.064
α_μ	-0.122	-0.380	-0.563	-0.839	-1.115
σ_d	0.079	0.224	0.328	0.484	0.640
σ_μ	0.368	0.368	0.368	0.368	0.368
σ_ν	0.208	0.208	0.208	0.208	0.208
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.

Table 6: GDP Deflator and *minus* Unemployment, Post-Volcker Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	0.933	0.772	0.657	0.485	0.312
γ_ℓ	0.003	0.003	0.003	0.003	0.003
γ_r	0.007	0.007	0.007	0.007	0.007
ϕ_d	0.436	0.436	0.436	0.436	0.436
ϕ_μ	-0.379	-0.379	-0.379	-0.379	-0.379
α_μ	-0.035	-0.123	-0.186	-0.280	-0.374
σ_d	0.062	0.190	0.281	0.419	0.556
σ_μ	0.347	0.347	0.347	0.347	0.347
σ_ν	0.117	0.117	0.117	0.117	0.117
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.

Table 7: CPI Deflator and *minus* Unemployment, Full Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	0.936	0.750	0.750	0.418	0.219
γ_ℓ	0.078	0.078	0.078	0.078	0.078
γ_r	0.144	0.144	0.144	0.144	0.144
ϕ_d	0.225	0.225	0.225	0.225	0.225
ϕ_μ	-1.134	-1.134	-1.134	-1.134	-1.134
α_μ	-0.116	-0.382	-0.382	-0.856	-1.141
σ_d	0.061	0.163	0.163	0.345	0.454
σ_μ	0.679	0.679	0.679	0.679	0.679
σ_ν	0.154	0.154	0.154	0.154	0.154
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.

Table 8: GDP Deflator and *minus* Unemployment, Post-Volcker Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	0.927	0.754	0.631	0.445	0.260
γ_ℓ	0.009	0.009	0.009	0.009	0.009
γ_r	0.015	0.015	0.015	0.015	0.015
ϕ_d	0.371	0.371	0.371	0.371	0.371
ϕ_μ	-0.250	-0.250	-0.250	-0.250	-0.250
α_μ	-0.020	-0.079	-0.120	-0.182	-0.245
σ_d	0.056	0.171	0.253	0.376	0.499
σ_μ	1.165	1.165	1.165	1.165	1.165
σ_ν	0.125	0.125	0.125	0.125	0.125
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.

Table 9: GDP Deflator and Total Hours, Full Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	1.005	0.983	0.967	0.943	0.919
γ_ℓ	-0.007	-0.007	-0.007	-0.007	-0.007
γ_r	0.051	0.051	0.051	0.051	0.051
ϕ_d	0.483	0.483	0.483	0.483	0.483
ϕ_μ	-0.995	-0.995	-0.995	-0.995	-0.995
α_μ	-0.094	-0.326	-0.491	-0.740	-0.988
σ_d	0.073	0.199	0.289	0.424	0.559
σ_μ	0.370	0.370	0.370	0.370	0.370
σ_ν	0.052	0.052	0.052	0.052	0.052
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.

Table 10: GDP Deflator and Total Hours, Post-Volcker Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	0.984	0.937	0.905	0.855	0.806
γ_ℓ	-0.001	-0.001	-0.001	-0.001	-0.001
γ_r	0.006	0.006	0.006	0.006	0.006
ϕ_d	0.448	0.448	0.448	0.448	0.448
ϕ_μ	-0.325	-0.325	-0.325	-0.325	-0.325
α_μ	0.009	-0.066	-0.119	-0.198	-0.278
σ_d	0.067	0.192	0.282	0.416	0.550
σ_μ	0.383	0.383	0.383	0.383	0.383
σ_ν	0.087	0.087	0.087	0.087	0.087
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.

Table 11: CPI Deflator and Total Hours, Full Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	1.013	1.009	1.005	1.000	0.996
γ_ℓ	-0.010	-0.010	-0.010	-0.010	-0.010
γ_r	0.103	0.103	0.103	0.103	0.103
ϕ_d	0.357	0.357	0.357	0.357	0.357
ϕ_μ	-1.088	-1.088	-1.088	-1.088	-1.088
α_μ	-0.079	-0.333	-0.514	-0.786	-1.058
σ_d	0.056	0.141	0.203	0.295	0.387
σ_μ	0.653	0.653	0.653	0.653	0.653
σ_ν	0.011	0.011	0.011	0.011	0.011
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.

Table 12: CPI Deflator and Total Hours, Post-Volcker Sample

$\alpha_r^\star$	0.100	0.333	0.500	0.750	1.000
α_ℓ	0.975	0.909	0.862	0.791	0.720
γ_ℓ	0.002	0.002	0.002	0.002	0.002
γ_r	0.010	0.010	0.010	0.010	0.010
ϕ_d	0.386	0.386	0.386	0.386	0.386
ϕ_μ	-0.243	-0.243	-0.243	-0.243	-0.243
α_μ	0.033	-0.023	-0.063	-0.123	-0.183
σ_d	0.065	0.185	0.271	0.399	0.528
σ_μ	1.171	1.171	1.171	1.171	1.171
σ_ν	0.125	0.125	0.125	0.125	0.125
RK	yes	yes	yes	yes	yes

Notes: This table shows the estimated coefficients obtained from estimating system (H.1), (H.2), (H.3) and (H.4) by Maximum Likelihood, and then using the mapping (G.6) to derive the structural parameters. In all these estimations, β is set to .99 and κ is normalised to 1. the star for α_r indicates that this coefficient is not estimated.